A STUDY OF FORCE IN A STEADY MAGNETIC FIELD WITH ASYMPTOTIC BOUNDARY CONDITIONS

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Abstract: In this paper there is performed a comparative study of the force that acts over a mobile coil pierced by current when it is placed in an external steady magnetic field. Firstly the determination of force is performed through analytic calculations and by experiments over a physical model from the Faculty of Electrotechnics, University of Craiova. The determined values are compared with those obtained through numerical calculations using a relatively new approach: the finite element method (FEM) in conjunction with asymptotic boundary conditions. From the comparative study of the magnetic force values one can formulate conclusions regarding the accuracy of the solutions.

Key words: magnetic field and forces, experiments, finite elements, asymptotic boundary conditions

1. ANALYTICAL DETERMINATION OF FORCE IN A STEADY MAGNETIC FIELD

One considers, for the exemplification of determinations, a physical system (Fig. 1), consisting of a magnetic circuit supplied by a fixed coil with N_1 turns and pearced by a d.c. current. The magnetic core is made from the nonlinear magnetic material heaving the lamination on the direction of magnetic flux and it an air gap is made in one of its columns. A mobile coil, made under the shape of a rectangular frame with N_2 turns is pearced by a d.c. current. This coil is suspended by means of two elastic springs in the air-gap upper side.

Calculation hypothesis. For the sake of simplicity, one considers the core permeability as infinite ($\mu \to \infty$). As a consequence, the magnetic field is concentrated within the air gap.

Under the above hypothesis used for calculation, the force is determined using one the known analytical equations, presented below.

1.1. Expression of the generalized magnetic force in magnetic fields at d.c. currents

The generalized forces theorem is applied:

$$F_m = \left.\frac{\partial W_m}{\partial x}\right|_{I=ct.}, \qquad (1)$$

where: W_m is the system magnetic energy, concentrated in the air gap, and *x* is the mobile coil displacement within the air gap.

The magnetic energy is calculated with respect to coils inductivities, using the expression:

$$W_m = \frac{1}{2}L_1 I_1^2 + \frac{1}{2}L_2 I_2^2 + M I_1 I_2, \qquad (2)$$

where L_1 and L_2 are the coils own inductances that do not depend on *x* displacement and M is coils' magnetic coupling inductance, that depends on *x*.

As a consequence the magnetic force can be expressed as follows, with respect to currents and mutual inductance:

$$F_m = I_1 I_2 \frac{\partial M}{\partial x}. \qquad (3)$$

1.2. Magnetic force calculated with Laplace's expression

The magnetic force that acts over a rectilinear conductor, pierced by current, placed along a direction perpendicular to an external uniform magnetic field lines is given the known relation:

$$F_m = BIl. \qquad (4)$$

in which, for the considered system, *B* is the magnetic inductance in the air gap, produced by the current I_1, $I = N_2 I_2$ is the total current through the mobile coil, and $l = b$ is the length of the coil side place in the air gap, that is perpendicular to the magnetic field lines.

The final expression used for the magnetic force calculation, obtained from the eq. (3) or (4) is:

$$F_m = k\, I_1\, I_2 \, , \qquad (5)$$

where we used the constant value:

$$k = \mu_0 N_1 N_2 \frac{b}{\delta} \qquad (6)$$

For the system consisting of the coils with N_1 turns and N_2 turns respectively, the magnetic core thickness, *b*, and the magnetic circuit air gap, δ, the magnetic force is proportional to the product of currents induced into the coils.

An analytical determination of force, with an improved accuracy, can be magnetic circuit approch. This approch will be studed in a future paper when the core saturation is taken into account.

1.3. Magnetic force calculated with Lorentz's expression

In general, the flux magnetic density is not constant over the whole section of the conductor or of the coil. In this case, the magnetic force can be numerical computed, with an improved accuracy, with the folowing integral over the volum of coil:

$$\vec{F} = \int_V (\vec{J} \times \vec{B})\, dv \, . \qquad (7)$$

2. EXPERIMENTAL PROCEDURE FOR THE MAGNETIC FORCE DETERMINATION

In order to perform an experimental determination of the magnetic force, in the laboratory of Electrical Engineering Department it was realized the device and experimental schematic shown in Fig. 1. The corresponding experimental procedure is presented by (Pu] ca] u, S. and col., 1990).

The constructive features of the magnetic circuit and the constants from the equations (5) and (8) are presented in table 1.

Table 1: Constructive features of magnetic circuit

Fixed coil	N_1 = 4000 turns, axial section dimensions: $d \times e = (50 \times 20)$mm
Mobile coil	N_2 = 100 turns, springs' the elasticity constant, k_e= 63 N/m
Magnetic circuit	$k = 4{,}452 \frac{N}{A^2}$, $a = 70 \cdot 10^{-3}$m, $b = 53 \cdot 10^{-3}$m, $c = 147 \cdot 10^{-3}$m, $d = 25 \cdot 10^{-3}$m, $e = 200 \cdot 10^{-3}$m, $g = 50 \cdot 10^{-3}$m, $\delta = 6 \cdot 10^{-3}$m, (d × e) is the dimensions of the fixed coil.

The coils are connected by 2 d.c. sources, U_{e1} and U_{e2}, by means of 2 series variable resistors R_{h1} and R_{h2}. The currents determined in coils are measured with the ampermeters *A*1 and *A*2, with a precision class of 0.5. The sources polarities at coils terminals are chosen so as determine an attraction force in the mobile coil air gap. Within the coil springs an elastic force is generated, as a reaction to the magnetic force.

The elastic force is proportional to the displacement *x* of the mobile coil and has the expression:

$$F_e = k_e\, x \, , \qquad (8)$$

where k_e is springs' the elasticity constant.

When the coil is in an equilibrium state, both forces (with contrary senses) are balanced:

$$F_m = F_e \, . \qquad (9)$$

For various values of currents through coils, there was determined the springs elongation, *x*, using a ruler scaled in millimeters. The measured data are substituted in the magnetic force expression (5) and in the elastic force expression (8), and the values are gathered in table 3. In principle, the calculated values of both forces must coincide.

3. NUMERICAL DETERMINATION OF MAGNETIC FORCE

The magnetic field produced in the magnetic circuit air gap can be computed with an improved accuracy. For this aim one uses the FEM analysis (Pei-bai Zhou, 1993, Silvester, P.P.,1990 and Pasare S., 1999) for a model closed to the real physical system. One starts from the differential equation of the vectorial magnetic potential for a bounded field domain around the magnetic circuit. The domain boundary and boundary conditions are chosen so as to get a better accuracy with respect to that corresponding to analytical solutions that use simplified models.

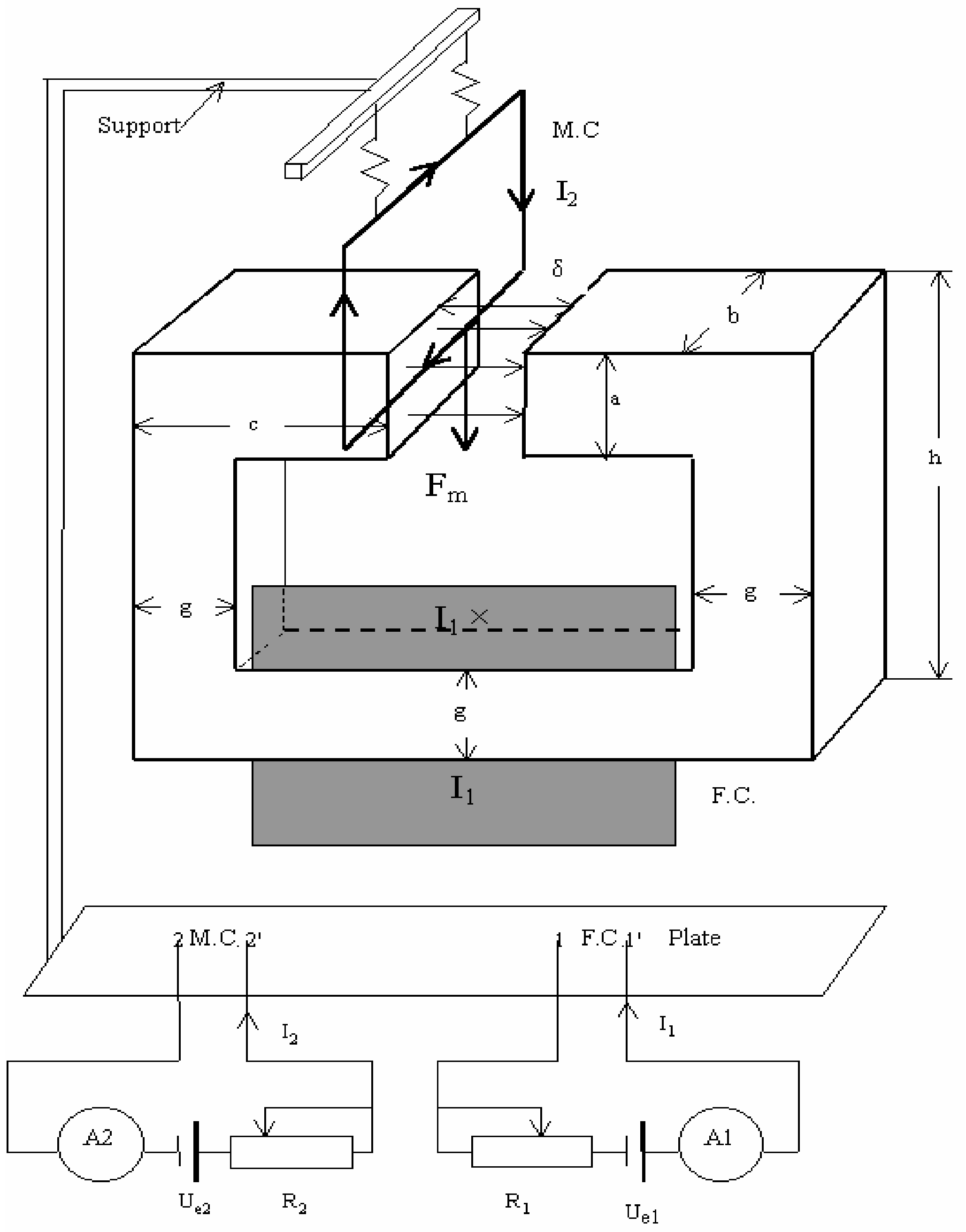

Fig. 1. Experimental schematic: M.C.-mobile coil, F.C.- fixed coil, F_m – magnetic force, $B = B(I_1)$ - magnetic flux density

3.1. Problem formulation and selection of mathematical model

Problem formulation. The analyzed physical system is represented by a non-linear magnetic circuit excited by a fixed coil fed to d.c. current. The dimensions of the system are known (see table 1). Also there are provided the current through the excitation coil and the non-linear magnetization characteristic of the ferromagnetic core (table 2 and Fig. 2). One determines the magnetic induction within the circuit air gap.

Problem type. The problem is considered to be a bidimensional problem for the analysis of magnetic fields with plane-parallel symmetry, reported to the cartesian system of coordinates (x,y) or to the polar system of coordinates (r,θ).

The analysis plane is perpendicular over the magnetic core width.

The operational differential equation for the vectorial magnetic potential is given by the general expression:

$$\nabla \times \left(\frac{1}{\mu(B)} \nabla \times \vec{A}\right) = \vec{J}\,, \qquad (10)$$

that can be written in particular forms (using cartesian or polar coordinates), for the 3 material media from field's domains: the magnetic core –

nonlinear area, the conducting winding and air, as linear areas (Fig. 3).
The magnetic potential *A* and the current density *J* are the components of the corresponding vector quantities along the Oz axis, perpendicular on the analysis plane.

Table 2

B(T)	0	0,4	0,5	0,6	0,8	1	1,2	1,3	1,4	1,5	1,52
H (A/m)	0	200	250	300	400	600	1000	1300	1830	2500	3000

Field domain boundary is chosen so as to have a circular shape, with the radius r_0 and the center in the central zone of the magnetic circuit, on which is employed a mixed condition of Robin type (asymptotic boundary condition). This boundary condition is employed because the field domain with infinite boundary must be limited to a finite domain with circular boundary.
Robin's condition for any potential function, *u*, has the general expression:

$$\frac{\partial u}{\partial n} + f_1 u = f_2 . \qquad (11\text{-a})$$

The same condition can be written as follows for the magnetic potential (Chen, Q. and Konrad, A.,1997):

$$\frac{\partial A}{\partial n} + \mu_0 c_0 A + \mu_0 c_1 = 0 , \qquad (11\text{-b})$$

where the coefficients are provided by the expressions:

$$c_0 = \frac{1}{\mu_0 r_0}, \quad c_1 = 0 . \qquad (12)$$

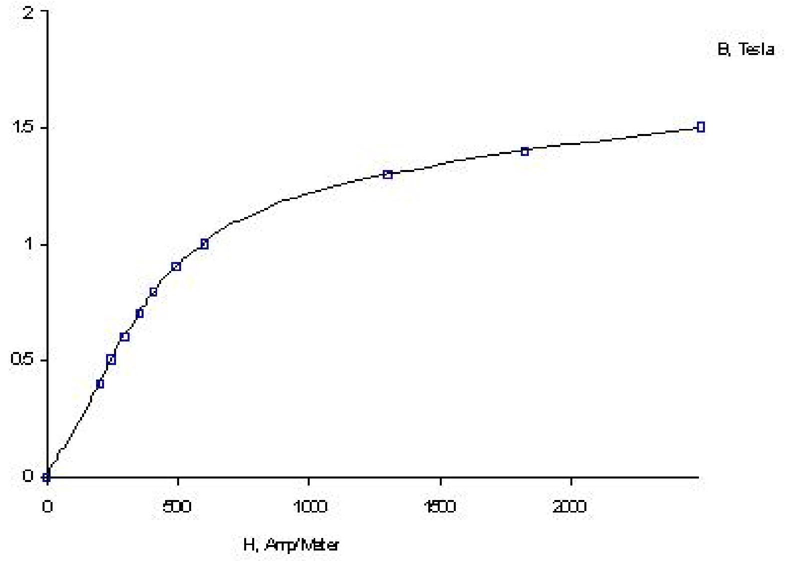

Fig. 2. Non-linear characteristic B(H) of magnetic material

The expression (11-b) results from the magnetic potential approximation, outside the circular boundary, using trigonometric series of harmonics from which one uses only the fundamental harmonic as being the most significant with respect to the other harmonics of higher orders:

$$A(r,\theta) \cong \frac{a}{r}\cos(\theta + \alpha), \quad r \geq r_0 . \qquad (13)$$

With $\mu_0 = 4\pi\, 10^{-7}\, \frac{H}{m}$ and $r_0 = 0{,}25\, m$, Robin's condition becomes:

$$\frac{10^7}{4\pi}\frac{\partial A}{\partial n} + 4\, A = 0 . \qquad (14)$$

3.2. Equations with finite elements and their solving

For the analysis of the steady magnetic field problem, in the plane domain with circular boundary and Robin conditions, one applies the variational finite element method. The differential equations of vector magnetic potential (10) are substituted by an algebraic equations equivalent system, with finite elements, obtained through the minimization of the associated energetic function.

The energetic functional for the non-linear bidimensional plane-parallel problem using cartesian coordinates has the expression:

$$W(A)=\int_{S_\Gamma}\left[\frac{\partial}{\partial x}\left(\frac{1}{\mu}\frac{\partial A}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{1}{\mu}\frac{\partial A}{\partial y}\right)\right]dxdy$$
$$-2\int_{S_\Gamma} J\,A\,dx\,dy+\int_\Gamma \frac{1}{\mu}\left(\frac{1}{2}f_1A^2-f_2A\right)dl \quad , \quad (15)$$

where: S_Γ is the circular plane surface of the field's domain, bounded by the circular boundary Γ, $\mu(B)$ is obtained from the nonlinear characteristic $B(H)$ given by Fig. 2, for the magnetic core, and the functions f_1 and f_2 are obtained through identification from (11a) and (11b).

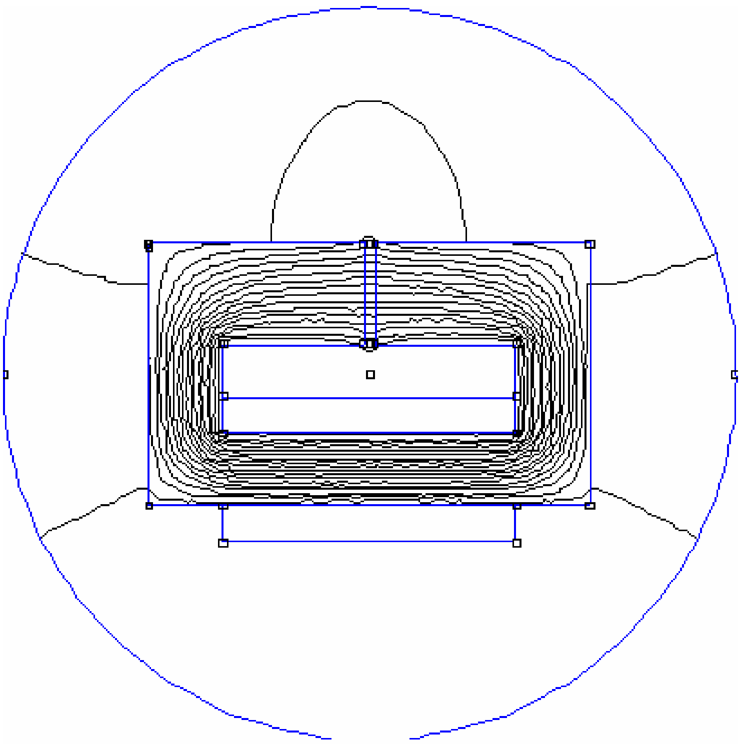

Fig. 3. Equipotential lines spectrum

In the linear subdomains, where the coils conductors are placed, and within the air gap (where $J=0$), the function expression is accordingly modified. The functional expression in polar coordinates is actually the expression in cylindrical coordinates in the plane rOz, where the only nonzero component of the magnetic potential is that along the Oz axis (as in the case of rectangular coordinates).

Field domain meshing is performed by means of linear triangular finite elements.

A convenient mesh step was selected, and so it was generated a mesh with 6238 linear triangular elements with 3192 nodes. The energetic functional, considering the FEM, is minimized with respect to the potentials associated to the mesh nodes and yields the nonlinear algebraic equations with FEM.

Field problem solving is performed by means of the FEMM program (Meeker, D., 2001). To solve the nonlinear equations one uses, in an iterative manner, the conjugated gradient method. The equipotential lines spectrum from the analyzed field domain is shown in Fig. 3.

3.3. Numerical computation of the magnetic force

For the determination of the magnetic force from the field problem numerical solution one must know the value of the magnetic flux density from the magnetic circuit air gap.

a). One assumes that the mobile coil fed to current does not influence the field from air gap.

The magnetic flux density from air gap is indeed considered for a uniform field, except for its margins

The magnetic force is determined by means of Laplace's expression (4) where the magnetic flux density, $B = B(I_1)$, obtained numerically for various values of the current I_1 is substituted. Table 3 presents the values of magnetic force determined by all the presented methods: analytically ($F_{m,a}$), experimentally ($F_{m,e}$) and numerically ($F_{m,n}$)-formula (4), for various values of the currents through coils and of numerically determined values of B (I_1).

b). **One assumes that the mobile coil fed to current does influence the field from air gap.** *The magnetic flux density* from air gap, in this case, is a ununiform field over the whole mobile coil (see Fig.4). *The magnetic force* is determined by means of Lorentz's integral expression (7) where the magnetic flux density, B = B(I_1, I_2), is obtained numerically for various values of the currents I_1 and I_2. In this case, the magnetic force can be computed, with an improved accuracy (see Table 4).

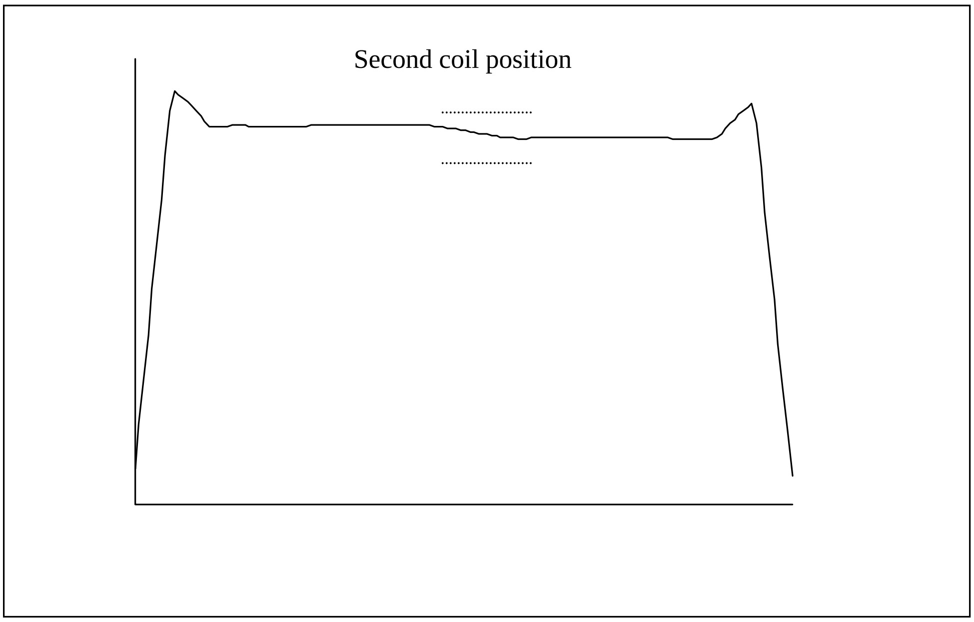

Fig.4. Magnetic flux density along of air gap , for I_1 = 1A, I_2 =0,3A

Table 3

Procedures applicated to determination of magnetic force					Formula (5)	Experi-mental values	Formula (4)	Absolute error	Relative error
Nr. crt.	I_1 (A)	I_2 (A)	B(I_1) (T)	$x \cdot 10^{-3}$ (m)	$F_{m,a}$ (N)	$F_{m,e} = F_e$ (N)	$F_{m,n}$ (N)	*E (N)	**E_r (%)
0	1	2	3	4	5	6	7	8	9
1	0,5	0,3	0,378	9	0,667	0,567	0,601	0,034	2,25
2	0,5	0,4	0,378	13	0,890	0,819	0,801	0,018	1,19
3	0,8	0,3	0,593	12	1,068	0,756	0,942	0,186	12,3
4	0,8	0,4	0,593	16	1,424	1,008	1,257	0,249	16,2
5	1	0,3	0,713	15	1,335	0,945	1,132	0,187	12,3
6	1	0,4	0,713	19	1,780	1,197	1,511	0,314	20,7

*E = $|F_{m,n}$- F_e (N)|, **E_r = 100 *E/$F_{m,n, max}$(%)

Table 4: Magnetic force calculated numerically

I_1 (A)	0,5	0,5	0,8	0,8	1	1
I_2 (A)	0,3	0,4	0,3	0,4	0,3	0,4
$F_{m,n}$ (N)-formula (4)	0,601	0,801	0,942	1,257	1,132	1,511
$F_{m,n}$ (N)-formula (7)	0,647	0,865	1,05	1,356	1,191	1,590

4. CONCLUSIONS

The comparative analysis concerning the values of the magnetic force is going to the following conclusions:

1.The differences between the force values determined analytically and experimentally respectively for identical currents, are significant and becomes higher for higher currents.
2.The columns 8 and 9 presents the absolute and respectively the relative errors with respect to the maximum numerical value of the force from table. For a maximum value of currents (I_1 =1A and I_2=0,4A), the percent error reaches a maximum (20,7 %). This situation is caused by the major differences between the models of calculus that uses major simplifying assumptions, and the real physical model.
3. One must mention, as an additional source of errors during the determination through calculations of the magnetic force with formula (4), the neglecting of the influence of current through the mobile coil over the resulting magnetic field from the magnetic circuit air gap.
4. The numerical determination of magnetic force using the formula (7) presents a very accuracy as it does take into account the flux leakage, the core saturation and the influence of current through the second coil.
5. The real problem of the magnetic field is three-dimensional. The definition of a problem on two-dimensional planar domain is a first step of the study of force in a steady magnetic field with asymptotic boundary conditions.

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