MODELLING AND ANALYSIS OF TEMPERATURE CONTROL PROCESS; A HYBRID PETRI NETS APPROACH

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Abstract: For the last past years, the formalism of classic Petri nets and their various extensions, offer powerful concepts and formal techniques for modeling, simulation and analysis of hybrid dynamic systems. In this way, this paper presents a Petri net approach to specify and simulate hybrid dynamic systems with autonomous commutation of the model generated by a hysteresis phenomenon, in order to validate their behavior for several simulation scenarios. An application to a thermostat with anticipative resistance used into a control temperature process is treated.

Keywords: Hybrid dynamic systems, Model approximation, Petri-nets, Simulation.

1. INTRODUCTION

As a rule, the hybrid systems are dynamical systems, where the behavior of interest is determined by interacting continuous and discrete dynamics. These systems typically contain variables or signals that take values from a continuous set and also variables that take values from a discrete, typically finite set. These continuous or discrete – valued variables or signals depend on independent variables such as time, which may also be continuous or discrete; the system can be event driven, but the variables can be also modified by events in an asynchronous manner.

Hybrid systems control has known recently considerable attention and many developments regarding theirs modeling and control have appeared in the literature (Grossman, et al., 1993; Demongodin et Koussoulas, 1998). The archetypal paradigm of a hybrid system is a supervisory control system whose fundamental structure consist of a high level (supervision) where a discrete – event system models decision making, and a low level (process) where a continuous system models the process and its local control loops. The role of the supervisor model is to describe the transition between the macro-states, which consist of the states of the continuous and the discrete – event system. Considering for a moment a Petri net model for the discrete – event part, the enabling and firing, obligatory or not, of a transition may represent symbolically the passage of the continuous system into a new operating regime. This higher level treats all sequences of events (starts and ends of operations, occurrences of unforeseen situations etc.) without complete information or knowledge regarding the evolution of the process in the elapsed time between events. The supervisor reacts to the state of the plant as perceived by it, issues commands and communicates them to the lower level. This kind of intervention may create problems in the operation of the continuous system, including deterioration of performance, non – attainability of the desired operation zones, and finally approach or event entrance to regions of instability (David et Alla, 2001).

In order to do a unitary conception of the hybrid systems representation, different approaches of modeling are used and at present there is already an abundance of such models, (Branicky, et al., 1994; Dubois, et al., 1994). They can be characterized and described along several dimensions. In broad terms, approaches differ with respect to the emphasis on or the complexity of the continuous and discrete dynamics, and on whether they emphasize analysis and synthesis results or analysis only or simulation only. On one end of the spectrum there are approaches to hybrid systems that represent extensions of systems theoretic ideas for systems that are described by ordinary differential equations to include discrete time and variables that exhibit jumps, or extend results to switching systems.

As a rule, is possible to describe a hybrid system through a formally expression:

$$\dot{\mathbf{x}}(t) = F(\mathbf{x}(t), \mathbf{q}(t), \mathbf{u}(t)), \quad \mathbf{x}(t_0) = \mathbf{x}_0, \mathbf{q}(t_0) = \mathbf{q}_0 , \quad (1)$$

where: $\mathbf{x}(t) \in X \subset \mathbb{R}^n$ is the state system vector, $\mathbf{q}(t) \in Q \subset \mathbb{N}^m$ and $\mathbf{u}(t) \in U \subset \mathbb{R}^c$ – the input vector (n, m, c – fixed). In our paper we are interested on a class of hybrid dynamical systems with autonomous commutation, which can be represented like an n- order differential system:

$$\dot{\mathbf{x}}(t) = \mathbf{A} \cdot \mathbf{x}(t) + \mathbf{B}(q) \cdot \mathbf{u}(t), \quad \mathbf{x}(t_0) = \mathbf{x}_0 , \quad (2)$$

with **A** – n - order square matrix and **B** – n ×c – order matrix.

Having a dual nature of a graphical tool and a mathematical object Petri nets can serve in both the practical and the theoretical camp. It is advantageous, if not indispensable, to be able to represent both continuous and discrete parts of a hybrid system in the same context. It looks rather obvious to choose the discrete – event domain as the foundation for this common representation. Therefore an extension for Petri nets is necessary to represent the continuous time dynamic components. In this work, our contribution was the representation such hybrid structure through a specific Petri nets extension - named *modified Petri Net* - in which for each continuous transition was assigned a firing speed as a function, whose arguments can be the token quantities of arbitrary places of the net, (Drath, 1997a, b, 1998; Drighiciu 2003, 2005).

2. CASE STUDY

2.1 Physical system structure

The system proposed for our study illustrates in fact a control process of the temperature into certain work chamber (Fig.1), using a bi – positional thermostat with anticipative resistance.

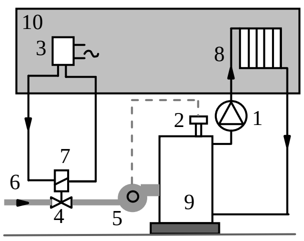

Fig.1 Temperature control process structure

As a rule, in the whole process the role of this device is very intuitive: from different variations of the temperature into the work chamber, the thermostat bi – metal is dilated or it is contracted in comparison with a reference value, so that the main contact c switches between „on" and „off" positions. For a temperature value, lower than that which is initially prescribed, the bi – metal is contracted and the contact *c* is putting on (Fig.2).

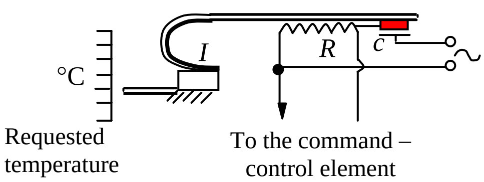

Fig.2 Illustrative for the work principle thermostat

Thus, the anticipative resistance is connected into the circuit, bringing the heating (many times indirectly) of the bi – metal. In the mean time, the relay (7 – Fig.1) turn on the supply valve (4 - Fig.1) and the gas – generating station, which induces the heating of the work chamber – is started. When the temperature rises and become greater than reference value, the anticipative resistance is, certainly, disconnected. In a qualitative representation of the control process (Fig.3), discrete variable „q" reaches two values (i.e. 1 and 0) according with one or other of steps θ_{t1} and θ_{t2}, considered as lower, respectively upon limits of the work chamber temperature.

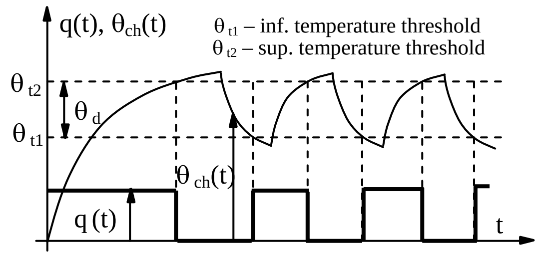

Fig.3 Illustrative to the temperature control process

Temperature fluctuations induced by thermal inertia of the device can be reduced using one anticipative resistance R. Hence, the exceeding of θ_d value (Fig.3) is restricted, the thermostat releasing before the moment when the reference value of temperature into the work chamber is reached. In this way, the temperature oscillations can be diminished, but the releasing frequency of the device may be increased.

2.2 The mathematical model

The control process of temperature into work chamber can be studied using a simplified mathematical model, established according to the Fourier's low of the heating process, supposing a proportional dependence between the heat flows and gradients of the temperature. Thus, the chamber temperature – θ_{ch} can be approximate like a linear dependence between the temperature values of the external environment - θ_e and the radiator - θ_r:

$$\frac{d\theta_{ch}(t)}{dt} = -c_1\big(\theta_{ch}(t)-\theta_e(t)\big)+c_2\big(\theta_r(t)-\theta_{ch}(t)\big) \quad (3)$$

Similarly, temperature of the thermostat - θ_{th} depends on heat changed between that device and the work chamber and on the thermal energy – Q_1 due to the anticipative resistance – R, when that one is connected to the power supply:

$$\frac{d\theta_{th}(t)}{dt} = -c_3\big(\theta_{th}(t)-\theta_{ch}(t)\big)+q(\theta_{th})\cdot Q_1 \quad (4)$$

On the other hand, temperature of the radiator – θ_r depends on the thermal changing between that and the chamber and on the thermal energy – Q_2 provided by radiator itself:

$$\frac{d\theta_r(t)}{dt} = -c_4\big(\theta_r(t)-\theta_{ch}(t)\big)+q(\theta_{th})\cdot Q_2 \quad (5)$$

In (3), (4) and (5), c_i (i = 1, …4) denotes the global coefficients of heat transfer, and „q" is a discrete variable who can reaches only two different values (0 or 1), according to the histeresis thermostat cycle (Fig.4) and controls the starting and the stopping process of the heating system.

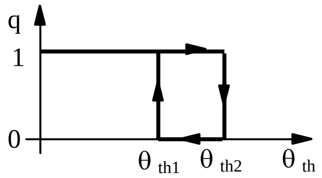

Fig. 4 Histeresis thermostat cycle

Considering as state vector of the system $\boldsymbol{\theta}^t = (\theta_{ch}\ \ \theta_{th}\ \ \theta_r)^t$ and as input vector $\mathbf{u}^t = (\theta_e\ \ Q_1\ \ Q_2)^t$, the simplified mathematical model can be represented through a linear system of equations, in accordance to (2):

$$\begin{pmatrix} \dot{\theta}_{ch} \\ \dot{\theta}_{th} \\ \dot{\theta}_r \end{pmatrix} = \begin{pmatrix} -(c_1+c_2) & 0 & c_2 \\ c_3 & -c_3 & 0 \\ c_4 & 0 & -c_4 \end{pmatrix} \cdot \begin{pmatrix} \theta_{ch} \\ \theta_{th} \\ \theta_r \end{pmatrix} +$$

$$+ \begin{pmatrix} c_1 & 0 & 0 \\ 0 & q & 0 \\ 0 & 0 & q \end{pmatrix} \cdot \begin{pmatrix} \theta_e \\ Q_1 \\ Q_2 \end{pmatrix} \qquad (6)$$

Those two values of the discrete variable „q" (1 and 0, respectively) induce to the whole system two distinct operated services: ON (for q = 1) and OFF (q = 0). The autonomous commutation of the hybrid system is released when the state space vector $\boldsymbol{\theta}$ (and, more precisely it θ_{th} component) reaches at first time the threshold value θ_{th1} (q = 0 and $d\theta_{th}/dt < 0$), then the other threshold value θ_{th2} (q = 1 and $d\theta_{th}/dt > 0$).

2.3 Hybrid Petri net model

The Petri net model for the control process of temperature was obtained starting at the previously observation in connection with the autonomous commutation of the system, due to hysteresis cycle threshold values θ_{th1} and θ_{th2} respectively. Hence, from initial state, until the temperature of thermostat become θ_{th2}, the whole dynamic of the system is described by (4) equation in which q = 1. Then, during [θ_{th2}, θ_{th1}] interval of temperature, when the anticipative resistance R and the gas – generating station are turned off, the behavior of the process is described by the same mathematical equations system, with q = 0. That commutation of the model is a periodical process, generated by the achievement of θ_{th1} and θ_{th2} values. Because the mathematical model of the process was considered as hybrid representation, for Petri net structure of that system, a hybrid topology was adopted (Fig.5).

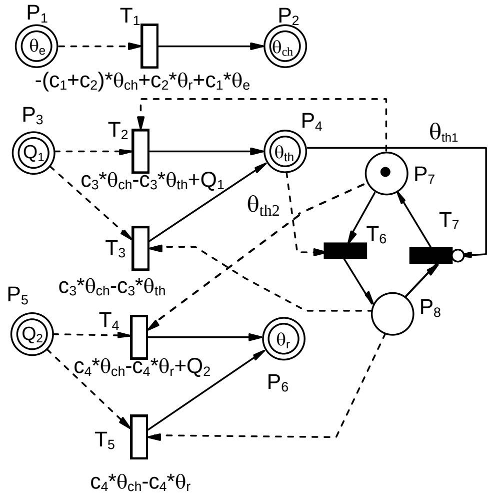

Fig.5 The Petri net model of the system

That is organized by a direct connection between specific elements of continuous Petri net with variable firing speed of transitions formalism and some elements of T-timed discrete Petri nets. The linear equation of the initial mathematical model was represented into continuous sub – net, formed with $P_1 \div P_6$ places and $T_1 \div T_5$ transitions. That sub-net models the behavior of the system between commutation processes. The changing of Petri net structure according a commutation process is possible to be made thanks to discrete sub – net (P_7, P_8 places and T_6, T_7 transitions), which activate or no – through test arcs – some of continuous transitions. The Petri net switches permanently between two similar structures, that behavior been induced for the weight of arcs $P_4 - T_6$ (test arc with θ_{th2} weight) and $P_4 - T_7$ (inhibitor arc with θ_{th1} weight) respectively. Specification of alls Petri net elements was made according to the hybrid model and, also, respecting the evolution rules dues to the specific net formalism: P_1 is assigned with the value of external environment temperature, P_2 – work chamber current temperature, P_3 – thermal energy produced by R, P_4 – thermostat current temperature, P_5 – thermal energy due to the gas – generating station, P_6 – radiator current temperature. The marking non – null of P_7 activate the firing of T_2 and T_4 transitions (q = 1) and, similarly, the marking non – null of P_8 activate the continuous firing of T_3 and T_5 (q = 0). The firing speed of continuous transitions is not constant, but having the same (more or less) expression with the state equation of the mathematical model.

Petri net model was constructed and then validated from point of view of its behavioral properties with Visual Object Net++ tool (Drath, 1997, 1998). That leaves various simulation scenarios – thanks to a useful interface and its flexibility – for different initial conditions and values of model parameters.

The values for global coefficients of heat transfer c_1, and, also, for Q_1 and Q_2 was the same used in (Cébron, 2000; Jolly, 2004). The reference value of work chamber temperature can be adopted and eventually changed by fixing the threshold values of the thermostat. Hence, in Fig.6 it is shown a set of simulation scenarios obtained upon the Petri net model, for: $c_1 = 10^{-4}$ s^{-1}, $c_2 = 5{\cdot}10^{-5}$ s^{-1}, $c_3 = 5{\cdot}10^{-3}$ s^{-1}, $c_4 = 10^{-3}$ s^{-1}, $Q_1 = 0.0132$ °C/s, $Q_2 = 0{,}06$ °C/s, $\theta_1 = 20$ °C; $\theta_2 = 22$ °C, $\theta_e = 15$ °C. Thermostat threshold values established for θ_1 and θ_2 ensures for the work chamber temperature - θ_{ch} a 20 °C reference value.

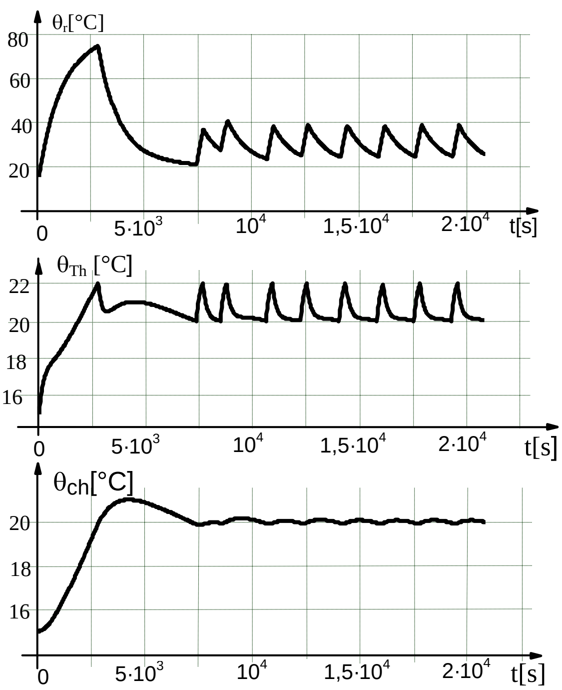

Fig.6 Petri net simulation results for $c_1 = 10^{-4}$ s^{-1}, $c_2 = 5{\cdot}10^{-5}$ s^{-1}, $c_3 = 5{\cdot}10^{-3}$ s^{-1}, $c_4 = 10^{-3}$ s^{-1}, $Q_1 = 0.0132$ °C/s, $Q_2 = 0{,}06$ °C/s, $\theta_{th1} = 20$ °C; $\theta_{th2} = 22$ °C, $\theta_e = 15$ °C.

During a simulation time interval, all initials value of state vector components was considered identical with external environment temperature value. Moreover, the effect of external perturbation on whole dynamic process was too neglected, the behavior of entire system being roughly the same whit that of an isolated system. Starting the initial hybrid Petri net structure (Fig.5), for various environmental initial conditions and more references values of work chamber temperature, different simulation scenarios results was obtained (Fig.7, Fig.8).

This proposed hybrid Petri net framework is able to make a quantitative evaluation of behaviorist properties and, obviously, it is a modular structure at which other sub – nets (continuous, discrete or hybrid) can be added, in order to detect the consequences of external perturbations of whole system behavior.

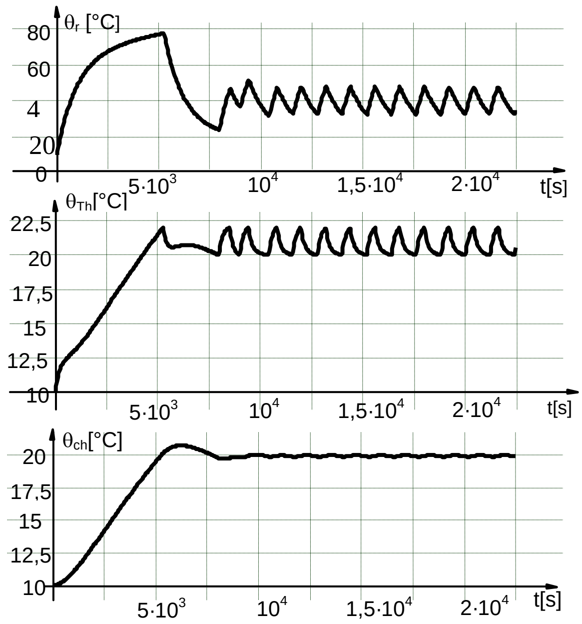

Fig.7 Petri net simulation results for $c_1 = 10^{-4}$ s^{-1}, $c_2 = 5{\cdot}10^{-5}$ s^{-1}, $c_3 = 5{\cdot}10^{-3}$ s^{-1}, $c_4 = 10^{-3}$ s^{-1}, $Q_1 = 0.012$ °C/s, $Q_2 = 0{,}06$ °C/s, $\theta_{th1} = 20$°C, $\theta_{th2} = 22$ °C, $\theta_e = 10$ °C.

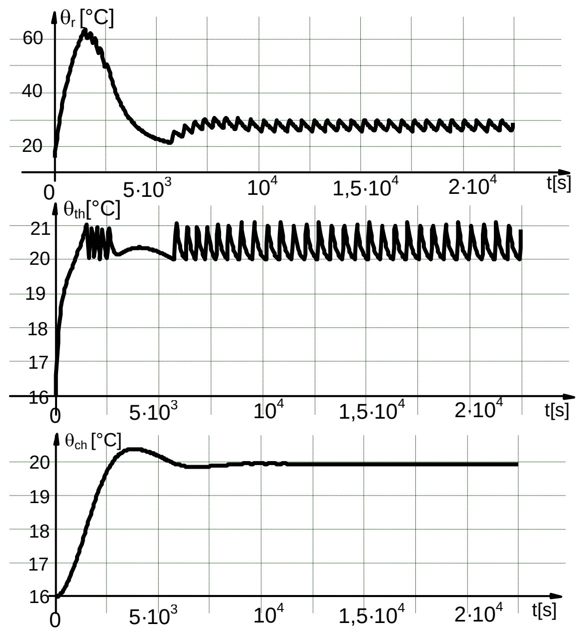

Fig.8 Petri net simulation results for $c_1 = 10^{-4}$ s^{-1}, $c_2 = 5{\cdot}10^{-5}$ s^{-1}, $c_3 = 5{\cdot}10^{-3}$ s^{-1}, $c_4 = 10^{-3}$ s^{-1}, $Q_1 = 0.0168$ °C/s, $Q_2 = 0{,}06$ °C/s, $\theta_{th1} = 20$°C, $\theta_{th2} = 22$ °C, $\theta_e = 16$ °C.

Thus, if during the process of temperature control, the work chamber is not considered an isolated system, but interacting with the external environment, the level of temperature (the reference value) may increase or decrease, in accordance with the thermal flow modifications. So, for a qualitative (at least) description of the whole process, the topology of initial Petri net model will be extended by addition a new specific elements (Fig.9). The complementary sub – net is a hybrid structure too, with a continuous transition (T_8) and four discrete elements (P_9, P_{10}, T_9, T_{10}). At the initial moment, the this firing, the non – null marking of P_9 authorize the continuous firing, with „*v*" speed of T_8 and the value of work chamber temperature associated to P_2 place will be modified. In Fig.10 is shown the results of simulation scenario, in this situation.

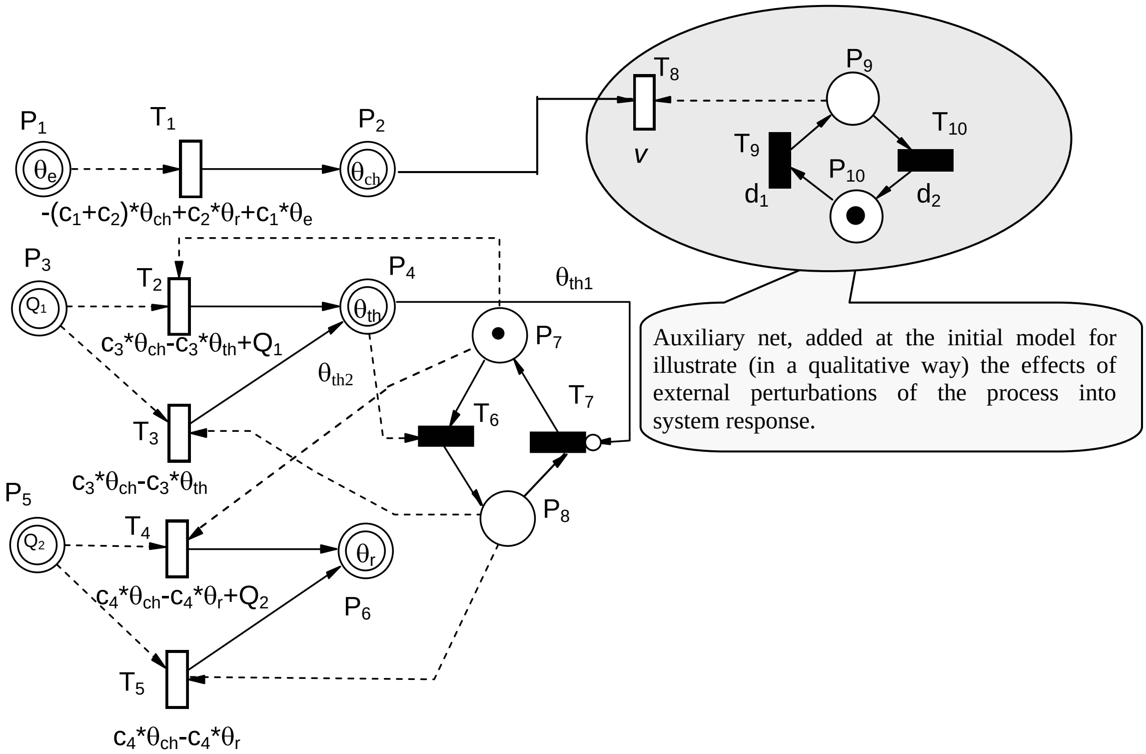

Fig.9 Modified Petri net model

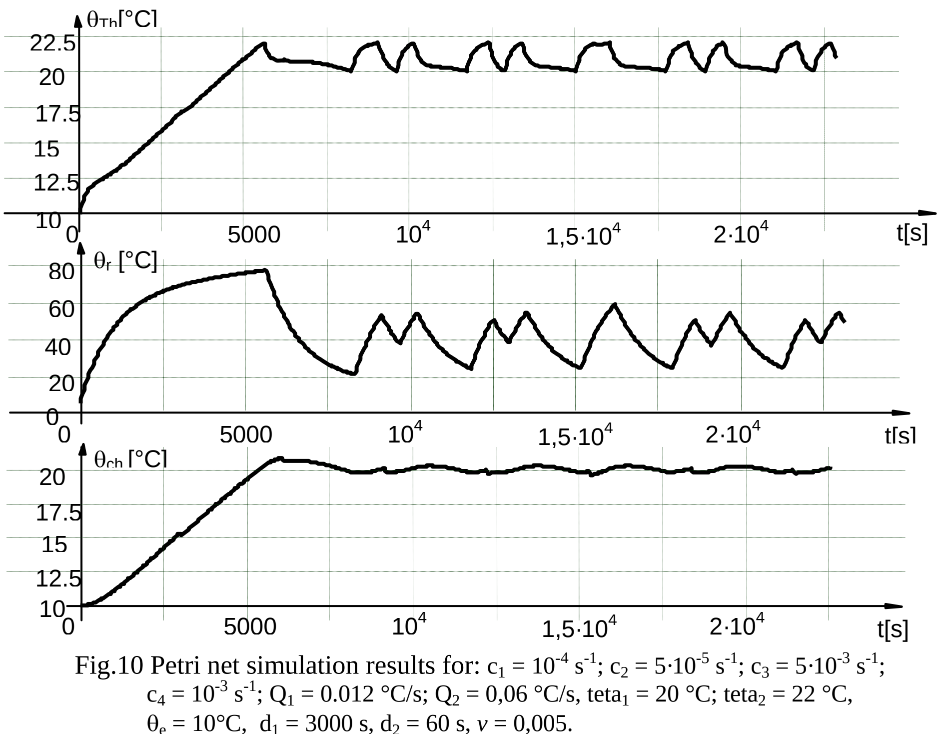

Fig.10 Petri net simulation results for: $c_1 = 10^{-4}\ s^{-1}$; $c_2 = 5\cdot 10^{-5}\ s^{-1}$; $c_3 = 5\cdot 10^{-3}\ s^{-1}$; $c_4 = 10^{-3}\ s^{-1}$; $Q_1 = 0.012$ °C/s; $Q_2 = 0,06$ °C/s, $teta_1 = 20$ °C; $teta_2 = 22$ °C, $\theta_e = 10$°C, $d_1 = 3000$ s, $d_2 = 60$ s, $v = 0,005$.

marking of P_{10} validate the T_9 transition, which will be fired after it laps of time - d_1 – associated. After Another possibility of modelling the influence of the external perturbations in the behavior of the system is to use for the sub – nets added a discrete or hybrid stochastic elements, which allows obtaining the quantitative interesting results. The most mathematical, textual or graphical approaches to describe hybrid systems are currently usable for small examples. Models of complex systems are unwieldy. Therefore a hierarchical concept to structure a model is needed. In order to solve the mentioned handling problems arising from the system complexity, in (Drath 1997a, 1998; Drighiciu 2003) was proposed a object oriented paradigm for the analysis of the Hybrid Petri Nets, resulting in a new method to describe both continuous and discrete event systems with reduced effort. One of the important advantages of the using this concept is the ability to describe a larger system by the decomposition into interacting objects. Because of the properties of objects, the modification of the system model could be easier achieved. The object-oriented concept unites the advantages of the modules and hierarchies and adds useful concepts like *reuse*, *encapsulation* and *information hiding*. In this way we get more flexibility.

3. CONCLUSIONS

The main idea of this paper was to consider an temperature control process such a particular hybrid system, with autonomous state commutation and, starting of this point, to find a specific frame for describing his behavior. A mathematical model of such a process has thus to be a hybrid model involving discrete variables (integers or with a domain in a finite set) and continuous variables (real numbers). Both dynamics (discrete and continuous) have to be modeled: a discrete event based dynamics for discrete variables (sequence of operations) and a continuous time dynamics for continuous variables (differential algebraic equations). The general approaches have strong similarities with hybrid automata, but the discrete dynamics is represented by Hybrid Petri Nets instead of automata, in order to address in an explicit way resource allocation policies. True concurrency is indeed required and the interleaving semantics of automata based approaches is not sufficient.

Adding news elements, with a grown power of analysis, may enrich the general model proposed. Hence, in the hybrid Petri Net model achieved, a first step to increase the design details was made by the use of variables delays associated to the discrete transitions. The concept “variable delay” is more often utilized in a determinist way. The delays are in any time defined in occurrence with an external event, a priori estimated, which may be generated by the decisional structure of the entire system. Another step in refining the initial model may be the use of powerful extensions of basic Petri Nets models, i.e. stochastic, colored or fuzzy Petri Nets. The integration of different process types in complex systems is rapid increased by the combination of dynamic influences of each process. Therefore, the hybrid object nets architecture can be used to get more flexibility compared with the common approximation principles.

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