MODELLING AND ANALYSIS OF TEMPERATURE CONTROL PROCESS;
A HYBRID PETRI NETS APPROACH

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Abstract: For the last past years, the formalism of classic Petri nets and their various
extensions, offer powerful concepts and formal techniques for modeling, simulation and
analysis of hybrid dynamic systems. In this way, this paper presents a Petri net approach
to specify and simulate hybrid dynamic systems with autonomous commutation of the
model generated by a hysteresis phenomenon, in order to validate their behavior for
several simulation scenarios. An application to a thermostat with anticipative resistance
used into a control temperature process is treated.

Keywords: Hybrid dynamic systems, Model approximation, Petri-nets, Simulation.

1. INTRODUCTION

As a rule, the hybrid systems are dynamical systems,
where the behavior of interest is determined by
interacting continuous and discrete dynamics. These
systems typically contain variables or signals that
take values from a continuous set and also variables
that take values from a discrete, typically finite set.
These continuous or discrete – valued variables or
signals depend on independent variables such as
time, which may also be continuous or discrete; the
system can be event driven, but the variables can be
also modified by events in an asynchronous manner.

Hybrid systems control has known recently
considerable attention and many developments
regarding theirs modeling and control have appeared
in the literature (Grossman, et al., 1993; Demongodin
et Koussoulas, 1998). The archetypal paradigm of a
hybrid system is a supervisory control system whose
fundamental structure consist of a high level
(supervision) where a discrete – event system models
decision making, and a low level (process) where a
continuous system models the process and its local
control loops. The role of the supervisor model is to
describe the transition between the macro-states,
which consist of the states of the continuous and the
discrete – event system. Considering for a moment a
Petri net model for the discrete – event part, the
enabling and firing, obligatory or not, of a transition
may represent symbolically the passage of the
continuous system into a new operating regime. This
higher level treats all sequences of events (starts and
ends of operations, occurrences of unforeseen
situations etc.) without complete information or
knowledge regarding the evolution of the process in
the elapsed time between events. The supervisor
reacts to the state of the plant as perceived by it,
issues commands and communicates them to the
lower level. This kind of intervention may create
problems in the operation of the continuous system,
including deterioration of performance, non –
attainability of the desired operation zones, and
finally approach or event entrance to regions of
instability (David et Alla, 2001).

In order to do a unitary conception of the hybrid
systems representation, different approaches of
modeling are used and at present there is already an
abundance of such models, (Branicky, et al., 1994;
Dubois, et al., 1994). They can be characterized and
described along several dimensions. In broad terms,
approaches differ with respect to the emphasis on or
the complexity of the continuous and discrete
dynamics, and on whether they emphasize analysis
and synthesis results or analysis only or simulation
only. On one end of the spectrum there are
approaches to hybrid systems that represent
extensions of systems theoretic ideas for systems that
are described by ordinary differential equations to
include discrete time and variables that exhibit
jumps, or extend results to switching systems.

As a rule, is possible to describe a hybrid system
through a formally expression:
\[ \dot{x}(t) = F(x(t), q(t), u(t)), \quad x(t_0) = x_0, \quad q(t_0) = q_0, \quad (1) \]
where:
- \( x(t) \in X \subseteq \mathbb{R}^n \) is the state system vector,
- \( q(t) \in Q \subseteq \mathbb{N}^m \) and \( u(t) \in U \subseteq \mathbb{R}^c \) – the input
  vector (\( n, m, c \) – fixed). In our paper we are
interested on a class of hybrid dynamical systems
with autonomous commutation, which can be
represented like an n- order differential system:
\[ \dot{x}(t) = A \cdot x(t) + B(q) \cdot u(t), \quad x(t_0) = x_0, \quad (2) \]
with \( A – n \) - order square matrix and \( B – n \times c \) – order
matrix.
Having a dual nature of a graphical tool and a mathematical object Petri nets can serve in both the practical and the theoretical camp. It is advantageous, if not indispensable, to be able to represent both continuous and discrete parts of a hybrid system in the same context. It looks rather obvious to choose the discrete – event domain as the foundation for this common representation. Therefore an extension for Petri nets is necessary to represent the continuous time dynamic components. In this work, our contribution was the representation such hybrid structure through a specific Petri nets extension - named modified Petri Net - in which for each continuous transition was assigned a firing speed as a function, whose arguments can be the token quantities of arbitrary places of the net, (Drath, 1997a, b, 1998; Drighiciu 2003, 2005).

2. CASE STUDY

2.1 Physical system structure

The system proposed for our study illustrates in fact a control process of the temperature into certain work chamber (Fig.1), using a bi – positional thermostat with anticipative resistance.

As a rule, in the whole process the role of this device is very intuitive: from different variations of the temperature into the work chamber, the thermostat bi – metal is dilated or it is contracted in comparison with a reference value, so that the main contact c switches between „on” and „off” positions. For a temperature value, lower than that which is initially prescribed, the bi – metal is contracted and the contact c is putting on (Fig.2).

Thus, the anticipative resistance is connected into the circuit, bringing the heating (many times indirectly) of the bi – metal. In the mean time, the relay (7 – Fig.1) turn on the supply valve (4 - Fig.1) and the gas – generating station, which induces the heating of the work chamber – is started. When the temperature rises and become greater than reference value, the anticipative resistance is, certainly, disconnected. In a qualitative representation of the control process (Fig.3), discrete variable „q” reaches two values (i.e. 1 and 0) according with one or other of steps \( \theta_{t1} \) and \( \theta_{t2} \), considered as lower, respectively upon limits of the work chamber temperature.

\[
\frac{d\theta_{a}(t)}{dt} = -c_1(\theta_{a}(t) - \theta_{a,t1}) + c_2(\theta_{a,t2} - \theta_{a}(t))
\]

(3)

Similarly, temperature of the thermostat - \( \theta_h \) depends on heat changed between that device and the work chamber and on the thermal energy – \( Q_1 \) due to the anticipative resistance – \( R \), when that one is connected to the power supply:

\[
\frac{d\theta_h(t)}{dt} = -c_1(\theta_h(t) - \theta_{a,t1}) + q(\theta_h) \cdot Q_1
\]

(4)

On the other hand, temperature of the radiator – \( \theta_r \) depends on the thermal changing between that and the chamber and on the thermal energy – \( Q_2 \) provided by radiator itself:

\[
\frac{d\theta_r(t)}{dt} = -c_1(\theta_r(t) - \theta_{a,t2}) + q(\theta_h) \cdot Q_2
\]

(5)
In (3), (4) and (5), $c_i$ (i = 1, ...4) denotes the global coefficients of heat transfer, and $\cdot \cdot \cdot q \cdot$ is a discrete variable who can reaches only two different values (0 or 1), according to the hysteresis thermostat cycle (Fig.4) and controls the starting and the stopping process of the heating system.

![Hysteresis thermostat cycle](image)

Fig. 4 Hysteresis thermostat cycle

Considering as state vector of the system $\theta' = (\theta_{th}, \theta_{tho}, \theta_t)$ and as input vector $u' = (Q, Q_2)$, the simplified mathematical model can be represented through a linear system of equations, in accordance to (2):

$$
\begin{align*}
\begin{bmatrix}
\dot{\theta}_{th} \\
\dot{\theta}_{tho} \\
\dot{\theta}_t
\end{bmatrix} &= 
\begin{bmatrix}
-(c_1 + c_2) & 0 & c_2 \\
c_1 & -c_3 & 0 \\
c_4 & 0 & -c_4
\end{bmatrix}
\begin{bmatrix}
\theta_{th} \\
\theta_{tho} \\
\theta_t
\end{bmatrix} + 
\begin{bmatrix}
c_1 \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
Q \\
0 \\
0
\end{bmatrix} + 
\begin{bmatrix}
0 & q & 0 \\
0 & 0 & q
\end{bmatrix}
\begin{bmatrix}
Q_2 \\
Q_3
\end{bmatrix} \\
\end{align*}
$$

Those two values of the discrete variable $\cdot \cdot \cdot q \cdot$ (1 and 0, respectively) induce to the whole system two distinct operated services: ON (for $q = 1$) and OFF (q = 0). The autonomous commutation of the hybrid system is released when the state space vector $\theta$ (and, more precisely its $\theta_{th}$ component) reaches at first time the threshold value $\theta_{th1}$ (q = 0 and $d\theta_{th}/dt < 0$), then the other threshold value $\theta_{th2}$ (q = 1 and $d\theta_{th}/dt > 0$).

2.3 Hybrid Petri net model

The Petri net model for the control process of temperature was obtained starting at the previously observation in connection with the autonomous commutation of the system, due to hysteresis cycle threshold values $\theta_{th1}$ and $\theta_{th2}$ respectively. Hence, from initial state, until the temperature of thermostat become $\theta_{th2}$, the whole dynamic of the system is described by (4) equation in which $q = 1$. Then, during $[\theta_{th2}, \theta_{th1}]$ interval of temperature, when the anticipative resistance R and the gas – generating station are turned off, the behavior of the process is described by the same mathematical equations system, with $q = 0$. That commutation of the model is a periodical process, generated by the achievement of $\theta_{th1}$ and $\theta_{th2}$ values. Because the mathematical model of the process was considered as hybrid representation, for Petri net structure of that system, a hybrid topology was adopted (Fig.5).

![Petri net model](image)

Fig.5 The Petri net model of the system

That is organized by a direct connection between specific elements of continuous Petri net with variable firing speed of transitions formalism and some elements of T-timed discrete Petri nets. The linear equation of the initial mathematical model was represented into continuous sub – net, formed with $P_1 + P_6$ places and $T_1 + T_5$ transitions. That sub-net models the behavior of the system between commutation processes. The changing of Petri net structure according a commutation process is possible to be made thanks to discrete sub – net ($P_7$, $P_8$ places and $T_6$, $T_7$ transitions), which activate or no – through test arcs – some of continuous transitions. The Petri net switches permanently between two similar structures, that behavior been induced for the weight of arcs $P_4$ – $T_6$ (test arc with $\theta_{th2}$ weight) and $P_4$ – $T_7$ (inhibitor arc with $\theta_{th1}$ weight) respectively. Specification of alls Petri net elements was made according to the hybrid model and, also, respecting the evolution rules dues to the specific net formalism: $P_1$ is assigned with the value of external environment temperature, $P_2$ – work chamber current temperature, $P_3$ – thermal energy produced by R, $P_4$ – thermostat current temperature, $P_5$ – thermal energy due to the gas – generating station, $P_6$ – radiator current temperature. The marking non – null of $P_7$ activate the firing of $T_2$ and $T_4$ transitions (q = 1) and, similarly, the marking non – null of $P_8$ activate the continuous firing of $T_3$ and $T_5$ (q = 0). The firing speed of continuous transitions is not constant, but having the same (more or less) expression with the state equation of the mathematical model.

Petri net model was constructed and then validated from point of view of its behavioral properties with Visual Object Net++ tool (Drath, 1997, 1998). That leaves various simulation scenarios – thanks to a useful interface and its flexibility – for different initial conditions and values of model parameters.
The values for global coefficients of heat transfer $c_1$, and, also, for $Q_1$ and $Q_2$ was the same used in (Cébron, 2000; Jolly, 2004). The reference value of work chamber temperature can be adopted and eventually changed by fixing the threshold values of the thermostat. Hence, in Fig.6 it is shown a set of simulation scenarios obtained upon the Petri net model, for: $c_1 = 10^{-4} \text{s}^{-1}$, $c_2 = 5 \cdot 10^{-5} \text{s}^{-1}$, $c_3 = 5 \cdot 10^{-3} \text{s}^{-1}$, $c_4 = 10^{-3} \text{s}^{-1}$, $Q_1 = 0.0132 ^\circ \text{C/s}$, $Q_2 = 0.06 ^\circ \text{C/s}$, $\theta_1 = 20 ^\circ \text{C}$; $\theta_2 = 22 ^\circ \text{C}$, $\theta_e = 15 ^\circ \text{C}$. Thermostat threshold values established for $\theta_1$ and $\theta_2$ ensures for the work chamber temperature - $\theta_{ch}$ a 20 °C reference value.

During a simulation time interval, all initials value of state vector components was considered identical with external environment temperature value. Moreover, the effect of external perturbation on whole dynamic process was too neglected, the behavior of entire system being roughly the same with that of an isolated system. Starting the initial hybrid Petri net structure (Fig.5), for various environmental initial conditions and more references values of work chamber temperature, different simulation scenarios results was obtained (Fig.7, Fig.8).

This proposed hybrid Petri net framework is able to make a quantitative evaluation of behaviorist properties and, obviously, it is a modular structure at which other sub – nets (continuous, discrete or hybrid) can be added, in order to detect the consequences of external perturbations of whole system behavior.

Thus, if during the process of temperature control, the work chamber is not considered an isolated system, but interacting with the external environment, the level of temperature (the reference value) may increase or decrease, in accordance with the thermal flow modifications. So, for a qualitative (at least) description of the whole process, the
The topology of the initial Petri net model will be extended by adding new specific elements (Fig. 9). The complementary sub-net is a hybrid structure too, with a continuous transition \( T_8 \) and four discrete elements \( P_9, P_{10}, T_9, T_{10} \). At the initial moment, the marking of \( P_{10} \) validates the \( T_9 \) transition, which will be fired after it laps time \( -d_1 \) associated. After this firing, the non-null marking of \( P_9 \) authorizes the continuous firing, with \( v \) speed of \( T_8 \) and the value of work chamber temperature associated to \( P_2 \) place will be modified. In Fig. 10 is shown the results of the simulation scenario, in this situation.

![Fig.9 Modified Petri net model](image)

**Fig.9 Modified Petri net model**

Another possibility of modeling the influence of the external perturbations in the behavior of the system is
to use for the sub–nets added a discrete or hybrid stochastic elements, which allows obtaining the quantitative interesting results. The most mathematical, textual or graphical approaches to describe hybrid systems are currently usable for small examples. Models of complex systems are unwieldy. Therefore a hierarchical concept to structure a model is needed. In order to solve the mentioned handling problems arising from the system complexity, in (Drath 1997a, 1998; Drighiciu 2003) was proposed a object oriented paradigm for the analysis of the Hybrid Petri Nets, resulting in a new method to describe both continuous and discrete event systems with reduced effort. One of the important advantages of the using this concept is the ability to describe a larger system by the decomposition into interacting objects. Because of the properties of objects, the modification of the system model could be easier achieved. The object-oriented concept unites the advantages of the modules and hierarchies and adds useful concepts like reuse, encapsulation and information hiding. In this way we get more flexibility.

3. CONCLUSIONS

The main idea of this paper was to consider an temperature control process such a particular hybrid system, with autonomous state commutation and, starting of this point, to find a specific frame for describing his behavior. A mathematical model of such a process has thus to be a hybrid model involving discrete variables (integers or with a domain in a finite set) and continuous variables (real numbers). Both dynamics (discrete and continuous) have to be modeled: a discrete event based dynamics for discrete variables (sequence of operations) and a continuous time dynamics for continuous variables (differential algebraic equations). The general approaches have strong similarities with hybrid automata, but the discrete dynamics is represented by Hybrid Petri Nets instead of automata, in order to address in an explicit way resource allocation policies. True concurrency is indeed required and the interleaving semantics of automata based approaches is not sufficient.

Adding news elements, with a grown power of analysis, may enrich the general model proposed. Hence, in the hybrid Petri Net model achieved, a first step to increase the design details was made by the use of variables delays associated to the discrete transitions. The concept “variable delay” is more often utilized in a determinist way. The delays are in any time defined in occurrence with an external event, a priori estimated, which may be generated by the decisional structure of the entire system. Another step in refining the initial model may be the use of powerful extensions of basic Petri Nets models, i.e. stochastic, colored or fuzzy Petri Nets. The integration of different process types in complex systems is rapid increased by the combination of dynamic influences of each process. Therefore, the hybrid object nets architecture can be used to get more flexibility compared with the common approximation principles.

4 REFERENCES


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