MULTI-LAYERED SPHERICAL MAGNETIC SHIELDING

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***Abstract* – Carefully planned magnetic shielding can protect objects and beings from the influence of magnetic fields in the whole frequency spectrum. In the paper it is shown that the most efficient magnetic shielding can be performed by a sphere of more ferromagnetic layers with non-magnetic material in between, which reduces the quantity of ferromagnetic material to minimum.**

Keywords*: *electromagnetic fields, ferromagnetics, shielding.

1. INTRODUCTION

Many of electromagnetic equipment generate strong magnetic fields in the environment. Therefore, it is often necessary to protect electrical appliances, as well as human beings from such strong fields [1].

The efficient electromagnetic shielding is performed by placing a conductive material of the certain thickness around the object that has to be protected. The shield thickness depends on frequency of the electromagnetic energy and on the conductivity of its material. The appropriate magnetic shielding is achieved exclusively by ferromagnetic materials. Not only diverse electrical appliances, but also human beings have to be protected when exposed to very strong magnetic fields. The most efficient protection can be ensured by a spherical shape but also using some other closed surfaces.

The exact solution of a ferromagnetic sphere in a homogeneous magnetic field is calculated in [2] and that of an ellipsoid and cylinder in [3]. The solutions are based on Laplace equation $\nabla^2\varphi = 0$, where φ is scalar magnetic potential.

The ferromagnetic sphere in a homogeneous magnetic field H_0 behaves like a magnetic dipole [4]. The sphere divides the space in two parts, each with their own permeability μ_1 and μ_2 (Fig. 1).

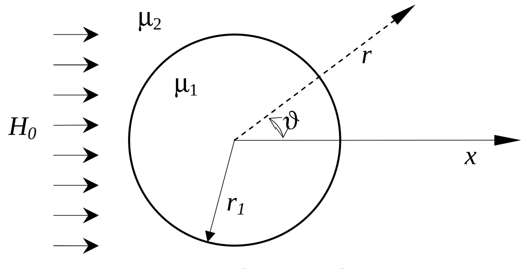

Fig.1: Feromagnetic sphere in a homogenous magnetic field H_0.

Magnetic field inside the sphere ($0 \le r \le r_1$) is also homogeneous and less than H_0, actually it is $H_0 + H_1$. The scalar magnetic potential can be written:

$$\varphi_1 = -(H_0 + H_1)\, r \cos\vartheta \tag{1}$$

Magnetic scalar potential for the outer space ($r \ge r_1$) can be written on the basis of the analytical solution as:

$$\varphi_2 = -H_0\, r \cos\vartheta + \frac{p\, \cos\vartheta}{4\pi\, \mu_2\, r^2} \tag{2}$$

where p is a dipole magnetic moment of a magnetic sphere, taken analoguely to a dielectric sphere:

$$p = \frac{\mu_1 - \mu_2}{\mu_1 + 2\mu_2} 4\pi\, r^3 H_0 \tag{3}$$

Direction of a homogeneous magnetic field H_0 is the direction of the dipole magnetic moment.

The exact solution of a hollow ferromagnetic sphere in a homogeneous magnetic field is given in [3,2]. Of course, if the approach with the dipoles is applied, it will result in the same solution. The protection factor of other closed objects can be calculated.

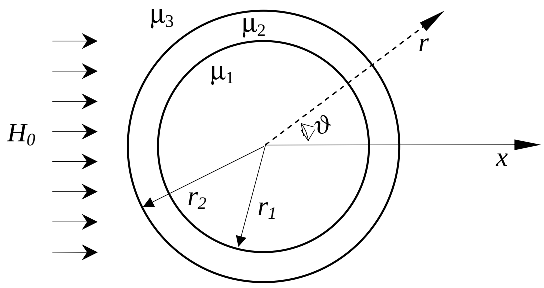

Fig.2: Hollow feromagnetic sphere in a homogenous magnetic field H_0.

The hollow magnetic sphere divides the space with its shells in three parts, with permeabilities μ_1, μ_2 and μ_3 (Fig. 2). The hollow sphere consists of two concentric layers; its behaviour can be seen as a superposition of the two adjacent layers. This means that the both spheres become dipoles, when influenced by the outer magnetic field. Their dipole moments are p_1 and p_2. So, in the area 1, with $r < r_1$, the homogeneous magnetic field has the value of $H_0 + H_1 + H_2$; in the area 2, where $r_1 \le r \le r_2$ the magnetic field is $H_0 + H_2$

with the superposition of the field of the dipoles p_1 of the inner sphere. In the area 3, where $r \geq r_2$ the field is the sum of the homogeneous field H_0 and the fields of the dipoles p_1 and p_2 of the both spheres.

Scalar magnetic potentials for the three areas are given as follows:

$$\varphi_1 = -(H_0 + H_1 + H_2) r \cos\vartheta \ , \ r \leq r_1 \qquad (4a)$$

$$\varphi_2 = -(H_0 + H_2) r \cos\vartheta + \frac{p_1 \cos\vartheta}{4\pi\mu_2 r^2} \ , \ r_1 \leq r \leq r_2 \qquad (4b)$$

$$\varphi_3 = -H_0 r \cos\vartheta + \frac{(p_1 + p_2)\cos\vartheta}{4\pi\mu_3 r^2} \ , \ r \geq r_2 \qquad (4c)$$

In order to determine the assumed quantities H_1, H_2, p_1 and p_2, the boundary conditions valid on $r = r_1$ and $r = r_2$ have to be enforced. The chosen boundary conditions refer to the equality of the normal components of the magnetic induction ($\vec{n} \cdot \mu \cdot \partial\varphi/\partial r$), as well as of the tangential components of the magnetic field ($\partial\varphi/r\partial\vartheta$) on the both sides of the boundary.

Thus, the unknown quantities of H_1, H_2, p_1 and p_2 are given:

$$H_1 = 3H_0 \left(\frac{r_2}{r_1}\right)^3 \frac{\mu_3(\mu_2 - \mu_1)}{D} \qquad (5a)$$

$$H_2 = -H_0 \frac{(\mu_2 - \mu_3)\left[\left(\frac{r_2}{r_1}\right)^3 (\mu_1 + 2\mu_2) - 2(\mu_2 + \mu_1)\right]}{D} \qquad (5b)$$

where D is

$$D = \left(\frac{r_2}{r_1}\right)^3 (\mu_1 + 2\mu_2)(\mu_2 + 2\mu_3) - 2(\mu_2 - \mu_1)(\mu_2 - \mu_3)$$

and V_2 is the outer sphere volume

$$V_2 = \frac{4\pi}{3} r_2^3$$

If we let $r_1 \to 0$, i.e. that the inner sphere dissappears, then the result for alone sphere is obtained.

2. MULTILAYERED FERROMAGNETIC SPHERE IN A HOMOGENEOUS MAGNETIC FIELD

It will be shown that the most efficient protection from the magnetic fields is the layered shielding. The efficiency of the protection is proven by an application of multilayered concentric spheres (Fig. 3). Sphere shells 1,2,3..n represent boundaries between ferromagnetic layers. Outer homogeneous magnetic field H_0 polarizes the concentric spheres so that they begin to behave like magnetic dipoles. As it is already shown by equation (4), the field in the boundary sphere consists of a homogeneous field as well as of a dipole field of all the spheres, that are inside the observed sphere.

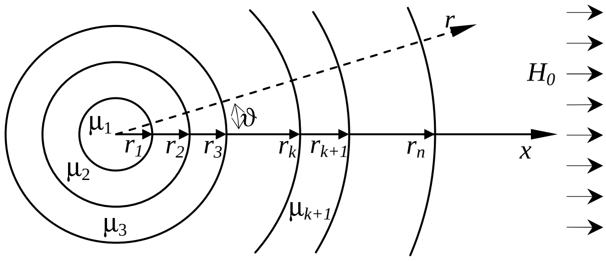

Fig.3: Multilayered ferromagnetic sphere in a homogenous magnetic field H_0.

For example, in the k-th layer the total field is a sum of a homogeneous field H_0 and of homogeneous fields of all the k-th to n-th spheres, as well as of the dipole fields of all the inner spheres (from 1 to (k-1)).

Therefore, the scalar potential of the k-th layer can be written as:

$$\varphi_k = -\left(H_0 + \sum_{i=k}^{n} H_i\right) r \cos\vartheta + \frac{\sum_{i=1}^{k-1} p_i}{4\pi\mu_k r^2} \cos\vartheta, \ r_{k-1} \leq r \leq r_k \qquad (7)$$

Let the permeabilities of the layers be different, having indices of the outer boundary sphere. The equation (7) is valid for all the layers from 1 to n, as well as for the space beyond the n-th layer. The unknowns are magnetic fields H_i and dipoles p_i (for i = 1,...n).

In order to determine 2n unknowns, it is necessary to solve 2n equations, which are given from the boundary conditions at all n boundaries:

$$\frac{\partial\varphi_k}{r\partial\vartheta} = \frac{\partial\varphi_{k+1}}{r\partial\vartheta} \qquad (8a)$$

$$\mu_k \frac{\partial\varphi_k}{\partial r} = \mu_{k+1} \frac{\partial\varphi_{k+1}}{\partial r} \qquad (8b)$$

with $r = r_k$.

The 2n linear equations thus obtained can be written in the matrix form [5]

$$\begin{bmatrix} A & B \\ \tilde{B} & C \end{bmatrix} \cdot \begin{bmatrix} H \\ p \end{bmatrix} = -H_0 \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad (9)$$

The matrices $A = |a_k|_n^n$ and $C = |c_k|_n^n$ are diagonal matrices of the n-th order with diagonal terms

$$a_{kk} = \frac{3V_k \mu_k \mu_{k+1}}{\mu_k - \mu_{k+1}} \tag{10}$$

$$c_{kk} = \frac{-2}{3V_k(\mu_k - \mu_{k+1})} \tag{11}$$

where V_k is a volume of the k-th sphere, μ_k permeability of the layer in the k-th sphere and μ_{k+1} permeability of the layer out of the k-th sphere.

The matrix **B** is also triangular matrix of the n-th order with the terms

$$b_{kk} = \frac{\mu_k}{\mu_k - \mu_{k+1}},\ b_{jk} = 1(\forall\ j > k), b_{jk} = 0(\forall\ j < k) \tag{12}$$

Matrix $\tilde{B}$ is a transposed matrix **B**.

Matrices $H = |H_k|_n^1$; $p = |p_k|_n^1$; $1 = |1_k|_n^1$; $0 = |0_k|_n^1$ are vectors with n components. The elements of vectors **H** and **p** are unknown homogeneous magnetic fields (H_i) and unknowns magnetic dipoles (p_i). By solving equation (9) the following is given

$$p = -B^{-1}AH \tag{13}$$

$$SH = -H_0 1 \tag{14}$$

where

$$S = B - CB^{-1}A \tag{15}$$

the auxiliary quadratic matrix of n-th order

$$s_{kk} = \frac{\mu_k + 2\mu_{k+1}}{\mu_k - \mu_{k+1}},\ s_{jk} = 1(\forall\ j < k),\ s_{jk} = \frac{-2 \cdot V_k}{V_j}(\forall\ j < k) \tag{16}$$

V_k and V_j are sphere volumes.

The unknown quantities of the field and dipoles are calculated by solving equations (13) and (14). The magnetic field protection factor is defined as a ratio of the homogeneous field inside the sphere and outer homogeneous field H_0:

$$Z = H_0 + \sum_{i=1}^{n} H_i \Big/ H_0 \tag{17}$$

The inner space is the one that has to be protected from the outer strong magnetic field H_0.

3. FERROMAGNETIC LAYERED SPHERE

The layered sphere consists of more concentric ferromagnetic spherical layers with air in between. Air is also inside the first and outside the last layer. It is assumed that the relative permeability of all the ferromagnetic layers is the same and very large (μ' >>1). In this case the equations can be simplified, because for the odd-layered spheres (k = 1,3,5,..,2n-1) permeability $\mu_k = \mu_0$, and for all the even layered (k = 2,4,6,...,2n) the permeability is $\mu_k = \mu' \mu_0$.

Thus, the diagonal members of the S-matrix (16) are

$$s_{kk} = \frac{1+2\mu}{1-\mu} \qquad (k = 1,3,5, ..., 2n\text{-}1)$$

$$s_{kk} = \frac{\mu+2}{\mu-1} \qquad (k = 2,4,6, ..., 2n) \tag{18}$$

with the unchanged other members (s_{jk}) of the matrix (16).

For example, in the case of one ferromagnetic layer, there are two boundary spheres V_1 and V_2, so the equation (14) can be written as

$$\begin{bmatrix} \frac{1+2\mu'}{1-\mu'} & 1 \\ -\frac{2V_1}{V_2} & \frac{\mu'+2}{\mu'-1} \end{bmatrix} \cdot \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} = -H_0 \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{19}$$

By combining (19) and (17) with the condition μ' >> 1, the magnetic protection factor for the sphere (eq. 17) with one ferromagnetic layer is given

$$Z_1 = \frac{9}{2\mu'} \frac{1}{1 - \frac{V_1}{V_2}} \tag{20}$$

Sphere with two ferromagnetic layers of the same material has four boundary spheres, so its S matrix is:

$$S = \begin{bmatrix} \frac{1+2\mu'}{1-\mu'} & 1 & 1 & 1 \\ -\frac{2V_1}{V_2} & \frac{\mu'+2}{\mu'-1} & 1 & 1 \\ -\frac{2V_1}{V_3} & -\frac{2V_2}{V_3} & \frac{1+2\mu'}{1-\mu'} & 1 \\ -\frac{2V_1}{V_4} & -\frac{2V_2}{V_4} & -\frac{2V_3}{V_4} & \frac{\mu'+2}{\mu'-1} \end{bmatrix} \tag{21}$$

The protection factor for the two ferromagnetic layers can be calculated in the same way as before, with the same condition μ' >> 1

$$Z_2 = \left(\frac{9}{2\mu'}\right)^2 \frac{1}{\left(1-\frac{V_1}{V_2}\right)\left(1-\frac{V_2}{V_3}\right)\left(1-\frac{V_3}{V_4}\right)} \tag{22}$$

From (22) it can be seen that the efficient magnetic protection can be performed by ferromagnetic materials of a high relative permeability μ'. Also, it is obvious that the layers thicknesses influence the protection. If the magnetic protection in the hollow one-layered sphere should be improved, then the new ferromagnetic layers with air in between should be added.

If the same ratio of the neighbouring spheres radii is assumed

$$\frac{r_1}{r_2} = \frac{r_2}{r_3} = \frac{r_3}{r_4} \qquad (23)$$

then the protection factor can be written as

$$Z_2 = \left(\frac{9}{2\mu'}\right)^2 \frac{1}{k^3} \qquad (24)$$

with the k as a ratio constant

$$k = 1 - \frac{V_1}{V_2} = 1 - \frac{V_2}{V_3} = 1 - \frac{V_3}{V_4} \qquad (25)$$

For example, if the outer field in a sphere with radius r_1 = 250 mm should be decreased to 1 % of its value by a ferromagnetic material of a relative permeability μ' = 1000, one-layered sphere of a thickness d_1 = 55 mm is needed. The layer thickness, which implies amount of material, could be smaller by taking ferromagnetic material of a higher permeability. Also, reduction of a material can be achieved by a two-layered ferromagnetic spheres. For the same protection factor and for the same material with the assumption (23), the ratio constant k = 0.1266. The total thickness of the both membranes is 24.2 mm, instead of 55 mm and the volume of the used magnetic material is 2.43 times less than in the first case.

Ferromagnetic materials are also very applicable for the electromagnetic fields protection at higher frequencies, because the penetration depth is decreasing with increasing relative permeability μ'. Thus, for the required relative permeability μ' = 1000, radius r_1 = 250 mm and electric conductivity σ = 1.5 x 10^4 S/m the penetration depth at frequency of 50 Hz is 1.83 mm. If the same field should be decreased to 1 % of its value, the ferromagnetic material thickness should be around 8.4 mm.

4. CONCLUSIONS

Magnetic shields are used for the protection of some electrical appliances and human beings from the strong magnetic fields. One of the most efficient object protection is achieved by shields shaped like a sphere. The recommended sphere consists of several ferromagnetic layers made from highly permeable magnetic materials. Combination of ferromagnetic and air (non-magnetic) layers can reduce magnetic fields inside the sphere, as required. If a ferromagnetic material is not a highly permeable enough, this deficiency can be avoided by several ferromagnetic layers. It is shown through a calculus that in order to achieve the same degree of protection factor, the expense of needed ferromagnetic material is considerably less for two layers. The magnetic shields are also more efficient at higher frequencies because the penetration depth is decreasing with increasing permeability.
The protection can be also performed by other closed shapes or surfaces, not only sphere, but all the three dimensions should be equal related to the sphere diameter.

References

[1] K. L. Kaiser, *Electromagnetic Shielding*, CRC Press, Boca Raton, 2005.

[2] J. D. Jackson, *Classical Electrodynamics*, John Wiley & Sons, New York, 1998.

[3] J. A. Stratton, *Electromagnetic Theory*, McGraw-Hill, New York, 1941.

[4] H. Kaden, *Wirbelströme und Schirmung in der Nachrichtentechnik*, Springer, Berlin, 2006.

[5] E. Kreyszig, *Advanced Engineering Mathematics*, John Wiley & Sons, New York, 1999.