A STUDY ABOUT THE AC SWITCHING REGIME OF A
SINGLE-PHASED TRANSFORMER CHARACTERIZED BY
A NON-LINEAR MAGNETIZATION CURVE WITH
PIECEWISE-LINEARISED HYSTERESIS CYCLE

Cristian-George CONSTANTINESCU 1, Constantin STRÎMBU 2

1 “Henri Coandă” Air Force Academy Brașov, Romania, ccg_cristi@yahoo.com
2 “Henri Coandă” Air Force Academy Brașov, Romania, strimbu_c@yahoo.com

Abstract - The goal of this paper is the steady state study of the periodical a.c. switching mode of a single-phase RL loaded transformer. It was considered a non-linear magnetic core with hysteresis cycle, its modelling starting from the fundamental magnetisation curve, experimentally determined in laboratory conditions. Three kinds of algorithms were developed in this purpose. The first one generates the hysteresis cycle, considered to consist in curves parallel with the fundamental one, shifted in the right or left side. The second one calculates the magnetic quantities and the third one the electric quantities, as well as the switch state (On or OFF). The resulted waveforms (the primary and secondary currents), accompanied by comparisons with experimental tests, are inserted in the paper as a conclusion. The mathematical support is MathCAD PLUS 6.0.

Keywords: ac switching, transformer, non-linear magnetisation curve, hysteresis cycle.

LIST OF SYMBOLS USED IN THE PAPER

(1) αH - the initial angle (moment) for a line of the magnetisation curve;
(2) β - the turn-off angle of the switch;
(3) 11α - the initial value of the primary current in the αH moment;
(4) 12α - the initial value of the secondary current in the αH moment;
(5) Ucα - the initial value of the capacitor voltage in the αH moment;

1. INTRODUCTION

In a.c. switching regime, the operation of the transformers differs from the "stationary" (no switched) one, because it implies non-symmetrical and no-sinusoidal currents and consecutively dc component of the magnetic field. Owing this, a premagnetisation in the core will be achieved, so the transformer may work in the saturation zone of the magnetisation curve.

Figure 1: The current rectifier

Figure 2: Hysteresis modelling

Regarding these reasons, the paper will present only the operation of the transformer with a single-pulse rectifier in the secondary coil, which the worst case concerning the currents waveforms consists in. The circuit diagram is shown in Fig. 1. It was considered a thyristor-type switch.

2. GENERATING THE HYSTERESIS CYCLE

The procedure developed for generating the hysteresis cycle will be shortly presented. Its input data is the fundamental magnetization curve of the core material, experimentally determined and piecewise linearised. The hysteresis cycle width is assumed to be proportional with the magnetic induction corresponding to the turning point (P, see Fig. 2; the sense of tracing the curve is changing).

Figure 2: Hysteresis modelling
It results a narrow hysteresis cycle corresponding to the quasi-linear zone and a broad one for the non-linear (including saturation) zones;

The hysteresis cycle is assumed as curves parallel with the fundamental one, the distance between them being in concordance with the above-mentioned (the red curve in Fig. 2, then the blue one).

An intermediate line (PP1 or QQ1 in Fig. 2) links these curves. A slope proportional with the magnetic permeability in the turning point characterizes this line.

Fig. 2 shows how this algorithm works: the point will trace the curve OP, then the red curve is generated, it is computed the point P1 or P2, and the portion PP1Q is traced. Here is generated a new curve (the blue one) and the portion QQ1(3) will be traced (in ascendant sense) and so on.

2.1 PROCEDURES USED TO GENERATE THE HYSTERESIS CYCLE

The fundamental magnetisation curve was linearised using the procedure CoefLines, whose logical diagram is shown in Fig. 3.

```
START CoefLines

f

k ← 0
i ← 1

YES

i ≤ last(f)

NO

STOP

m ← \frac{f_i - f_{i+1}}{\mu_i - \mu_{i+1}}

\mu_i ← \frac{f_i - f_{i+1}}{\mu_i - \mu_{i+1}}

A ← \begin{bmatrix} a_1 & a_2 & \cdots & a_n \\ b_1 & b_2 & \cdots & b_n \end{bmatrix}

k ← k+1
i ← i+1

STOP
```

Figure 3: Logical diagram of the linearization procedure

The input data is a two-column matrix, f, containing the experimentally determined values: the magnetic field in the first column and the magnetic induction in the second one. The procedure returns a matrix, containing the magnetic field values in the first column and the lines parameters (slope and intercept) in the next two ones. These parameters have dimensions: the slope is a magnetic permeability and the intercept is a magnetic induction.

The fundamental magnetisation curve has an inflexion in the origin point, which has to be eliminated for the hysteresis cycle. This goal is achieved using the procedure SmoothCurve, shown in Fig. 4.

In this way, the point on the magnetisation curve will step the fundamental curve only in the first period. The biasing voltage being sinusoidal, the magnetic induction will be likely, so the magnetic field time variation will be characterised by peak values. When the magnetic field is reaching a peak value, the sense of the curve pursuit is changing, so it is necessary to generate the hysteresis curve. As mentioned above, this curve is considered to be parallel with the fundamental one, excepting the inflexion in the origin zone. So, the curve that will be shifted is the one returned by the procedure SmoothCurve.

```
START SmoothCurve

M

i ← 0; \mu ← (M_i)

YES

i ≤ max(M)

NO

STOP

j ← i+1; b ← (M_j)

YES

j ≤ max(M)

NO

STOP

h ← \frac{m+b}{n}

\mu ← \frac{m+b}{n}

STOP
```

Figure 4: Logical diagram of the procedure eliminating the inflexion in origin area

Its input data are M, a matrix containing the linearization of the fundamental curve (returned by the
procedure CoefLines and Bhpoz, a matrix containing the positive values of the fundamental curve.

Fig. 5 shows the logical diagram of the procedure (NewCurve) that shifts the magnetisation curve. The input data of this procedure are \( A \), the matrix returned by the function SmoothCurve, \( I_p \), a variable associated with the sense of movement on the curve (\( I_p = 1 \) for the ascendant sense and \( I_p = -1 \) for the descendent one) and \( \Delta h \), which is the magnitude of the curve shifting. For \( \Delta h \) a linear variation between 0 and coercive value \( H_c \) is proposed, assuming that the value \( H_c \) is corresponding to the saturation zone (\( B_{sat} = 1.7T \)). The proportionality constant is the value of magnetic induction in the turning point:

\[
\Delta h = \frac{B}{B_{sat}} - H_c
\]

The former and the new curves have to be linked. This will be made by a linking line, starting from the turning point on the former curve, with a \( n_\mu \) times lower slope (magnetic permeability) and ending when it is crossing a line of the new curve. This task has to be fulfilled by the procedure NewLine, whose logical diagram is shown in Fig. 6.
Finally, the main function that manages the magnetisation curve with hysteresis cycle can be built, as the logical diagram from Fig. 7 is showing.

This function is calling the above-mentioned procedures and returns a matrix containing the linearization of the further “Work Curve”, starting from or ending in the turning point, according to the iP value (i.e. according to the sense of the movement on the curve.)

```
START WorkCurve

WL, B1, iP, Δh, h, b, np

H ← NowCurve(WL, iP, Δh)
N1 ← CoalLines(N)
FL ← NowLine(b, Δh, iP, np, E1, N1)
A ← submatrix(Fl, 0, rows(Fl) - 1, 0, cols(Fl) - 2)

YES

iP = 1

NO

A1 ← submatrix(N1, 0, (FL−1)h, 0, cols(N1) - 1)
M ← stack(A1, A)

ΔA1 ← submatrix(N1, (FL−1)h + 1, rows(N1) - 1, 0, cols(N1) - 1)
M ← stack(A, ΔA1)

STOP
```

Figure 7: Logical diagram of the main function to update the magnetisation curve

3. PRIMARY CURRENT WAVEFORMS

Once having a linearised magnetisation curve, the transformer will be a linear one on each line of the curve, so the equations will be the same with ones inserted in [7] and [9]. Although, the curve is non-linear (only piecewise linearised), so a continuation method is necessary. It is the same with the one described in [10]. Basically, there are two subjects in discussion: to find out the moments when the point reaches the end of the actual line of the magnetisation curve (task fulfilled by the procedure NextStep) and to evaluate the electrical quantities initial values and the switch state (task fulfilled by the procedure Eval).

Only the main function, that manages the full calculus process will suffer a little change, because it has to call an additional function, WorkCurve, at the right times (i.e., when the NextStep function signalise that a peak value of the magnetic field was reached).

Either in [7, 9] or in [10], the load current theoretical waveform is satisfactory enough if it is compared with an experimentally determined one.

This is the reason of non-introducing in this paper such waveforms, but only the primary current ones, which the main problem consists in. Some of the obtained waveforms are inserted in Fig. 8, as comparisons between theoretical and experimentally determined ones.
4. CONCLUSIONS

The experiments prove the accuracy of our calculus. Comparative with [9] and [10], the primary current presents a significantly lower error (the maximum one is about 5%), regarding the turn-off angles and (specially) amplitudes. Certain improvements in the theoretical waveform shape can be observed as well. For instance, in the waveform shown in Fig. 8b, it can be observed the approach between the two waveforms in the second part of the turn-off interval. But it still remains a problem: the primary current values (I_{II}) in the switch turn-off moment does not match at all with the experimentally determined ones. It is not an easy problem, because it has many directions to face:

- To not consider the switch ideal anymore. (In this direction, only a non-null value of the holding current, I_{H}, was considered).
- To find another models of the magnetisation curve, instead the piecewise linearised one.

References