ABOUT MATHCAD MODELING OF BUCK PWM AC REGULATORS

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Abstract – **This paper is focused on predicting the power transfer in Buck PWM AC regulators. The circuit simulation and results prediction were carried out using specific computational tools such MathCAD. The commutation process is studied and the article presents a method to establish the mathematically model of the power transfer. Comparative evaluations with experimentally determinations are presented.**

1. INTRODUCTION

AC regulators (AC-AC converters) are used to obtain variable ac voltage from a fixed ac source. AC regulators with thyristor and transistors are commonly used in industrial practice. Thyristor phase control has major disadvantages like: generation of high harmonics in the source current, generation of sub harmonics at integral control and a displacement power at phase angle. One method to eliminate these unfavorable properties is a PWM or APWM AC line power control. The equivalent power circuits [1] of PWM Buck regulator in two operating modes are shown in fig.1.

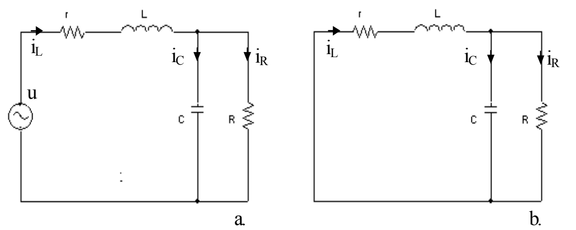

Fig. 1. The equivalent power circuits of PWM Buck regulator

2. CONTROL MODES OF LOAD CURRENT AND VOLTAGE

When control signal is applied to the AC converter switches, a PWM output voltage appears. To determine the power transfer in AC-AC converters it is used the MathCAD functions. The calculus is based on frequency switching and duty factor functions. The switching frequency is 20kHz and 200 is number of turn on commands.

The algorithm is presented in fig.2.

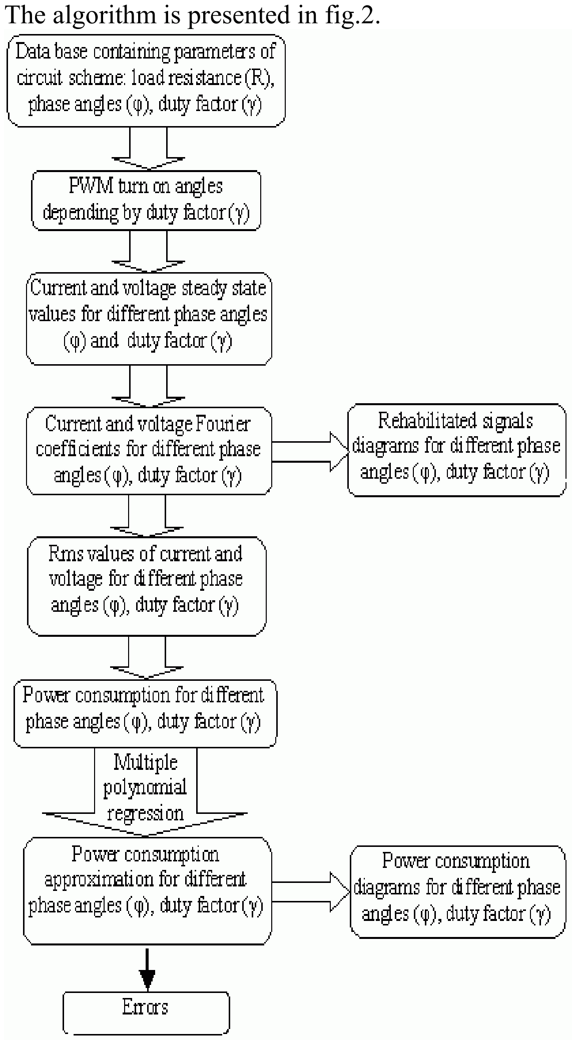

Fig. 2. The calculus algorithm

The power regulation can be made with phase angle control for one constant duty factor [2], fig. 3 or with duty factor variation for constant phase angle.

3. MATHEMATICAL DETERMINATION

The circuit operating is not easy to characterize with mathematical expressions. It is necessary to determine the initial steady state values. The MathCAD function is "CondInit (M, Φ, ε)". The MathCAD function in fig.6. is presented. "M" is the matrix that returns the turn on and turn off angles. "Φ" is the phase angles matrix. "ε" is the precision.

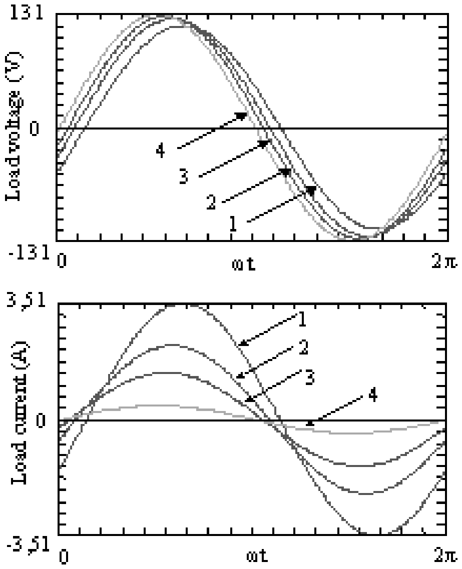

Fig. 3. The load waveforms for duty factor $\gamma = 0,4$and different phase angles: 1) $\varphi = 6^o$; 2) $\varphi = 10^o$; 3) $\varphi = 16^o$; 4) $\varphi = 40^o$.

The experiments are realized to verify the mathematical determination. Fig. 5. and 6. presents the comparative evaluation between analytical determined and experimental capacitor and inductive waveforms.
348V is the experimental peak-to-peak load voltage value and 356V is the peak-to-peak load voltage value determined with MathCAD function.
The error, 2.24%, is admissible because the mathematical model circuit is ideal.
The load voltage waveforms collation in fig. 7. is presented.
Because the load current (fig.4, fig.5) and the load voltage are not sinusoidal, to estimate the power transfer it's necessary to determine the load current and voltage Fourier coefficients for 100 harmonics.

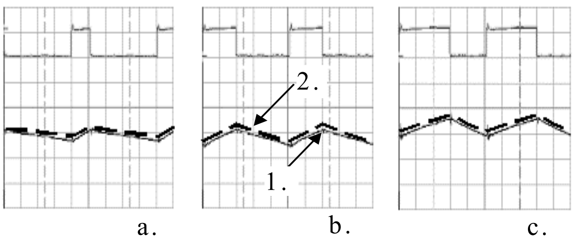

Fig. 4. The capacitive current waveforms a) $\gamma = 0,2$; b) $\gamma = 0,4$; c) $\gamma = 0,6$and 1.experimental waveform; 2.analitical determined waveform.

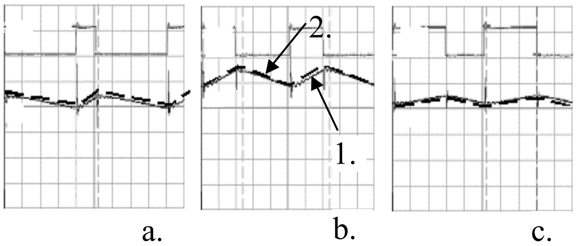

Fig. 5. The inductive current waveforms a) $\gamma = 0,2$; b) $\gamma = 0,4$; c) $\gamma = 0,6$and 1.experimental waveform; 2.analitical determined waveform.

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CondInit(M, Φ, ε) := δ ← ε + 1
  for i ∈ 0..rows(M) − 2
    for j ∈ 0..2·length(Φ)·cols(M) − 1
      I_{i,j} ← 0
  while δ > ε
    c ← 0
    for n ∈ 0..cols(M) − 1
      for f ∈ 0..last(Φ)
        Iα ← (I^<c>)_0
        Uα ← (I^<c+1>)_0
        for i ∈ 0..rows(M) − 2
          if mod(i, 2) = 0
            I1_{i+1,c} ← iLαβ(M_{i+1,n}·ω^-1, r, L, C, Φ_f, M_{i,n}, Iα, Uα, Sαβ(Φ_f))
            I1_{i+1,c+1} ← uCαβ[(M^<n>)_{i+1}·ω^-1, r, L, C, Φ_f, (M^<n>)_i, Iα, Uα, Sαβ(Φ_f)]
          if mod(i, 2) = 1
            Iα ← iLβα(M_{i+1,n}·ω^-1, r, L, C, Φ_f, M_{i,n}, I1_{i,c}, I1_{i,c+1}, Sβα(Φ_f))
            Uα ← uCβα(M_{i+1,n}·ω^-1, r, L, C, Φ_f, M_{i,n}, I1_{i,c}, I1_{i,c+1}, Sβα(Φ_f))
            if i ≠ rows(M) − 2
              I1_{i+1,c} ← Iα
              I1_{i+1,c+1} ← Uα
            if i = rows(M) − 2
              I1_{0,c} ← Iα
              I1_{0,c+1} ← Uα
        c ← c + 2
    δ1 ← |max(I1 − I)|
    δ2 ← |max(I − I1)|
    δ ← if(δ2 > δ1, δ2, δ1)
    I ← I1
  I
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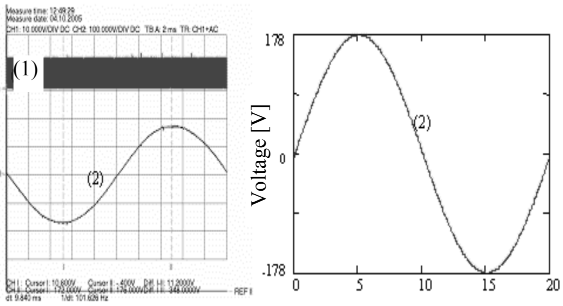
Fig. 6. The MathCAD function



a. b.

Fig. 7. Load voltage waveforms collation a. experimental waveform; b. analitical determined waveform.

Fourier coefficients are:

$$u(t) = U_0 + \sum_{1}^{\infty} U_k \cos(k\omega t + \varphi_{uk}) \qquad (1)$$

$$i(t) = I_0 + \sum_{1}^{\infty} I_k \cos(k\omega t + \varphi_{ik}) \qquad (2)$$

$$\begin{cases} U_k = \sqrt{S_{uk}^2 + C_{uk}^2} \\ \varphi_{uk} = -arctg \dfrac{S_{uk}}{C_{uk}} \end{cases} \qquad (3)$$

$$\begin{cases} I_k = \sqrt{S_{ik}^2 + C_{ik}^2} \\ \varphi_{ik} = -arctg \dfrac{S_{ik}}{C_{ik}} \end{cases} \qquad (4)$$

MathCAD relationships for Fourier coefficients are:

$$C := \begin{cases} \displaystyle\sum_{k=0}^{last(Uc)-1} 2 \cdot f \cdot \left[\int_{\frac{Uc_k}{\omega}}^{\frac{Uc_{k+1}}{\omega}} fct1 \cdot \cos(n \cdot \omega \cdot t) \cdot (1 - mod(k,2))\, dt + \int_{\frac{Uc_k}{\omega}}^{\frac{Uc_{k+1}}{\omega}} fct2 \cdot \cos(n \cdot \omega \cdot t) \cdot mod(k,2)\, dt \right] & \text{if } n>0 \\ \displaystyle\sum_{k=0}^{last(Uc)-1} f \cdot \left[\int_{\frac{Uc_k}{\omega}}^{\frac{Uc_{k+1}}{\omega}} fct1 \cdot (1 - mod(k,2))\, dt + \int_{\frac{Uc_k}{\omega}}^{\frac{Uc_{k+1}}{\omega}} fct2 \cdot mod(k,2)\, dt \right] & \text{otherwise} \end{cases}$$

$$S := \sum_{k=0}^{last(Uc)-1} 2 \cdot f \cdot \left[\int_{\frac{Uc_k}{\omega}}^{\frac{Uc_{k+1}}{\omega}} fct1 \cdot \sin(n \cdot \omega \cdot t) \cdot (1 - mod(k,2))\, dt + \int_{\frac{Uc_k}{\omega}}^{\frac{Uc_{k+1}}{\omega}} fct2 \cdot \sin(n \cdot \omega \cdot t) \cdot mod(k,2)\, dt \right]$$

where $Uc = M^{<c>}$ are the turn on and turn off angles values.

Function "fct1" can be the load current expression, the source current or load and source voltage expressions in turn on time. Function "fct2" is the turn off expression of current and voltage of different electrical quantities. The rehabilitated load waveforms, with (5), denotes the uprightness of Fourier coefficients calculus.

$$Sr = \sqrt{2} \sum_{k=1}^{lastM} M_k^i \sin(k\omega t + M_k^{i+1}) + M_0^i \qquad (5)$$

where "M" is coefficients Fourier matrix (table 1) and M_0 is continuous component:

M =

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	1.571	$1.766 \cdot 10^{-6}$	1.571	0	1.571	0	1.571	-0.077	1.571	0	1.571	0
1	20.725	-0.459	0.619	-0.459	0.063	1.085	0.624	-0.358	39.033	0.041	0.175	-0.466	21.685
2	0	1.7	$2.567 \cdot 10^{-6}$	1.7	0	-1.441	0	-1.441	0.112	-1.441	0.001	-1.441	0
3	0	1.701	$2.567 \cdot 10^{-6}$	1.701	0	-1.441	0	-1.441	0.112	-1.441	0.001	-1.441	0
4	0	1.701	$2.569 \cdot 10^{-6}$	1.701	0	-1.441	0	-1.441	0.112	-1.441	0.001	-1.441	0
5	0	1.7	$2.57 \cdot 10^{-6}$	1.7	0	-1.441	0	-1.441	0.112	-1.441	0.001	-1.441	0
6	0	1.697	$2.566 \cdot 10^{-6}$	1.697	0	-1.441	0	-1.441	0.112	-1.441	0.001	-1.441	0
7	0	1.701	$2.57 \cdot 10^{-6}$	1.701	0	-1.441	0	-1.441	0.112	-1.441	0.001	-1.441	0
8	0	1.702	$2.572 \cdot 10^{-6}$	1.702	0	-1.442	0	-1.442	0.112	-1.441	0.001	-1.441	0
9	0	1.7	$2.568 \cdot 10^{-6}$	1.7	0	-1.441	0	-1.441	0.112	-1.441	0.001	-1.441	0

The matrix has sub matrixes with:
Column i = harmonics amplitudes corresponding to each signal;
Column i+1 = phase angle corresponding to each signal;
It wants to achieve the power balance. It must calculate the active power, P, reactive power, Q, apparent power, S. In fig.8. apparent power diagram is presented.
To achieve the equivalent circuit, it is necessary to determine one function, being a polynomial regression like expression (6).

$$p(\Gamma, \Phi) = \sum_{i=1}^{n} \left(\Gamma^i \cdot \sum_{j=1}^{m} b_j \Phi^j \right) \qquad (6)$$

Γ is matrix of duty factor values;
m, n are degree of polynomial regression;
b_j is coefficients of polynomial degree, like:

The ratio (7)

$$\eta = \frac{\textbf{load power}}{\textbf{source power}} \qquad (7)$$

represents the power transfer.

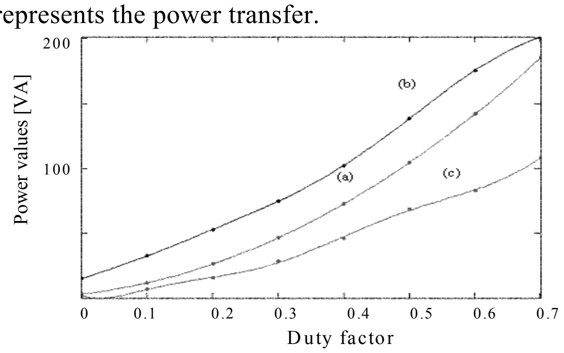


Fig. 8. Apparent power diagram: a. load resistance apparent power; b. source apparent power; c. capacitor apparent power

Coefficients of load apparent power, m=5, n=5:

$$CoefPuteriSR = \begin{bmatrix} 19.17710614 & -93.57487869000001 & 185.9675903 & -38.69723511 & -304.5333405 & 253.6326065 \\ -465.3423462 & 2270.834717 & -4514.127197 & 943.9150390999999 & 7382.684082 & -6150.942627 \\ 6807.371399 & -38441.38184 & 107539.9414 & -146894.6445 & 81001.99609 & -6994.716797 \\ -19486.2644 & 95040.54102 & -188624.3828 & 38226.3125 & 310997.9844 & -258519.3672 \\ 49003.55371 & -239161.2188 & 475584.0469 & -100093.3438 & -776463.7188 & 647231.8594 \\ -68355.4043 & 333968.1563 & -666259.0938 & 148800.6875 & 1.070059563\cdot 10^{6} & -896140.7813 \end{bmatrix}$$

Coefficients of source apparent power, m=4, n=5:

$$CoefPuteriS = \begin{bmatrix} -11221.45184 & 121363.3929 & -607811.2504 & 1.579755829\cdot 10^{6} & -2.000149175\cdot 10^{6} & 959403.0878 \\ 272440.3045 & -2.943024828\cdot 10^{6} & 1.473147165\cdot 10^{7} & -3.827657317\cdot 10^{7} & 4.845320604\cdot 10^{7} & -2.323855663\cdot 10^{7} \\ -2.483832518\cdot 10^{6} & 2.68274407\cdot 10^{7} & -1.342778889\cdot 10^{8} & 3.48879608\cdot 10^{8} & -4.41626648\cdot 10^{8} & 2.118047796\cdot 10^{8} \\ 1.133892927\cdot 10^{7} & -1.224301221\cdot 10^{8} & 6.127249045\cdot 10^{8} & -1.591872965\cdot 10^{9} & 2.014979647\cdot 10^{9} & -9.663627934\cdot 10^{8} \\ -2.841897399\cdot 10^{7} & 3.066697639\cdot 10^{8} & -1.534440936\cdot 10^{9} & 3.98598307\cdot 10^{9} & -5.045010953\cdot 10^{9} & 2.419408991\cdot 10^{9} \end{bmatrix}$$

In fig. 9., power transfer surface is presented. It perceives that the ratio η is nearly 1. In fig 10 power error approximation is shown.

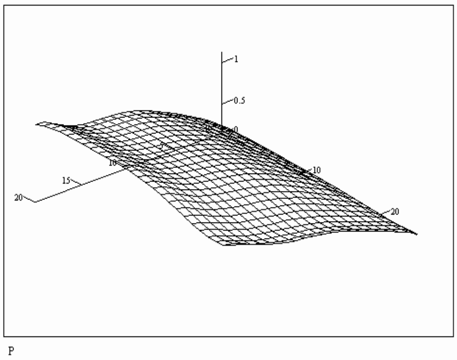


Fig. 9. Power surface for all duty factor and phase angles Φ=ct.

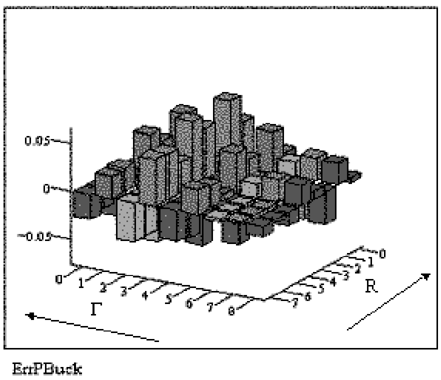


Fig. 10. Error values between power approximation and analytical determined power.

Fig. 11 shows the approximation of power transfer with the function deriving from (6).

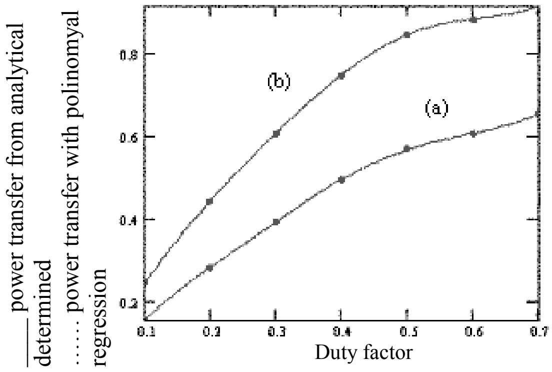


Fig. 11. Power transfer values: (1) Φ= 6°; (2) Φ= 35°

The errors between power from analytical determined and polynomial regression are smaller from 1%, like it sees in fig.12.

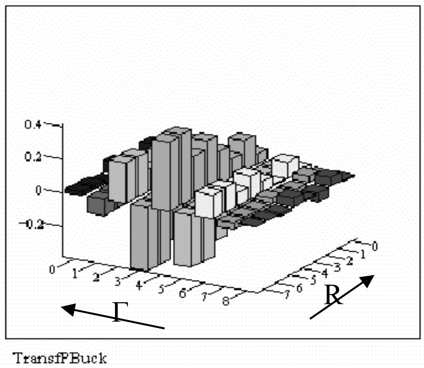


Fig. 12. Error values between power transfer approximation and analytical determined power transfer.

4. CONCLUSIONS

$-0.076\ \% \le \varepsilon \le 0.066\ \%$ is the power error approximation and $-0.3\% \le \varepsilon \le 0.4\%$ is the power transfer error approximation.

Power transfer polynomial regression is a function of phase angle and duty factor.

The error values are small, so the approximation is useful in regulator modeling.

References

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