RADIANT ENERGY DENSITY CALCULATION ON AN INCLINED SURFACE

Cristian BRATU, Felicia IVAN

University of Craiova, Faculty of Electric Engineering
cbratu@elth.ucv.ro, ely_felicia@yahoo.com

Abstract **- Solar energy is a renewable energy source that can solve (supply) the energy demand (power and heat) in a lot of places on earth.**
To design solar installation it is useful to know the radiant energy density.
The paper presents a methodology to evaluate the radiant energy density (annually) for an inclination surface.
Also, the paper includes a computer application developed on Visual Basic 6.0 that allow to calculate the direct, diffuse and reflected density energy that is on an incline surface.

Keywords: *radiant energy density, declination, sunrise hour angle*

1. INTRODUCTION

The energy is a mondial economy problem. The exhaustion reserve for oil, natural gases, coal, nuclear combustible and the irrational exploitation woods generates.
A challenge for the XXI century is the opportunity for each man to use the clean energy for a reasonable cost [1].
In many places on Earth, the Sun has capacity to solve the energy problems that are growing up along with a better life standard, but simultaneously with the exhaustion of the conventional sources [2].
In order to capitalization the radiant solar energy, it must be known the quantity that a surface can receive during a day, a month or for different geographical zone.
A disadvantage for the solar energy is that the Sun executes a nocturnal movement, and from there the captors need to be watched all the time.
On the other hand, this movement is not the same every day, because the Earth revolution around the Sun. Even the use of a plane captor that is usually fixed, it is useful to know the movement to evaluate the captor optimal inclination through the horizontal position.
Because the nocturnal Sun movement, the solar sunray are captained by Earth for different angles, changing from area to area, hour to hour, from season to season.
Among the meteorological factors, a particular influence on solar ground radiation it's being exerted by: the atmosphere transparency, turbidity, the sort of clouds, their thickness and position. Thus, the radiant energy can be:
- direct radiation B; this represents the radiation received from the Sun without being spread by the atmosphere.
- diffuse radiation D; emerging when the solar beam passing through the atmosphere is being spread, in other words, it's being diffuse in all directions.
- albedo or reflected radiation R; it is the radiation reflected by the earth surface, which drops on the solar collector.
- total solar radiation or global radiation G; it represents the sum of previous three components, that fell on a certain surface.

2. INCIDENT DENSITY RADIATION CALCULATION FOR AN ORIENTATED PLANE

The optimum solution could be the pursuance of the Sun in it's apparently movement on the celestial arch, so that the beam falls perpendicularly on the interest surface [1].
Solar systems with flat surfaces are built without pursuance. They are oriented to the south and installed under an optimal angle from the horizon for the specific location and the exploitation period along the year.
The optimum bent angle is approximated precisely sufficient with the relation:

$$\theta_{\hat{i}ncl} = \varphi - \delta \tag{1}$$

$\theta_{\hat{i}ncl}$ - the solar panel bent angle, [°];
φ - the geographical latitude of the location, [°];
δ - the Sun declination calculated with relation 3, [°];

Generally, the tables and graphics regarding solar radiation present values for incident radiant energy through a horizontal surface.
To design a solar system, we must know the radiant energy density for a horizontal surface, the incident radiant energy density through a horizontal surface.
The radiant density energy B_0 received, during one day, by a unit horizontal area outside the Earth's atmosphere is calculated with relation:

$$B_0 = \frac{24}{\pi} \cdot E_0 \cdot \left[1 + 0{,}33 \cdot \cos\left(\frac{2\pi \cdot n}{365}\right)\right] \cdot \left(\cos\varphi \cdot \cos\delta \cdot \sin\omega_s + \omega_s \cdot \sin\varphi \cdot \sin\delta\right)\left[Wh/m^2 \cdot zi\right] \tag{2}$$

where:
E_0=1367 W/m^2 – is solar constant;
n – number of the days in the year (for example n=1 for 1 January);
ω- hour angle for the sunrise, [°];

Solar declination for a usually day "n" it can be calculated using an Cooper approximate formula:

$$\delta = 23{,}45 \cdot \sin\left(360^\circ \frac{284+n}{365}\right)\left[^\circ\right] \quad (3)$$

The sunrise solar hour angle is:

$$\omega_s = \cos^{-1}(-\tan\varphi \cdot \tan\delta)\left[^\circ\right] \quad (4)$$

The sunset hour angle is:

$$\omega_s = -\cos^{-1}(-\tan\varphi \cdot \tan\delta)\left[^\circ\right] \quad (5)$$

The clearness index k_T is calculated for each month of the year using the relation:

$$k_T = \frac{G}{B_0} \quad (6)$$

G- global radiant density energy on a horizontal plane, [$Wh/m^2 \cdot day$].

The clearness index k_T describes the (average) attenuation of solar radiation by atmosphere at a given site during a given month.

Diffuse energy density D for a horizontal plane is approximated using the equation:

$$\frac{D}{G} = 1 - 1{,}13 \cdot k_T \Rightarrow D = G \cdot (1 - 1{,}13 \cdot k_T)\left[Wh/m^2 \cdot zi\right] \quad (7)$$

The beam energy density B, through a horizontal plane is:

$$B = G - D\left[Wh/m^2 \cdot zi\right] \quad (8)$$

The beam energy density $B(\theta_{incl})$ on a south-facing panel inclined at an angle θ_{inclin} to the horizontal surface is:

$$B(\theta_{incl}) = B \cdot \frac{\cos(\varphi - \theta_{incl}) \cdot \cos\delta \cdot \sin\omega_0 + \omega_0 \cdot \sin(\varphi - \theta_{incl}) \cdot \sin\delta}{\cos\varphi \cdot \cos\delta \cdot \sin\omega_s + \omega_s \cdot \sin\varphi \cdot \sin\delta} + \frac{\omega_0 \cdot \sin(\varphi - \theta_{incl}) \cdot \sin\delta}{\cos\varphi \cdot \cos\delta \cdot \sin\omega_s + \omega_s \cdot \sin\varphi \cdot \sin\delta}\left[Wh/m^2 \cdot z\right] \quad (9)$$

where:

$$\omega_0 = \min\{\omega_s, \omega_s'\} \quad (10)$$

ω_s - is the sunrise angle above the horizon, given by the relation (4)

$$\omega_s' = \cos^{-1}\left[-\tan(\varphi - \theta_{incl}) \cdot \tan\delta\right]\left[^\circ\right] \quad (11)$$

ω'_s - is the sunrise angle above a plane inclined at angle θ_{incl} to the orizontal.

Assuming that the diffuse radiation is distributed isotropically over the sky dome, the diffuse density radiation on the inclined surface is given by:

$$D(\theta_{incl}) = \frac{1}{2} \cdot (1 + \cos\theta_{incl}) \cdot D\left[Wh/m^2 \cdot zi\right] \quad (12)$$

The reflected energy density is generally small, and a simple isotropic model is usually sufficient, thus can be determine with the relation:

$$R(\theta_{incl}) = \frac{1}{2} \cdot (1 - \cos\theta_{incl}) \cdot \rho \cdot G\left[Wh/m^2 \cdot zi\right] \quad (13)$$

ρ - reflectivity od the surrounding area.

Ground cover	Reflectivity, ρ
dry bare ground	0,2
dry grassland	0,3
desert sand	0,4
snow	0,5÷0,8

Table 1. Typical reflectivity values for some ground covers

In this case we can evaluate the global radiant density $G(\theta_{incl})$ on an incline surface as sum of direct radiation, reflected and diffuse:

$$G(\theta_{incl}) = B(\theta_{incl}) + D(\theta_{incl}) + R(\theta_{incl})\left[Wh/m^2 \cdot zi\right] \quad (14)$$

3. APPLICATION C.D.E.R. PROGRAM

For Craiova, the geographical coordinate, are: the latitude: $\varphi = 44{,}23°$, longitude $\lambda = 23{,}87°$.

The average declination calculation for each month is established for each day for which the declination is more closely to the monthly mean declination:

For example, for the 1 January (n=1) the declination is:

$$\delta = 23{,}45 \cdot \sin\left(360^\circ \frac{284+n}{365}\right) = 23{,}45 \cdot \sin\left(360^\circ \frac{284+1}{365}\right) = -23{,}01^\circ$$

Day in January	Declination δ [°]	Day in January	Declination δ [°]
1	-23,0116	16	-21,0963
2	-22,9305	**17**	**-20,917**
3	-22,8427	18	-20,7314
4	-22,748	19	-20,5397
5	-22,6466	20	-20,3419
6	-22,5385	21	-20,138
7	-22,4237	22	-19,9282
8	-22,3023	23	-19,7125
9	-22,1742	24	-19,491
10	-22,0396	25	-19,2636
11	-21,8985	26	-19,0306
12	-21,7509	27	-18,7919
13	-21,5968	28	-18,5477
14	-21,4363	29	-18,2979
15	-21,2695	30	-18,0428
		31	-17,7823
Monthly mean declination	**-20,8472**		

Table 2. The declination calculation for January

The declination value calculated for the 17th January day is the most closely to the monthly mean declination. Similarly it can be determined the rest of the annual values:

Month	Day	Monthly mean declination, δ [°]
January	17 (n=17)	-20,9
February	16 (n=47)	-13,0
March	16 (n=75)	-2,4
April	15 (n=105)	9,4
May	15 (n=135)	18,8
June	11 (n=162)	23,1
July	17 (n=198)	21,2
August	16 (n=228)	13,5
September	15 (n=258)	2,2
October	15 (n=288)	-9,6
November	14 (n=318)	-18,9
December	10 (n=344)	-23,0

Table 3. Monthly mean declination values for the year

The global radiant energy density on an inclined surface for January is calculated knowing the radiant energy density for a horizontal surface (table 4).

The monthly mean radiant energy density	Jan.	Feb.	Mar.	Apr.	May	Jun
G [kWh/m^2·zi]	1,44	2,25	3,42	4,58	5,58	6,19
	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
	6,53	5,59	4,06	2,69	1,72	1,22

RetScreen Data Base [4]

Table 4. Radiant energy density for an horizontal plane

Through the presented methodology it was developed a C.D.E.R program (**C**alculul **D**ensităţii **E**nergiei **R**adinte - Evaluating the Radiant Energy Density) with Visual Basic 6.0. This program allows us to calculate the radiant energy density for a inclined surface with a corresponding angle, also the possibility to generate a ratio using the Microsoft Excel. We will calculate the generated energy density for a inclined surface for all the monthly years' (table 5).

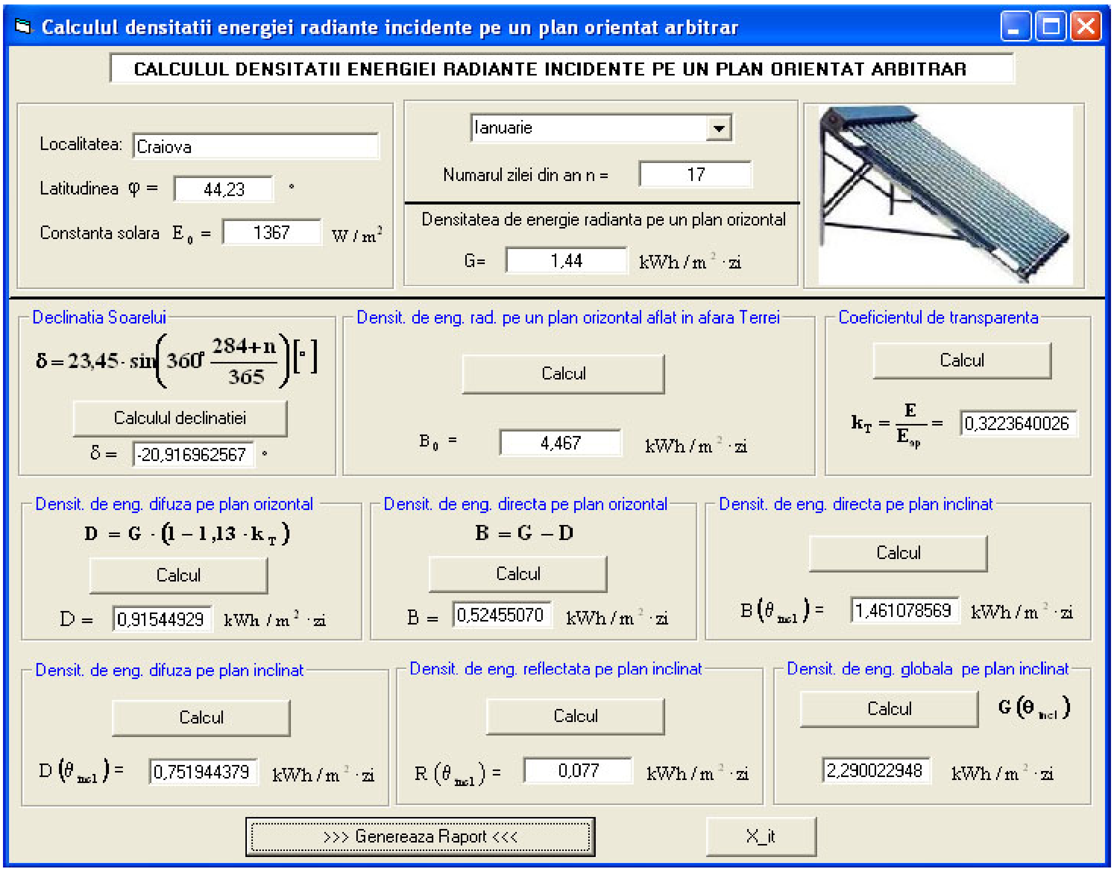

Figure 1. Program C.D.E.R.

Month	B_0 [kWh/m^2·zi]	k_T	The global radiant energy density for a inclined surface [kWh/m^2·zi]									
			0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
Jan.	4,46	0,322	1,440	1,682	1,895	2,070	2,204	2,290	2,327	2,314	2,251	2,140
Feb.	6,029	0,373	2,250	2,523	2,750	2,924	3,039	3,092	3,080	3,005	2,869	2,676
Mar.	7,638	0,447	3,420	3,721	3,951	4,102	4,169	4,151	4,049	3,864	3,603	3,274
Apr.	8,627	0,530	4,580	4,790	4,910	4,932	4,856	4,683	4,420	4,073	3,656	3,180
May	8,643	0,645	5,580	5,655	5,631	5,501	5,267	4,934	4,512	4,016	3,464	2,880
June	8,265	0,748	6,190	6,164	6,037	5,802	5,458	5,016	4,487	3,891	3,252	2,606
Julie	7,198	0,824	6,530	6,568	6,488	6,278	5,940	5,482	4,918	4,267	3,556	2,821
Aug.	7,767	0,719	5,590	5,287	5,942	5,929	5,786	5,517	5,129	4,637	4,055	3,407
Sep.	7,225	0,561	4,060	4,403	4,654	4,805	4,851	4,791	4,627	4,363	4,007	3,571
Oct.	6,017	0,447	2,690	3,039	3,329	3,550	3,696	3,762	3,747	3,649	3,473	3,225
Nov.	4,626	0,371	1,720	2,021	2,284	2,500	2,663	2,768	2,813	2,795	2,714	2,575
Dec.	3,933	0,310	1,220	1,444	1,641	1,807	1,934	2,021	2,064	2,061	2,014	1,924
Mean annual			**3,773**	**3,941**	**4,126**	**4,183**	**4,155**	**4,042**	**3,848**	**3,578**	**3,243**	**2,857**

Table 5. Generated radiation density for a bender surface

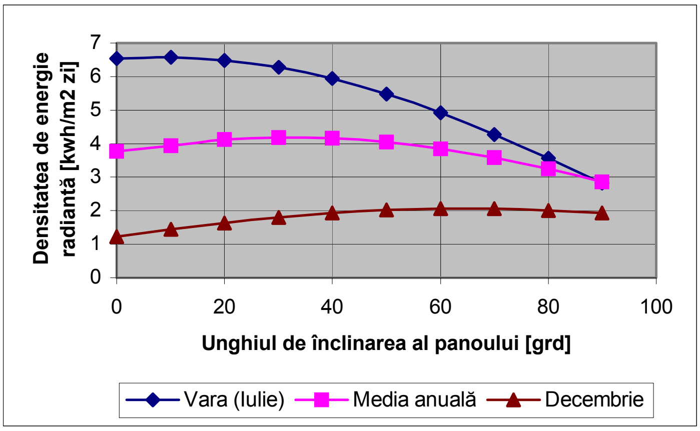

Figure 2. The mean radiant energy density for different period of the years on a inclined surface for Craiova.

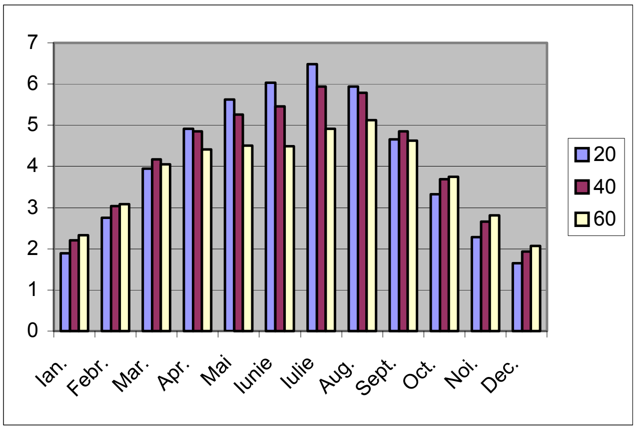

Figure 3. Daily radiation for Craiova over the year for selected angles of panel inclination

With the help using the value from the table 5 we can represent the average radiant energy density graphic for the warmest month of the year (July) for the coldest month of the year and the average for Craiova.

Also the graphic represent the year daily radiation for Craiova corresponding to the bended angle of the solar panel.

4. CONCLUSIONS

Having the determined methodology for the radiant energy density using the CDER program allows establishing the optimum bended angle of the solar panel in order to collect a high-energy radiant quantity.
Knowing this energy quantity we can determine the analyzed solar panel energy efficiency indicators.

References

[1] www.utm.md/master/curs/sre.html (Surse regenerabile de energie, Sobor Ion, Dinov Daniela, ş.a.).

[2] Al. Dănescu, S. Bucurenciu, St. Petrescu, Utilizarea Energiei Solare, Editura Tehnică, Bucureşti, 1980.

[3] Tomas Markvart, Solar Electricity (Second edition), John Wiley & Sons, ltd, Iulie 2001.

[4] www.RetScreen.net