AIRCRAFT'S LONGITUDINAL MOTION COMMAND EFFICIENCY CONTROL LAWS

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***Abstract* – Based on the longitudinal static stability's coefficients, with respect to the vertical overload and on the vertical overload's gradients, one presents some formulas for the longitudinal command chain's transmission rate's calculation, as well as for the stick effort's rigidity, which are non-linear functions with respect to the dynamic pressure and flight altitude.**
One also presents a non-linear model for an automatic control system for the longitudinal command's efficiency with a relay–type regulator (with non-sensitivity zone and hysterezis). This kind of systems is a follower system type with respect to the dynamic pressure and flight altitude, which parameters are chosen based on the absolute stability conditions.

Keywords*: *longitudinal, command law, hysterezis.

1. LONGITUDINAL STABILITY AND MANEVRABILITY PARAMETERS

The longitudinal static stability of the aircraft can be expressed using the stability coefficients for speed, σ_V, or for overload, σ_n, defined based on the function $m_y(\alpha, \omega_y, V, \delta_p)$, which express the dependence of the aerodynamic moment coefficient M_y, acting relative to $Oy-$axis, on the aircraft's attach angle, the pitch angular speed ω_y, the flight speed, as well as on the tail plane's angle δ_p [1], [2]

$$\sigma_V = \left.\frac{\mathrm{d}m_y}{\mathrm{d}\alpha}\right|_{n_z=1} = \frac{\partial m_y}{\partial \alpha} + \frac{\partial m_y}{\partial V}\frac{\mathrm{d}V}{\mathrm{d}\alpha} = m_y^{\alpha} + m_y^{V}\frac{\mathrm{d}V}{\mathrm{d}\alpha}, \quad (1)$$

$$\sigma_n = \left.\frac{\mathrm{d}m_y}{\mathrm{d}c_z}\right|_{V=ct} = \frac{\partial m_y}{\partial c_z} + \frac{\partial m_y}{\partial \omega_y}\frac{\mathrm{d}\omega_y}{\mathrm{d}c_z} = m_y^{c_z} + m_y^{\omega_y}\frac{\mathrm{d}m_y}{\mathrm{d}c_z}, \quad (2)$$

where n_z is the vertical load, c_z-lift coefficient. If $\sigma_V < 0$ and $\sigma_n < 0\left(m_y^{\alpha} \approx m_y^{c_z} < 0, m_y^{V} < 0, m_y^{\omega_y} < 0\right)$, the plane is static stable.
The longitudinal static manoeuvrability is expressed by the balance curves, which has the implicit forms

$$x_m = f_1(V, H, n_z), F_m = f_2(V, H, n_z) \quad (3)$$

or

$$x_m = f_1(M, H, n_z), F_m = f_2(M, H, n_z) \quad (4)$$

or

$$x_m = g_1(q, H, n_z), F_m = g_2(q, H, n_z), \quad (5)$$

where H is the flight altitude, M-flight Mach number, $q = \rho\frac{V^2}{2}-$dynamic pressure, x_m-stick's longitudinal displacement, F_m-stick's effort (load), as well as by the gradients with respect to the overload [2]

$$x_m^n = \left(\frac{\mathrm{d}x_m}{\mathrm{d}n_z}\right)_0 = -\frac{1}{k_{mp}}\frac{G}{S}\frac{1}{q}\frac{1}{m_y^{\delta_p}}(\sigma_n)_0, \quad (6)$$

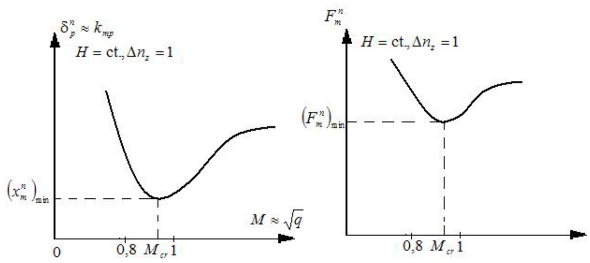
$$F_m^n = \left(\frac{\mathrm{d}F_m}{\mathrm{d}n_z}\right)_0 = k_{mp}b_pS_p\frac{m_s^{\delta_p}}{m_x^{\delta_p}}(\sigma_n)_0; \quad (7)$$

the terms which have "0" as index are for the neutral stick's position, $G-$aircraft's weight, S_p-tail plane's span, $m_s^{\delta_p}-$tail plane command's transmission ratio;

$$\delta_p = k_{mp}x_m; \delta_p^n = k_{mp}x_m^n. \quad (8)$$

$$F_m = k_m x_m; F_m^n = k_m x_m^n; \quad (9)$$

k_m is the longitudinal stick effort rigidity.



a. b.
Figure 1: Balance curves

The balance curves' shapes from fig.1 are confirmed by the next assumptions concerning the aircraft's flight dynamics [1], [2]:
1) for law flight speed values ($V<100$km/h, $M<M_{cr}$) and law flight altitudes, $(\sigma_n)_0 \approx \mathrm{const.} < 0$ and $m_y^{\delta_p} \cong const. < 0$;

2) when H grows, for the same V value, M grows too, over M_{cr} value, so $(\sigma_n)_0$ grows and $m_y^{\delta_p}$ decreases.

2. LONGITUDINAL COMMAND'S EFFICIENCY CONTROL LAWS

The longitudinal command's efficiency control law's is non-linear, expressed by

$$\delta_m^n \approx k_{mp} = -\frac{G}{S}\frac{(\sigma_n)_0}{\left(x_m^n\right)_{imp} m_y^{\delta_p}}\frac{1}{q}. \tag{10}$$

obtained of the equation (8) balance function, for $x_m^n = \left(x_m^n\right)_{imp}$;

The pilot must push/pull the stick with same ratio effort/overload unit, that means $F_m^n = \text{const}, \forall q$. Consequently,

$$k_m \approx \frac{1}{x_m^n} \approx q. \tag{11}$$

The system in fig.2 [3] realizes the force's arm's length (l_3) on the load mechanism, as well as the amplifier's arm's length (l) modification: for $H \le H_0$ (H_0 – altitude's limit until $(\sigma_n)_0$ and $m_y^{\delta_p}$ remain constant),

$$l \approx k_{mp} \approx \delta_p \approx \frac{1}{q}, \tag{12}$$

$$l_3 \approx \sqrt{q} \approx \sqrt{k_m}, \tag{13}$$

relations which will be demonstrated. The annotations in fig.2 significance are: 1-stick, 2-acting lever, 3-electromechanism's rod, 4-hydro-amplifier (booster), 5-mechanism's spring (MS) and trimmer effect mechanism (MT), 6-tail plane (stabilizer), 7-command block.

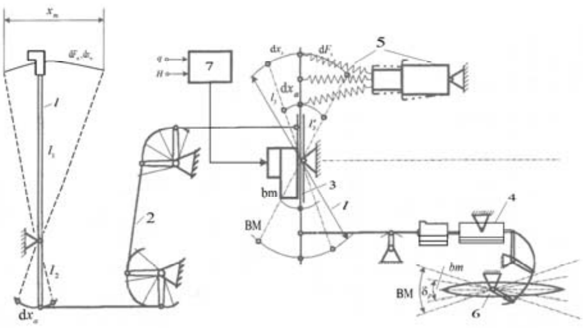


Figure 2: Force's arm's length (l_3) modification system

The energy's balance law

$$\mathrm{d}F_m \cdot \mathrm{dx_m} = \mathrm{d}F_s \cdot \mathrm{d}x_s, \tag{14}$$

gives

$$k_t = \frac{\mathrm{d}x_s}{\mathrm{d}x_m} = \frac{\mathrm{d}F_m}{\mathrm{d}F_s}, \tag{15}$$

$$\mathrm{dF_m} = k_t \mathrm{d}F_s = k_t k_s \mathrm{d}x_s = k_t^2 k_s \mathrm{d}x_m, \tag{16}$$

where k_s is the MS rigidity constant. Consequently and considering also the first equation in (9), it results

$$k_m = \frac{\mathrm{d}F_m}{\mathrm{d}x_m} = k_t^2 k_s, k_s = k_s(l_3). \tag{17}$$

According to fig.2, one can express

$$\frac{\mathrm{d}x_a}{\mathrm{d}x_m} = \frac{l_2}{l_1}, \frac{\mathrm{d}x_a}{\mathrm{d}x_s} = \frac{l_3'}{l_3}, \tag{18}$$

and consequently

$$k_t = \frac{\mathrm{d}x_s}{\mathrm{d}x_m} = \frac{l_2}{l_1 l_3'} l_3 = k_a l_3, \tag{19}$$

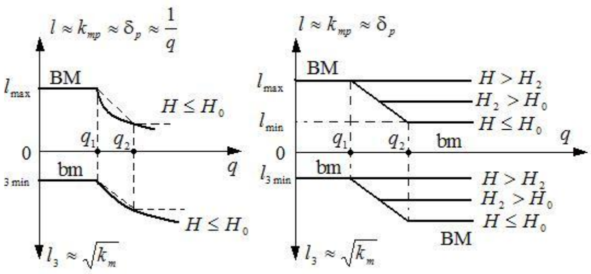
where $k_a = l_2 / l_1 l_3' = \text{const.}$

From equations (17) and (18) it results

$$k_m = \frac{\mathrm{d}F_m}{\mathrm{d}x_m} = \frac{F_m^n}{x_m^n} = c_3 l_3^2. \tag{20}$$

where $c_3 = c_3(l_3) = k_s k_a^2$, which is equivalent to equation (12).

The relations (12) and (13), for $H \le H_0$ and $q \ge q_1$, are the expressions for curves in fig.3.a. Because of the l, l_3 limited dimensions and $l + l_3 = l_t = \text{const.}$, it results that, for $q \le q_1$, they become horizontal. These limits are established for each aircraft's command chain. Those curves can be approximated, with enough small errors, by the ones in fig.3.b.



a. b.

Figure 3: Arm's modification with respect to q

For $H > H_0$, these characteristics are corrected according to the $(\sigma_n)_0$ and $m_y^{\delta_p}$ coefficients modifications as well as to equation (10).

According to fig.3, for high values of q and $H \le H_0$, l – small (small arm b_m) and booster and

l_3 – big (big arm BM) on MS. Consequently, F_s is small (small effort on MS) and, consequently, $F_m \approx k_m$ – big (big effort on stick). When H grows ($H > H_0$), l grows and l_3 decreases; F_m decreases too (small effort on stick) and F_s grows (big effort on MS) [3].

An option for longitudinal command efficiency controller is also known as ARU-3V [3], which has: $l_{\max} = 100\text{mm}, l_{\min} = 50\text{mm}, H \cong 5\text{km}, q_1 \cong 70\text{mmHg}$, $q_2 \cong 370\text{mmHg}$ and the slope $m = 0{,}17\text{mm/mmHg}$.

In fig.4 is represented a modelling block diagram for an automatic control system for the longitudinal command efficiency.

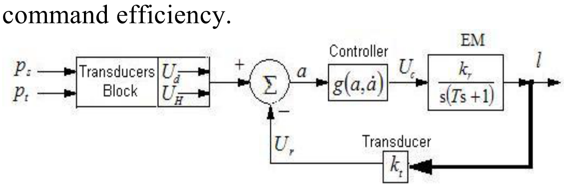


Figure 4: Block diagram for an automatic control system for the longitudinal command efficiency.

The command law is non-linear [4];

$$U_c = g(a,\dot a) = \begin{cases} -\eta, a < -a_0, a \in (-a_0, -\lambda a_0)\text{cu } \dot a > 0, \\ 0, a \in [-\lambda a_0, \lambda a_0], a \in (-a_0, -\lambda a_0) \text{ cu } \dot a < 0, \\ \eta, a > a_0, a \in (\lambda a_0, a_0)\text{cu } \dot a < 0. \end{cases} \quad (21)$$

The system's error could have, e.g., the next form

$$a = U - U_r = \begin{cases} U_d - U_r \;\;, \delta U \le \delta U_0 (H \le H_0), \\ U_d + \delta U - U_r = U_H - U_r, \delta U > \delta U_0 (H > H_0), \end{cases} \quad (22)$$

where $\delta U = U_H - U_d; U_d$ – dynamic pressure's transducer's output voltage; U_H – altitude's transducer's output voltage.

The k_r and T parameters' choice can be made according to the imposed absolute stability conditions or to the self–oscillations existence conditions [4], [5].

The linear sub–system in fig.4 has the transfer function

$$H_L(\text{s}) = \frac{l(\text{s})}{U_c(\text{s})} = \frac{k_r}{\text{s}(T\text{s}+1)}, \quad (23)$$

consequently

$$\frac{\dot l(\text{s})}{U_c(\text{s})} = \frac{k_r}{T\text{s}+1}. \quad (24)$$

The time constant T can be chosen as $T = 0{,}01 \div 0{,}05\text{s}$. From the EM rod's displacement speed value, for a steady – state regime $(V_s = \dot l_s)$, it results $k_r = \frac{\dot l_s}{U_{cs}} = \frac{\dot l_s}{\eta}$.

The coefficient k_t can be calculated as

$$k_t = \frac{\Delta U_r}{\Delta l} \overset{a \to 0}{=} \frac{\Delta U}{\Delta l} = \frac{\Delta U_d}{l_{\max} - l_{\min}} = \frac{U_{d1} - U_{d2}}{l_{\max} - l_{\min}}. \quad (25)$$

Advanced control systems, having the block diagram description as the one in fig.4, are presented in [6] and [7].

3. CONCLUSIONS

Some relations for the longitudinal command chain's transmission ratio calculation, as well as for the stick rigidity, described as functions of the dynamic pressure and flight altitude are presented. One also presents the non – linear model of the longitudinal command's efficiency control system, with a relay – type controller with insensibility zone and hysterezis. The system follows the dynamic pressure's and flight altitude's variations.

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