# PARAMETERS ESTIMATION OF THE AIRCRAFT'S LONGITUDINAL AND LATERAL MOVE 

Mihai LUNGU<br>PhD student - Avionics Division, Faculty of Electrical Engineering, University of Craiova, ROMANIA, E-mail: Lma1312@yahoo.com


#### Abstract

This paper deals with an analysis regarding identification and control of stability processes of an aircraft's longitudinal and lateral move. The estimation of the transfer function for a stabile flight of an aircraft (longitudinal move) isn't made using the most important estimation method (the least square method). Other three methods (origin pole compensation method, instrumental variables method and prediction error method - with the two variants) are used here to estimate all the coefficients of the transfer function of the longitudinal move of an aircraft. All these methods estimates the transfer function by the continue minimization of the error function. The lateral move model's parameters are estimated using the least square method. Thus, a simulation program for discrete transfer function determination of the lateral move, for stabilization analyses and for time characteristics plotting of the leading system and estimated system output in random perturbation conditions is made; estimation errors are being calculated.


Keywords: estimation, residues, longitudinal, lateral, transfer function

## 1. INTRODUCTION

We consider the longitudinal move of an aircraft. One presents in the following pages some methods used for aircraft's longitudinal and lateral moves identification. These methods calculate the coefficients of the transfer function in closed loop. It doesn't matter if the system is stable or not. The most important thing is superpose of the estimated system's response and the leading system's response.

## 2. THE ORIGIN POLE COMPENSATION (CMMPM)

A state estimator must assure the controllability of the system whose parameters are estimated, indifferent to adaptive structure. The least square method doesn't always give models characterized by controllability. That's why in some cases it must be modified.
The system, whose parameters must be determined, is described by the equation

$$
\begin{equation*}
L\left(\mathrm{z}^{-1}\right) y(t)=z^{-q} M\left(\mathrm{z}^{-1}\right) u(t)+C\left(\mathrm{z}^{-1}\right) e(t)+d, \tag{1}
\end{equation*}
$$

where $\mathrm{z}^{-1}$ is the delay operator and the polynomials $L\left(\mathrm{z}^{-1}\right)$ and $M\left(\mathrm{z}^{-1}\right)$ are

$$
\begin{align*}
& L\left(\mathrm{z}^{-1}\right)=1+a_{1} z^{-1}+a_{2} \mathrm{z}^{-2}+\ldots+a_{n} \mathrm{z}^{-n},  \tag{2}\\
& M\left(\mathrm{z}^{-1}\right)=b_{0}+b_{1} \mathrm{z}^{-1}+b_{2} \mathrm{z}^{-2}+\ldots+b_{m} \mathrm{z}^{-m}
\end{align*}
$$

The estimated model $\hat{A}$ of the leading system $A$ (aircraft), obtained by an parametric identification method, may be described by equation

$$
\begin{equation*}
\hat{L}\left(\mathrm{z}^{-1}\right) y(t)=\mathrm{z}^{-q} \hat{M}\left(\mathrm{z}^{-1}\right) u(t)+\hat{C}\left(\mathrm{z}^{-1}\right) \hat{e}(t)+d \tag{3}
\end{equation*}
$$

where $\hat{e}(t)$ is the noise applied to the model and the polynomials $\hat{L}\left(\mathrm{z}^{-1}\right), \hat{M}\left(\mathrm{z}^{-1}\right)$ and $\hat{C}\left(\mathrm{z}^{-1}\right)$ have expressions

$$
\begin{align*}
& \hat{L}\left(\mathrm{z}^{-1}\right)=1+\hat{a}_{1} \mathrm{z}^{-1}+\hat{a}_{2} \mathrm{z}^{-2}+\ldots+\hat{a}_{n} \mathrm{z}^{-n} \\
& \hat{M}\left(\mathrm{z}^{-1}\right)=\hat{b}_{0}+\hat{b}_{1} \mathrm{z}^{-1}+\hat{b}_{2} \mathrm{z}^{-2}+\ldots+\hat{b}_{m} \mathrm{z}^{-m}  \tag{4}\\
& \hat{C}\left(\mathrm{z}^{-1}\right)=1+\hat{c}_{1} \mathrm{z}^{-1}+\hat{c}_{2} \mathrm{z}^{-2}+\ldots+\hat{c}_{n} \mathrm{z}^{-n}
\end{align*}
$$

CMMP algorithm (least square algorithm) modification is based upon the discrete transfer function modification through origin pole $(z=0)$ compensation. The modified CMMP algorithm (CMMPM) builds a convergent vector $v(t)$ and with it the vector of the estimated parameters [1]

$$
\begin{equation*}
\hat{b}^{\prime}(k)=\hat{b}(k)+P(k) v(k) . \tag{5}
\end{equation*}
$$

Thus, the coefficient $\hat{b}^{\prime}$ is almost non-null.
The control law may be chosen of general form

$$
\begin{equation*}
u(k)=R\left(\mathrm{z}^{-1}, \hat{b}^{\prime}\right) u(k)+S\left(\mathrm{z}^{-1}, \hat{b}^{\prime}\right) y(k) \tag{6}
\end{equation*}
$$

with the polynomials

$$
\begin{align*}
& R\left(\mathrm{z}^{-1}, \hat{b}^{\prime}\right)=\sum_{i=1}^{\alpha} \mathrm{z}^{-i} r_{i}\left(\hat{b}^{\prime}\right), \\
& S\left(\mathrm{z}^{-1}, \hat{b}^{\prime}\right)=\sum_{i=0}^{\beta} \mathrm{z}^{-i} s_{i}\left(\hat{b}^{\prime}\right) . \tag{7}
\end{align*}
$$

The closed loop system is described by equation [1]

$$
W(k+1)=D\left(z^{-1}, \hat{b}^{\prime}\right) W(k)+\left[\begin{array}{c}
e(k+1)  \tag{8}\\
0
\end{array}\right],
$$

where

$$
D\left(\mathrm{z}^{-1}, \hat{b}^{\prime}\right)=\left[\begin{array}{cc}
z\left[1-\hat{L}\left(\mathrm{z}^{-1}, \hat{b}^{\prime}\right)\right.  \tag{9}\\
z S\left(\mathrm{z}^{-1}, \hat{b}^{\prime}\right) & \hat{M}\left(\mathrm{z}^{-1}, \hat{b}^{\prime}\right) \\
\mathrm{z} R\left(\mathrm{z}^{-1}, \hat{b}^{\prime}\right)
\end{array}\right],
$$

and

$$
\begin{equation*}
e(k+1)=x^{T}(k)\left[b(k)-b^{T}(k)\right]+n(k+1), \tag{10}
\end{equation*}
$$

$n(k+1)$ is white noise.

## 3. INSTRUMENTAL VARIABLES METHOD (MVI)

This method is a generalization of CMMP. It gives the estimated parameters only for the determinist part of the model $\hat{A}$ and not for the parameters of the polynomial $\hat{C}\left(\mathrm{z}^{-1}\right)$ associated to the random perturbation. The leading system model ( $A$ ) is described by the equation (1) and the one of the estimated model $(\hat{A})$ by equation (3); in this equation one considers $C\left(\mathrm{z}^{-1}\right)=1$. The equation equivalent to equation (3) is

$$
\begin{equation*}
y(k)=x^{T}(k) \hat{b}+\hat{e}(k) \tag{11}
\end{equation*}
$$

By multiplication of this equation with $W(k)-$ instrumental variable vector (whose elements haven't physic significations, they are only necessary "instruments" for the $\hat{b}$ estimation), one obtains the equation of estimator $\hat{b}$.

$$
\begin{equation*}
\sum_{k=1}^{N} W(k) y(k) \cong\left(\sum_{k=1}^{N} W(k) x^{T}(k)\right) \hat{b} \tag{12}
\end{equation*}
$$

or

$$
\begin{equation*}
\hat{b}=\left[\sum_{k=1}^{N} W(k) x^{T}(k)\right]^{-1}\left[\sum_{k=1}^{N} W(k) y(k)\right] \tag{13}
\end{equation*}
$$

where $N$ is the measurements number; the vector $W$ may be chosen different ways. Let vector $W$ be [2]
$W(k)=F\left(z^{-1}\right)\left[\begin{array}{llll}u(k-1) & u(k-2) & \ldots & u\left(k-n_{W}\right)\end{array}\right]^{T}, \quad(14)$
where $n_{W}=m+n$; if $\hat{L}\left(\mathrm{z}^{-1}\right)$ and $\hat{M}\left(\mathrm{z}^{-1}\right)$ are the $L\left(\mathrm{z}^{-1}\right)$ and $M\left(\mathrm{z}^{-1}\right)$ polynomials estimations, one chooses

$$
\begin{equation*}
F\left(\mathrm{z}^{-1}\right)=\hat{L}^{-1}\left(\mathrm{z}^{-1}\right) \tag{14}
\end{equation*}
$$

Let the transfer function associated to the longitudinal move of an aircraft be
$H_{\delta_{p}}^{\theta}\left(z^{-1}\right)=z^{-1} \frac{1,449-1,2248 z^{-1}-0,0513 z^{-2}+0,0485 z^{-3}}{1-1,604 z^{-1}+0,277 z^{-2}+0,302 z^{-3}+0,035 z^{-4}}(15)$
The calculus program is presented in Appendix 1. The input $u$ and perturbation $e$ of the leading
system are random type. For the parameters estimation of the vector $\hat{b}$ one uses, in Matlab medium, the operator iv4. Using the instruction resid the residues associated to this method are calculated and plotted (figure 1). In figure 2 frequency characteristics (amplitude-frequency and phasefrequency) for the leading system and for the estimated system (using CMMP and MVI methods) are plotted.


Figure 1: Residues of CMMPM method


Figure 2: Frequency characteristics for CMMP and MVI methods

The discrete transfer function is
$\hat{H}_{\delta_{p}}^{\theta}\left(z^{-1}\right)=z^{-1} \frac{1,278-0,810 z^{-1}-0,288 z^{-2}+0,081 z^{-3}}{1-1,356 z^{-1}-0,108 z^{-2}+0,3262 z^{-3}+0,149 z^{-4}}$

## 4. THE PREDICTION ERROR METHOD (MEP)

This method is more complicated than the others, but it is more precisely. MEP calculates the coefficients of the polynomials $M\left(\mathrm{z}^{-1}\right), L\left(\mathrm{z}^{-1}\right)$ and the coefficients of the polynomials that "modify" perturbation which affects the leading system. Starting from an initial estimation, one calculates the parameter of the system through successive iterations till the convergence criteria is reached. The initial estimations used by MEP may be obtained using one of the previous methods [2].
Thleprediction error is the perturbation $e$

$$
\begin{equation*}
e=y-\hat{y} . \tag{17}
\end{equation*}
$$

The leading system is described by equation

$$
\begin{equation*}
L\left(\mathrm{z}^{-1}\right) y=M\left(\mathrm{z}^{-1}\right) u+C\left(\mathrm{z}^{-1}\right) e \tag{18}
\end{equation*}
$$

Thus, the residue is

$$
\begin{equation*}
e=\frac{1}{C\left(\mathrm{z}^{-1}\right)}\left[L\left(\mathrm{z}^{-1}\right) y-M\left(\mathrm{z}^{-1}\right) u\right] . \tag{19}
\end{equation*}
$$

The estimated parameters (the vector $\hat{b}$ ) are determined through the sum's minimization of the square prediction errors

$$
\begin{equation*}
\hat{b}=\arg \left(\min _{b} \sum_{k=1}^{N} e^{2}(k)\right), \tag{20}
\end{equation*}
$$

where $N$ is the available data number. An estimation algorithm is the following one [2]:

1) one makes an initial estimation of the coefficients of $C\left(\mathrm{z}^{-1}\right)$ using a CMMP type method, and thus it results $\hat{C}\left(\mathrm{z}^{-1}\right)$,
2) using the previous estimation $(\hat{C})$ one calculates filtering signals

$$
\begin{align*}
& u_{f}=\hat{C}\left(\mathrm{z}^{-1}\right) u, \\
& y_{f}=\hat{C}\left(\mathrm{z}^{-1}\right) y . \tag{21}
\end{align*}
$$

one determines the estimations $\hat{L}\left(\mathrm{z}^{-1}\right)$ and $\hat{M}\left(\mathrm{z}^{-1}\right)$ of the polynomials $L\left(\mathrm{z}^{-1}\right)$ and $M\left(\mathrm{z}^{-1}\right)$;

$$
\begin{align*}
& \hat{L}\left(\mathrm{z}^{-1}\right), \hat{M}\left(\mathrm{z}^{-1}\right)=\arg \left(\left.\min _{L, M} \sum_{k=1}^{n} e^{2}(k)\right|_{C=\hat{C}}\right)=  \tag{22}\\
& \quad=\arg \left[\min _{L, M} \sum_{i=1}^{N}\left(L\left(\mathrm{z}^{-1}\right) y_{f}-M\left(\mathrm{z}^{-1}\right) U_{f}\right)^{2}\right]
\end{align*}
$$

4) hereby a new estimation $\hat{C}\left(z^{-1}\right)$ is calculated [3]

$$
\begin{equation*}
C\left(\mathrm{z}^{-1}\right)=\arg \left[\min _{C} \sum_{k=1}^{N}\left(\frac{1}{C\left(\mathrm{z}^{-1}\right)} \hat{V}\right)^{2}\right] \tag{23}
\end{equation*}
$$

where $\hat{V}$ is expressed function of assessments $\hat{L}\left(\mathrm{z}^{-1}\right)$ and $\hat{M}\left(\mathrm{z}^{-1}\right)$ from previous step;

$$
\begin{equation*}
\hat{V}=\hat{L}\left(\mathrm{z}^{-1}\right) y-M\left(\mathrm{z}^{-1}\right) u \tag{24}
\end{equation*}
$$

The calculus formula for $\hat{b}$ is

$$
\begin{equation*}
\hat{b}=\left[\sum_{k=1}^{N} x^{F}(k)\left(x^{F}(k)\right)^{T}\right]^{-1}\left[\sum_{k=1}^{N} x^{T}(k) y^{F}(k)\right] . \tag{25}
\end{equation*}
$$

### 4.1. Parameters estimation of the output error model

Calculus program is presented in Appendix 2.

Instead of ARX, or iv4, one uses the instruction oe with a similar syntax.
Estimated transfer function is
$\hat{H}_{\delta_{p}}^{\theta}\left(\mathrm{z}^{-1}\right)=\mathrm{z}^{-1} \frac{1,03-0,271 \mathrm{z}^{-1}-0,2204 \mathrm{z}^{-2}-0,616 \mathrm{z}^{-3}}{1-0,877 \mathrm{z}^{-1}-0,373 \mathrm{z}^{-2}-0,004 \mathrm{z}^{-3}+0,248 \mathrm{z}^{-4}}$
The curves $y(t)=\theta(t)$ and $\hat{y}(t)=\hat{\theta}(t)$ are plotted in figure 3.


Figure 3: The outputs $y$ and $\hat{y}$

### 4.2. Parameters estimation of the ARMAX model

Calculus program is presented in Appendix 3. Instead of instruction oe one uses instruction ARMAX.
The estimate transfer function is

$$
\begin{equation*}
\hat{H}_{\delta_{p}}^{\theta}\left(z^{-1}\right)=z^{-1} \frac{1,127-1,247 z^{-1}-0,113 z^{-2}+0,084 z^{-3}}{1-1,64 z^{-1}+0,314 z^{-2}+0,338 z^{-3}-0,001 z^{-4}} \tag{27}
\end{equation*}
$$

The curves $y(t)=\theta(t)$ and $\hat{y}(t)=\hat{\theta}(t)$ are plotted in figure 4. The frequency characteristics [4,5] (amplitude - frequency and phase-frequency) are plotted in figure 5 ( 1 - CMMP, 2 - MVI, 3 ARMAX, 4 - frequency characteristics of the leading system $(A)$ ).


Figure 4: The outputs $y$ and $\hat{y}$


Figure 5: Comparative frequency characteristics

## 5. LATERAL MOVE ANALYSIS

Lateral move aircraft dynamics may be described by the matriceal equations with non-dimensional variables [6]

$$
\begin{align*}
& \dot{x}=A x+B u, \\
& y=C x+D u, \tag{28}
\end{align*}
$$

in which

$$
x^{T}=\left[\begin{array}{lll}
\beta & \omega_{z} & \omega_{x}
\end{array}\right) \varphi, u^{T}=\left[\begin{array}{ll}
\delta_{d} & \delta_{e}
\end{array}\right], y^{T}=\left[\begin{array}{ll}
\omega_{z} & \varphi \tag{29}
\end{array}\right] ;
$$

$\beta$ is the side-slip angle, $\omega_{z}$ and $\omega_{x}$ are respectively the yaw and roll angular velocities, $\varphi$ - the roll angle, $\delta_{d}$ - the rudder deflection, $\delta_{e}$ - the aileron deflection ( $\delta_{d}$ and $\delta_{e}$ are expressed in degrees).
For the following coefficients of an heavy aircraft [6], which flies at $H=8 \mathrm{~km}$, with $M=0.8$, using a Matlab program, one obtains matrices $A$ and $B$ of the linear system (28) while the vectors $C$ and $D$ are chosen from the second equation (28).
$x$ is the state vector, $\delta_{p}$ - elevator deflection. Choosing as output variable $y=\theta$ - the pitch angle, it results $C=\left[\begin{array}{llll}0 & 0 & 1 & 0\end{array}\right] ; D=0$.
With these values, using a Matlab program, is obtained matrix $A$ and the vectors $B, C, D$;
$A=\left[\begin{array}{cccc}-0.026 & 0.025 & -0.1 & 0 \\ -0.36 & -3 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0.4212 & -38.49 & 0 & -3.67\end{array}\right]$,
$B^{T}=\left[\begin{array}{llll}0 & 0 & 0 & 28\end{array}\right], C=\left[\begin{array}{llll}0 & 0 & 1 & 0\end{array}\right], D=0$.
The program also calculates the transfer functions
$H_{\mathrm{s}_{d}}^{\mathrm{o}_{z}}(\mathrm{~s})=\frac{-0.4750 \mathrm{~s}^{3}-0.2479 \mathrm{~s}^{2}-0.1187 \mathrm{~s}-0.0563}{1.00 \mathrm{~s}^{4}+0.6358 \mathrm{~s}^{3}+0.9389 \mathrm{~s}^{2}+0.5116 \mathrm{~s}+0.0037}$,
$H_{\delta_{d}}^{\varphi}(\mathrm{s})=\frac{0.1148 \mathrm{~s}^{2}-0.2004 \mathrm{~s}-1.3726}{1.00 \mathrm{~s}^{4}+0.6358 \mathrm{~s}^{3}+0.9389 \mathrm{~s}^{2}+0.5116 \mathrm{~s}+0.0037}$,
$H_{\delta_{e}}^{\omega_{z}}(\mathrm{~s})=\frac{0.1230 \mathrm{~s}^{3}+0.0303 \mathrm{~s}^{2}+0.0824 \mathrm{~s}+0.0419}{1.00 \mathrm{~s}^{4}+0.6358 \mathrm{~s}^{3}+0.9389 \mathrm{~s}^{2}+0.5116 \mathrm{~s}+0.0037}$,
$H_{\delta_{e}}^{\varphi}(\mathrm{s})=\frac{1.0729 \mathrm{~s}^{2}+0.2317 \mathrm{~s}+1.0237}{1.00 \mathrm{~s}^{4}+0.6358 \mathrm{~s}^{3}+0.9389 \mathrm{~s}^{2}+0.5116 \mathrm{~s}+0.0037}$.
The calculated eigenvalues are:
$p_{1,2}=-0.0329 \pm 0.946 \mathrm{i}, \quad p_{3}=-0.5627, \quad p_{4}=-0.0073$.
In fig. 6 Dirac impulse's responses $\omega_{z}\left(\delta_{d}\right), \omega_{z}\left(\delta_{e}\right)$, $\omega_{z}\left(\delta_{d}\right), \omega_{z}\left(\delta_{e}\right)$ are represented (all the amplitudes are expressed in radians).


Figure 6: Dirac impulse's responses
The program also calculates the matrices associated to the discrete mode

$$
\begin{align*}
& A_{d}=\left[\begin{array}{cccc}
0.8876 & -0.3081 & 0.0415 & 0.0198 \\
0.202 & 0.3973 & -0.0046 & 0.0024 \\
-1.2515 & 0.5106 & 0.7617 & 0.0024 \\
-0.3313 & 0.151 & 0.4407 & 0.9976
\end{array}\right],  \tag{35}\\
& B_{d}=\left[\begin{array}{cc}
0.4806 & -0.0013 \\
-1.5809 & 0.3387 \\
0.0599 & 4.839 \\
0.039 & 1.2585
\end{array}\right], C_{d}=\left[\begin{array}{llll}
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1
\end{array}\right], D_{d}=D=\left[\begin{array}{ll}
0 & 0 \\
0 & 0
\end{array}\right] .
\end{align*}
$$



Figure 7: Step responses of the discrete mode
In fig. 7 step responses of the discrete mode (with the amplitudes $\delta_{d}=0.0065 \mathrm{rad}$ and $\delta_{e}=0.008 \mathrm{rad}$ ) are represented. The discrete transfer functions are

$$
\begin{align*}
& H_{\delta_{d}}^{\omega_{z}}\left(\mathrm{z}^{-1}\right)=\mathrm{z}^{-1} \cdot \frac{1,2182 \mathrm{z}^{-3}-3,9394 \mathrm{z}^{-2}+4,2814 \mathrm{z}^{-1}-1,5809}{0,3403 \mathrm{z}^{-4}-1,7912 \mathrm{z}^{-3}+3,5019 \mathrm{z}^{-2}-3,0442 \mathrm{z}^{-1}+1},  \tag{36}\\
& H_{\delta_{d}}^{\varphi}\left(\mathrm{z}^{-1}\right)=\mathrm{z}^{-1} \cdot \frac{0,0732 \mathrm{z}^{-3}-0,1676 \mathrm{z}^{-2}-0,4514 \mathrm{z}^{-1}+0,039}{0,3403 \mathrm{z}^{-4}-1,7912 \mathrm{z}^{-3}+3,5019 \mathrm{z}^{-2}-3,0442 \mathrm{z}^{-1}+1},  \tag{37}\\
& H_{\delta_{e}}^{\omega_{z}}\left(\mathrm{z}^{-1}\right)=\mathrm{z}^{-1} \cdot \frac{-0,3406 \mathrm{z}^{-3}+1,0157^{-2}-1,0484 \mathrm{z}^{-1}+0,3880}{0,3403 \mathrm{z}^{-4}-1,7912 \mathrm{z}^{-3}+3,5019 \mathrm{z}^{-2}-3,0442 \mathrm{z}^{-1}+1},  \tag{38}\\
& H_{\delta_{e}}^{\varphi}\left(\mathrm{z}^{-1}\right)=\mathrm{z}^{-1} \cdot \frac{0,4866 \mathrm{z}^{-3}-0,9573 \mathrm{z}^{-2}-0,384 \mathrm{z}^{-1}+1,2585}{0,3403 \mathrm{z}^{-4}-1,7912 \mathrm{z}^{-3}+3,5019 \mathrm{z}^{-2}-3,0442 \mathrm{z}^{-1}+1}, \tag{39}
\end{align*}
$$

while the discrete eigenvalues are $z_{1}=0.8306$; $z_{2,3}=0.8384 \pm 0.2454 \mathrm{i} ; \quad z_{4}=0.5368$.
So, the discrete model of the aircraft's lateral move may be described by the following metriceal equation

$$
\left[\begin{array}{c}
\omega_{z}  \tag{40}\\
\varphi
\end{array}\right]=\left[\begin{array}{cc}
H_{\delta_{d}}^{\omega_{z}}\left(\mathrm{z}^{-1}\right) & H_{\delta_{e}}^{\omega_{z}}\left(\mathrm{z}^{-1}\right) \\
H_{\delta_{d}}^{\varphi}\left(\mathrm{z}^{-1}\right) & H_{\mathrm{\delta}_{e}}^{\varphi}\left(\mathrm{z}^{-1}\right)
\end{array}\right] \cdot\left[\begin{array}{c}
\delta_{d} \\
\delta_{e}
\end{array}\right] .
$$

## 6. PARAMETRIC IDENTIFICATION ALGORITM

Calculus algorithm for a parametric estimator is based on the least square method [7].The estimated parameters of the $\hat{A}$ model are permanently modified with an adaptation mechanism, based on the continue measure and the minimization of the error function $\hat{e}$. One makes the vector $Y^{T}=\left[\begin{array}{llllll}y(1) & y(2) & \ldots & y(k) & \ldots & y(N)\end{array}\right]$, the $X$ matrix and the vectors $e, \hat{e}$

$$
X=\left[\begin{array}{c}
x^{T}(1)  \tag{41}\\
x^{T}(2) \\
\vdots \\
x^{T}(k) \\
\vdots \\
x^{T}(N)
\end{array}\right], \hat{e}=\left[\begin{array}{c}
\hat{e}(1) \\
\hat{e}(2) \\
\vdots \\
\hat{e}(k) \\
\vdots \\
\hat{e}(N)
\end{array}\right], e=\left[\begin{array}{c}
e(1) \\
e(2) \\
\vdots \\
e(k) \\
\vdots \\
e(N)
\end{array}\right],
$$

The estimated parameters vector $\hat{b}$ may be obtained through minimisation of the square indicator

$$
\begin{align*}
& J=(Y-X \hat{b})^{T}(Y-X \hat{b})=  \tag{42}\\
& =Y^{T} Y-\hat{b}^{T} X^{T} Y-Y^{T} X \hat{b}+\hat{b}^{T} X^{T} X \hat{b} .
\end{align*}
$$

From minimum of $J$ condition [7], that means $\frac{\partial J}{\partial \hat{b}}=0$, one results

$$
\begin{equation*}
\hat{b}-b=\left(X^{T} X\right)^{-1} X^{T} e . \tag{43}
\end{equation*}
$$

One makes the notations

$$
X(k)=\left[\begin{array}{c}
x^{T}(1)  \tag{44}\\
x^{T}(2) \\
\vdots \\
x^{T}(k)
\end{array}\right], Y(k)=\left[\begin{array}{c}
y(1) \\
y(2) \\
\vdots \\
y(k)
\end{array}\right],
$$

where $x(k) \underline{\underline{\Delta}} x(k T)$ and $y(k) \underline{=} y(k T)$, and $T$ is the sample period and

$$
X(k+1)=\left[\begin{array}{c}
X(k)  \tag{45}\\
x^{T}(k+1)
\end{array}\right], Y(k+1)=\left[\begin{array}{c}
Y(k) \\
y(k+1)
\end{array}\right] .
$$

The recurrence relation between $P(k+1)(P$ the covariance matrix) and $P(k)$ is $[2,3]$

$$
\begin{equation*}
P(k+1)=P(k)\left[I_{m+n}-\frac{x(k+1) x^{T}(k+1)}{\lambda+x^{T}(k+1) P(k) x(k+1)} P(k)\right], \tag{46}
\end{equation*}
$$

where $\lambda$ represents the forget factor, and the recurrence relation between $\hat{b}(k+1)$ and $\hat{b}(k)$

$$
\begin{equation*}
\hat{b}(k+1)=\hat{b}(k)+P(k+1) x(k+1) \hat{e}(k+1) . \tag{47}
\end{equation*}
$$

The discrete transfer functions of $A$ are (31) $\div(34)$, while the discrete transfer functions of $\hat{A}$ may be obtained using a Matlab program

$$
\begin{align*}
& \hat{H}_{\mathrm{\delta}_{d}}^{\omega_{z}}\left(z^{-1}\right)=z^{-1} \cdot \frac{1,364 \mathrm{z}^{-3}-4,115 \mathrm{z}^{-2}+4,475 \mathrm{z}^{-1}-1,646}{0.4063 \mathrm{z}^{-4}-1,974 \mathrm{z}^{-3}+3,682 \mathrm{z}^{-2}-3,106 \mathrm{z}^{-1}+1},  \tag{48}\\
& \hat{H}_{\mathrm{\delta}_{d}}^{\varphi}\left(z^{-1}\right)=z^{-1} \cdot \frac{0,01383 \mathrm{z}^{-3}-0,2511 \mathrm{z}^{-2}-0,4213 \mathrm{z}^{-1}+0,001393}{0,2753 \mathrm{z}^{-4}-1,606 \mathrm{z}^{-3}+3,327 \mathrm{z}^{-2}-2,991 \mathrm{z}^{-1}+1},  \tag{49}\\
& \hat{\mathrm{H}}_{\mathrm{\delta}_{e}}^{\omega_{z}}\left(z^{-1}\right)=z^{-1} \cdot \frac{-0,3072 \mathrm{z}^{-3}+0,8676 \mathrm{z}^{-2}-0,9953 \mathrm{z}^{-1}+0,4401}{0,2814 \mathrm{z}^{-4}-1,671 \mathrm{z}^{-3}+3,423 \mathrm{z}^{-2}-3,028 \mathrm{z}^{-1}+1},  \tag{50}\\
& \hat{H}_{\mathrm{\delta}_{e}}^{\varphi}\left(\mathrm{z}^{-1}\right)=\mathrm{z}^{-1} \cdot \frac{0,5927 \mathrm{z}^{-3}-0,9603 \mathrm{z}^{-2}-0,4538 \mathrm{z}^{-1}+1,407}{0,4334 \mathrm{z}^{-4}-2,065 \mathrm{z}^{-3}+3,779 \mathrm{z}^{-2}-3,141 \mathrm{z}^{-1}+1} \tag{24}
\end{align*}
$$

Figure 8: Parametric identification scheme
In this program, first matrices $A$ and $B$ are formed (they contain the coefficients of the four transfer functions). The input $u$ and perturbation $e$ of the leaded system are chosen as random type. For the $\hat{b}$ parameters of model $\hat{A}$ estimation one uses ARX operator from Matlab, which has the following syntax $\mathbf{t h}=\mathbf{A R X}(\mathbf{z}, \mathbf{n n})$, where $\boldsymbol{z}=\left[\begin{array}{ll}y & u\end{array}\right]$ - output data matrix ( $y$ ) and the input $(u) ; \mathbf{n n}=\left[\begin{array}{lll}n a & n b & n c\end{array}\right]$ defines the denominator order ( $n a$ ), numerator order ( $n b$ ) and the model's delay ( $n c$ ); th returns the estimated parameters in theta format (the elements of the vector $\hat{b}$ ) using the least square method. The program plots the characteristics $y(t)$ and $\hat{y}(t)$, presented in fig. $9 ; y(t)$ is the output of the leaded system A and $\hat{y}(t)$ - the output of the estimated
model ( A ). In fig. 9 one also presents the perturbation [7].
In fig. 9 the following characteristics families are presented: $\omega_{z}(t), \hat{\omega}_{z}(t)$ as responses to random input signal $\delta_{d}$ and random perturbation $e_{11}(t)$-fig.9.a; $\varphi(t), \hat{\varphi}(t)$ as responses to random input signal $\delta_{e}$ and random perturbation $e_{12}(t)$-fig.9.b; $\omega_{z}(t), \hat{\omega}_{z}(t)$ as responses to random input signal $\delta_{d}$ and random perturbation $e_{21}(t)$ - fig.9.c; $\varphi(t), \hat{\varphi}(t)$ as responses to random input signal $\delta_{d}$ and random perturbation $e_{22}(t)$ - fig.9.d.


d.

Figure 9: Lateral Parametric identification
The Matlab program used for least square method identification is presented in Appendix 4.

## 7. APPENDIX

## Appendix 1

close all;clear all;
$\mathrm{A}=\left[\begin{array}{lllll}1 & -1.6041 & 0.2776 & 0.302 & 0.0352\end{array}\right]$; $\mathrm{B}=\left[\begin{array}{llll}1.2449 & -1.2248 & -0.0513 & 0.0485\end{array}\right]$; tho $=$ poly2th(A,B); $u=i \operatorname{dinput}(300$, 'rbs'); $\mathrm{e}=\mathrm{randn}(300,1) ; \mathrm{y}=\mathrm{idsim}([\mathrm{u}, \mathrm{e}]$, tho $) ; \mathrm{z}=[\mathrm{y}, \mathrm{u}]$; th=iv4(z,[4 40]);
\%Calculul si reprezentarea grafica a reziduurilor ee=resid(z,th); h=figure; plot(ee);
title('Reziduurile metodei variabilelor instrumentale'); xlabel('Timp [s]');grid;
\%Compararea functiilor de transfer obtinute prin iv4,arx si metoda de analiza spectrala
[GS,NSS] $=\operatorname{spa}(\mathrm{z}) ;$ [Ga,NSa] $=\operatorname{trf}($ tho $) ; \mathrm{Gi}=\operatorname{trf}(\mathrm{th})$;
h=figure; bodeplot([GS Ga Gi]);
grid; present(tho); present(th)

## Appendix 2

close all;clear all;
$\mathrm{F}=\left[\begin{array}{llll}1 & -1.6041 & 0.2776 & 0.302 \\ 0.0352\end{array}\right] ;$ $\mathrm{B}=\left[\begin{array}{llll}1.2449-1.2248 & -0.0513 & 0.0485\end{array}\right]$; tho $=$ poly2th( $1, \mathrm{~B}, 1,1, \mathrm{~F}, 1$ ); $\mathrm{u}=\mathrm{idinp} \mathrm{d}(100$, 'rbs'); $\mathrm{e}=\mathrm{randn}(100,1)$;
$\mathrm{y}=\mathrm{idsim}([\mathrm{u}, \mathrm{e}]$, tho $) ; \mathrm{z}=[\mathrm{y}, \mathrm{u}] ;$ th=oe(z,[ 440$]$ );
present(tho); present(th);
\%Reprezentarea grafica a raspunsurilor sistemului condus si modelului
yl=idsim([u e],th); $\operatorname{plot}(\mathrm{y})$;
title('Semnalele de iesire ale sistemului condus si modelului'); xlabel('Timp [s]');
hold on;plot(y1,'r');grid;

## Appendix 3

close all;clear all;
$\mathrm{A}=\left[\begin{array}{lllll}1 & -1.6041 & 0.2776 & 0.302 & 0.0352\end{array}\right]$;
$\mathrm{B}=[1.2449-1.2248-0.0513$ 0.0485];
tho $=$ poly2th(A,B);u=idinput(300,'rbs');
$\mathrm{e}=\operatorname{randn}(300,1) ; \mathrm{y}=\mathrm{idsim}([\mathrm{u}, \mathrm{e}]$, tho $) ; \mathrm{z}=[\mathrm{y}, \mathrm{u}]$;
th $=\operatorname{armax}\left(z,\left[\begin{array}{lll}4 & 4 & 0\end{array}\right]\right) ; i=i v 4\left(z,\left[\begin{array}{lll}4 & 1\end{array}\right]\right)$; res=resid(z,th); [GS,NSS]=spa(z);
[Ga,NSa] $=\operatorname{trf}($ tho $) ; \mathrm{Gi}=\operatorname{trf}(\mathrm{i}) ; \mathrm{Ge}=\operatorname{trf}(\mathrm{th})$
bodeplot([GS Ga Ge Gi]); grid; present(tho); present(th);
$\mathrm{y} 1=\mathrm{idsim}([\mathrm{u}, \mathrm{e}]$, th $) ; \mathrm{h}=$ figure; $\operatorname{plot}(\mathrm{y})$; grid;hold;plot(yl,'r'); xlabel('Timp [s]');

## Appendix 4

close all;clear all;
$\%$ Intrarea 1 , iesirea 1
$\mathrm{A}=\left[\begin{array}{llll}1 & -3.0442 & 3.5019 & -1.7912 \\ 0.3403\end{array}\right]$;
$\mathrm{B}=\left[\begin{array}{lll}-1.58 & 4.2814 & -3.9394 \\ 1.2182\end{array}\right]$;
tho $=$ poly2th $(\mathrm{A}, \mathrm{B})$;
u11=idinput(300,'rbs');e11=randn(300,1);
y11=idsim([u11,e11],tho);
$\mathrm{z}=[\mathrm{y} 11, \mathrm{u} 11] ;$ th $11=\operatorname{arx}(\mathrm{z},[440])$;
yc11=idsim([u11,e11],th11);
subplot(211);plot(y11); grid; hold; plot(yc11,'r');
title('Variatia in timp a iesirilor modelului si sistemului condus cu intrarea 1, iesirea 1');
subplot(212); plot(e11); grid; title('Perturbatia e11'); xlabel('Timp[s]');
\%Intrarea 1, iesirea 2
$\mathrm{A}=\left[\begin{array}{llll}1 & -3.0442 & 3.5019 & -1.7912 \\ 0.3403\end{array}\right]$;
$\mathrm{B}=\left[\begin{array}{lll}0.039-0.4514-0.1676 ~ 0.0732\end{array}\right]$;
tho $=$ poly 2 th $(A, B)$;
u12 $=$ idinput( 300 ,'rbs');e12=randn(300,1);
$\mathrm{y} 12=\mathrm{idsim}([\mathrm{u} 12, \mathrm{e} 12]$, tho $)$;
$\mathrm{z}=[\mathrm{y} 12, \mathrm{u} 12]$;th12= $\operatorname{arx}(\mathrm{z},[440])$;
yc12=idsim([u12,e12],th12);
$\mathrm{h}=$ figure; $\operatorname{subplot}(211) ; \operatorname{plot}(\mathrm{y} 12)$;
grid; hold; plot(yc12,'r');
subplot(212); plot(e12); grid;
title('Perturbatia e12'); xlabel('Timp[s]');
$\%$ Intrarea 2, iesirea 1
$\mathrm{A}=\left[\begin{array}{llll}1 & -3.0442 & 3.5019 & -1.7912 \\ 0.3403\end{array}\right]$;
$\mathrm{B}=\left[\begin{array}{llll}0.3880 & -1.0484 & 1.0157 & -0.3406\end{array}\right]$;
tho $=$ poly2th(A,B);
u21=idinput(300,'rbs'); e21=randn(300,1);
$\mathrm{y} 21=\mathrm{idsim}([\mathrm{u} 21, \mathrm{e} 21], \mathrm{tho}) ; \mathrm{z}=[\mathrm{y} 21, \mathrm{u} 21]$;
th21 $=\operatorname{arx}(\mathrm{z},[440]) ; \mathrm{yc} 21=\mathrm{idsim}([\mathrm{u} 21, \mathrm{e} 21]$, th21 $)$;
$\mathrm{h}=$ figure; $\operatorname{subplot}(211) ; \operatorname{plot}(\mathrm{y} 21)$; grid;
hold; $\operatorname{plot}\left(\mathrm{yc} 21, \mathrm{r}^{\prime}\right)$; subplot(212);plot(e21); grid;
title('Perturbatia e21'); xlabel('Timp[s]');
$\%$ Intrarea 2 , iesirea 2
$\mathrm{A}=\left[\begin{array}{llll}1 & -3.0442 & 3.5019 & -1.7912 \\ 0.3403\end{array}\right]$;
$\mathrm{B}=[1.2585-0.384-0.9573$ 0.4866];
tho $=$ poly2th(A,B); u22=idinput(300,'rbs');
$\mathrm{e} 22=$ randn $(300,1) ; \mathrm{y} 22=\mathrm{idsim}([\mathrm{u} 22, \mathrm{e} 22]$,tho $)$;
$\mathrm{z}=[\mathrm{y} 22, \mathrm{u} 22]$; th22 $=\operatorname{arx}(\mathrm{z},[440])$;
yc22=idsim([u22,e22],th22);
$\mathrm{h}=$ figure; subplot (211); plot(y22); grid;
hold; plot(yc22,'r');
title('Variatia in timp a iesirilor modelului si sistemului condus cu intrarea 2, iesirea 2');
subplot(212); plot(e22); grid;
title('Perturbatia e22');xlabel('Timp[s]');

## 8. CONCLUSIONS

The identification of the aircraft's longitudinal move is achieved using four methods in Matlab medium. One observes that all the methods are very good and the transfer functions $H_{\delta_{p}}^{\theta}$ and $\hat{H}_{\delta_{p}}^{\theta}$ are very nearby. Is not important if the system, describing longitudinal move of an aircraft, is stabile or not. The most important thing is that the time characteristics $y(t)$ and $\hat{y}(t)$ are almost the same. That may be proved by the fact that the coefficients of the transfer function $H_{\delta_{p}}^{\theta}$ are near-by to those of $\hat{H}_{\delta_{p}}^{\theta}$.
The identification of the aircraft's lateral move is achieved using the method of the least square and a program in Matlab medium, which has as most representative command the command ARX. One may observe that superpose of the system's outputs and leading system's outputs is almost perfect.

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