TRANSIENT PARAMETERS OF A COIL WITH A SCREENED IRON CORE BY A SUPERCONDUCTOR TUBE

Constantin MANTEA, Ioan PUFLEA

INCDIE ICPE-CA
mantea@icpe-ca.ro

Abstract – **The transient electromagnetic field of a coil with an iron core screened by a superconducting tube is studied based on London's equations. The transient parameters of the coil are calculated. The result of this study are useful for design and investigation of an inductive fault current limiter with a high temperature superconducting shield.**

Keywords: *High temperature superconducting fault current limiters (HTSFCL), transient parameters, superconducting shields.*

1. INTRODUCTION

The theory of transient parameters, [1], [2], of the liniar electric circuits with non filiform elements gives the integral equations satisfied by current and voltage which generalize differential equations of the theory of filiform circuits. Their kernels are the transient resistence $r(t)$ and transient inductance $l(t)$. These transient parameters are computed by solving adequate problem of transient electromagnetic field.

In this paper the transient parameters of a cylindrical coil with an iron core screened by a superconducting tube are studied using the Laplace transform (operational method) [4], [5].

The geometrical configuration is shown in figure 1. The cylinders are long compared with their diameters, so the magnetic fields are longitudinal and the electric fields are circumferential. One thought that the magnetic flux there is only in the iron core ($\mu_r >> 1$).

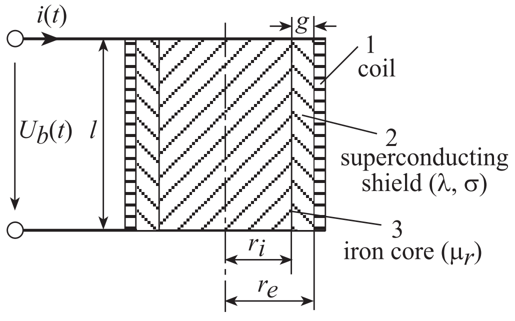

Figure 1

2. ELECTROMAGNETIC FIELD EQUATIONS

The assumptions that $\sigma = \infty$ and $\mu = 0$ are independent one of each other and neither one can give a correct description of superconductivity. The London`s equations realise the unification by describing the supercurrent $\bar{J}_s$ as being always determined by local electromagnetic field :

$$\nabla \times \bar{J}_s = -\frac{\bar{H}}{\lambda^2} \tag{1}$$

$$\frac{\partial \bar{J}_s}{\partial t} = \frac{\bar{E}}{\mu_0 \lambda^2} \tag{2}$$

The fact that the entropy changes continuously at the transition to the resistive state (no transition heat, only a discontinuity in the specific heat) shows that it is a gradual transition. Accordingly it is assumed that the total current $\bar{J}$ can be devided into two parts, the supercurrent $\bar{J}_s$ and the normal current $\bar{J}_n$:

$$\bar{J} = \bar{J}_s + \bar{J}_n \tag{3}$$

The normal current is connected with the electric field by Ohm`s law :

$$\bar{J}_n = \sigma \bar{E} \tag{4}$$

It`s further assumed the Maxwell`s equations :

$$\nabla \times \bar{H} = \bar{J} \tag{5}$$

$$\nabla \times \bar{E} = -\mu_0 \frac{\partial \bar{H}}{\partial t} \tag{6}$$

$$\nabla \cdot \bar{H} = 0 \tag{7}$$

$$\nabla \cdot \bar{E} = 0 \tag{8}$$

3. OPERATIONAL ELECTROMAGNETIC FIELD INSIDE OF SUPERCONDUCTING SHIELD

Combining London`s and Maxwell`s equations results:

$$\bar{H}(r,p)=\left[C_1\cdot I_0(\gamma r)+C_2\cdot K_0(\gamma r)\right]\cdot\bar{e}_z \quad (9)$$

where C_1 and C_2 are integral constants, I_0 and K_0 are modified Bessel`s functions of index zero and :

$$\gamma^2=\mu_0\cdot\sigma\cdot p+\frac{1}{\lambda^2} \quad (10)$$

$$\bar{E}(r,p)=-\left[C_1\cdot I_1(\gamma r)-C_2\cdot K_1(\gamma r)\right]\cdot \cdot\frac{\mu_0 p}{\gamma}\bar{e}_\varphi \quad (11)$$

where I_1 and K_1 are modified Bessel`s functions of index one.

4. OPERATIONAL ELECTROMAGNETIC FIELD INSIDE OF IRON CORE

Magnetic field is assumed constant :

$$\bar{H}_i=H_i\cdot\bar{e}_z \quad (12)$$

and results an electric field :

$$\bar{E}_i(r,p)=-\mu_0\mu_r\cdot\frac{rp}{2}\cdot H_i\cdot\bar{e}_\varphi \quad (13)$$

5. BOUNDARY CONDITIONS AND INTEGRAL CONSTANTS

$E(r,p)$ and $H(r,p)$ are continuous functions at the boundaries $r = r_i$ and $r = r_e$. Therefore, results the equations :

$$\begin{cases} C_1\cdot I_0(\gamma r_e)+C_2\cdot K_0(\gamma r_e)=H_e \\ C_1\cdot I_0(\gamma r_i)+C_2\cdot K_0(\gamma r_i)=H_i \\ C_1\cdot I_1(\gamma r_i)-C_2\cdot K_1(\gamma r_i)=D\cdot H_i \end{cases} \quad (14)$$

where H_e is the external magnetic field (magnetic field of the coil) and :

$$D=\frac{\mu_r\cdot\gamma\cdot r_i}{2}$$

From these equations one find the integral constants:

$$C_1=\frac{K_1(\gamma r_i)+D\cdot K_0(\gamma r_i)}{K_1(\gamma r_i)\cdot I_0(\gamma r_i)+K_0(\gamma r_i)\cdot I_1(\gamma r_i)} \quad (15)$$

$$C_2=\frac{I_1(\gamma r_i)-D\cdot I_0(\gamma r_i)}{K_1(\gamma r_i)\cdot I_0(\gamma r_i)+K_0(\gamma r_i)\cdot I_1(\gamma r_i)} \quad (16)$$

$$H_i=\frac{K_1(\gamma r_i)\cdot I_0(\gamma r_i)+K_0(\gamma r_i)\cdot I_1(\gamma r_i)}{I_0(\gamma r_e)\cdot\tilde{K}+K_0(\gamma r_e)\cdot\tilde{I}}\cdot H_e,$$
$$\tilde{K}\equiv K_1(\gamma r_i)+D\cdot K_0(\gamma r_i),$$
$$\tilde{I}\equiv I_1(\gamma r_i)-D\cdot I_0(\gamma r_i). \quad (17)$$

6. OPERATIONAL IMPEDANCE

Taking into account the asymptotic expressions of the Bessel functions :

$$I_0(x)=I_1(x)=\frac{e^x}{\sqrt{2\pi x}} \quad (18)$$

$$K_0(x)=K_1(x)=\sqrt{\frac{\pi}{2x}}\cdot e^{-x} \quad (19)$$

and :

$$H_e(p)=\frac{N\cdot I(p)}{l} \quad (20)$$

one obtains :

$$H_i(p)=\frac{1}{\mathrm{ch}\sqrt{p\tau+\alpha}+\frac{\sqrt{p\tau+\alpha}}{m}\cdot\mathrm{sh}\sqrt{p\tau+\alpha}}\cdot \cdot\sqrt{\frac{r_e}{r_i}}\cdot\frac{N\cdot I(p)}{l} \quad (21)$$

where :

$$m=\frac{2g}{\mu_r r_i};\qquad \alpha=\frac{g^2}{\lambda^2};\qquad \tau=\mu_0\sigma g^2$$

Then it was computed operational magnetic flux, terminal voltage and, finally, operational impedance :

$$Z(p)=\frac{U_b(p)}{I(p)}= =L_0\cdot\frac{p}{\mathrm{ch}\sqrt{p\tau+\alpha}+\frac{\sqrt{p\tau+\alpha}}{m}\cdot\mathrm{sh}\sqrt{p\tau+\alpha}} \quad (22)$$

with :

$$L_0=\mu_0\mu_r\cdot\frac{\pi r_i^2N^2}{l}\cdot\sqrt{\frac{r_e}{r_i}}\,.$$

7. TRANSIENT PARAMETERS OF COIL

According to the general theory of the transient parameters, [1], the transient resistence and inductance can be defined by their Lapalace transforms :

$$R(p) = \frac{Z(p)}{p} \tag{23}$$

$$L(p) = \frac{Z(p)}{p^2} \tag{24}$$

The transient resistence $r(t)$ and inductance $l(t)$ are obtained by performing the inverse Laplace transform (Heaviside`s formula) :

$$\frac{r(t)}{L_0} = \frac{1}{\tau} \cdot \sum_{k=1}^{\infty} \frac{2\xi_k m\sqrt{\xi_k^2 + m^2}}{\xi_k^2 + m^2 + m} \cdot e^{-\left(\xi_k^2+\alpha\right)\cdot\frac{t}{\tau}} \tag{25}$$

$$\frac{l(t)}{L_0} = \frac{1}{\operatorname{ch}\frac{g}{\lambda} + \frac{1}{m}\frac{g}{\lambda}\cdot\operatorname{sh}\frac{g}{\lambda}} - \sum_{k=1}^{\infty} \frac{2\xi_k m\sqrt{\xi_k^2 + m^2}}{\left(\xi_k^2 + \alpha\right)\left(\xi_k^2 + m^2 + m\right)} \cdot e^{-\left(\xi_k^2+\alpha\right)\cdot\frac{t}{\tau}} \tag{26}$$

in which ξ_k are the roots of the transcedental equation [3] :

$$m \cdot \operatorname{ctg} \xi_k = \xi_k \tag{27}$$

Must be point out that :

$$r(0) = \infty; \quad r(\infty) = 0 \tag{28}$$

$$l(\infty) = \frac{1}{\operatorname{ch}\frac{g}{\lambda} + \frac{1}{m}\cdot\operatorname{sh}\frac{g}{\lambda}} \tag{29}$$

8. CONCLUSIONS

The transient parameters of the coil are monotonic functions of time. The transient resistence is infinite at the first moment because of the surface distribution of the initial Meissner`s and eddy currents. In the steady state, due to the presence of the iron core, the coil is inductive.

References

[1] R. Radulet, *Introduction des parametres transitoires dans l`etude de circuit electriques ayant des elements non filiforme*, Rev. Roum. Sci. Techn. Electrotechn. Et Energie, vol. 11. nr. 4, 1966, pp. 565- 573

[2] R. Radulet, *O teorie generala a parametrilor lineici tranzitorii*, St. Cerc. Energ. Electr., vol. 16, nr. 3, 1966, pp. 417-421

[3] C. Bala, *Efectul pelicular tranzitoriu*, St. Cerc. Energ. Electr., vol. 17, nr. 3, 1966, pp. 621-632

[4] M. Ichikawa, *Inductive type FCL*, IEEE Trans. Appl. Superconductivity, vol. 13, no. 2, 2003, pp. 2004-212

[5] V. Sokolovsky, *Superconducting FCL: Design and Application*, Trans. Appl. Superconductivity, vol. 14, no. 3, 2004, pp. 1990-1996