

## INDUCTION MOTORS DYNAMIC REGIMES ANALYSIS METHODS

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**Abstract** – Induction motors dynamic regimes analysis importance is enforced by the intensive use of such motors in electrical drives applications. Thus, the knowledge and understanding of these phenomena may lead to a decrease in electrical energy consumption in the context of continuous diminishing of energy resources. This paper makes an overview of a developed simulation platform, created in the MATLAB-SIMULINK environment, for modeling of an induction motor. Based on this simulation platform, a dynamic regime study for the no-load and under-load starting processes may be carried out during the early stages of the motor's design.

**Keywords:** *induction motor, dynamic regime, motor model.*

### 1. THE INDUCTION MOTOR'S DYNAMIC REGIME MODEL

The electromagnetic/mechanic dynamic regime of an induction motor may be described by a mathematical model. This model is written supposing the following simplifying hypothesis [1]:

- the motor's iron parts magnetic permeability is considered infinite;
- the iron losses are neglected;
- the motor is considered to be symmetric from the electric, magnetic and constructive points of view;
- no magnetic fields harmonic interaction is considered;
- the motor's gap is considered to be constant;
- the slot effect is neglected;
- the supplying voltage is considered to be sinusoidal;
- the motor's rotor is considered to be wounded and short-circuited.

#### A. The induction motor's K reference frame model

The KRF (K Reference Frame) mathematical model of an induction motor is given by the following equations [1]:

$$\begin{aligned} u_{sK} &= R_1 \cdot i_{sK} + \frac{d\varphi_{sK}}{dt} + j \cdot \omega_K \cdot \varphi_{sK} \\ 0 &= R_2 \cdot i_{rK} + \frac{d\varphi_{rK}}{dt} + j(\omega_K - \omega) \varphi_{rK} \\ \left(\frac{J}{p}\right) \cdot \frac{d\omega}{dt} &= m - m_r - \left(\frac{F}{p}\right) \cdot \omega \\ m &= \frac{3}{2} \cdot p \cdot L_m \cdot \Im(i_{sK} \cdot i_{rK}^*) \\ \varphi_{sK} &= L_s \cdot i_{sK} + L_m \cdot i_{rK} = L_{\sigma 1} \cdot i_{sK} + \varphi_u \\ \varphi_{rK} &= L_r \cdot i_{rK} + L_m \cdot i_{sK} = L_{\sigma 2} \cdot i_{rK} + \varphi_u \\ \varphi_u &= L_m (i_{sK} + i_{rK}) = L_m \cdot i_{mK} \end{aligned} \quad (1)$$

where:

$\omega_K = p\Omega_K$ ,  $\omega = p\Omega$  – KRF/RRF (Rotor Reference Frame – fixed to the rotor) electrical speed, both referred to the SRF (Stator Reference Frame) – figure 1,

$p$  – the number of pole pairs,

$\Omega_K = \left(\frac{d\theta_K}{dt}\right)/p$ ,  $\Omega = \left(\frac{d\theta}{dt}\right)/p$  – KRF/RRF

mechanical speed, both referred to the SRF,

$\theta_K$ ,  $\theta$  – the KRF/RRF electrical angle (figure 1), both referred to the SRF.

$$\begin{aligned} u_{sK} &= u_s e^{-j\theta_K}; i_{sK} = i_s e^{-j\theta_K}; i_{rK} = i_r e^{-j(\theta_K - \theta)}; i_{mK} = \\ \varphi_{sK} &= \varphi_s e^{-j\theta_K}; \varphi_{rK} = \varphi_r e^{-j(\theta_K - \theta)} \end{aligned}$$

the voltage, stator/rotor/magnetizing current and stator/rotor flux space vectors expressed in the KRF,

$J$  – total inertial momentum,

$m$  – electromagnetic torque,

$m_r$  – the load's torque,

$R_1, R_2$  – stator/rotor winding resistance,

$L_m$  – mutual inductance (appendix A),

$L_s, L_r$  – stator/rotor self inductance (appendix A)

$L_{\sigma 1}, L_{\sigma 2}$  – stator/rotor leakage inductance (appendix A),

$F$  – the windage losses coefficient,

$\varphi_m$  – magnetizing flux.

As figure 1 shows the KRF is an arbitrary reference frame and so the presented model (1) may be written in different ways in accordance with the used reference frame or the chosen state variables.

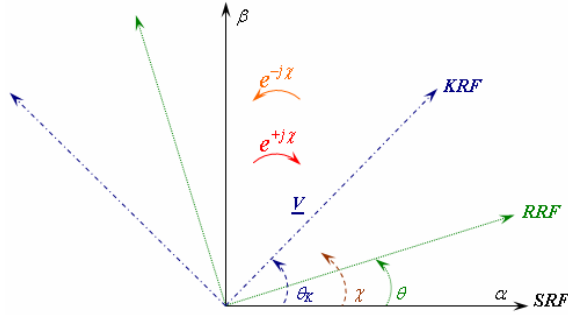


Figure 1. The used reference frames

## B. The induction motor's SRF model using currents as state variables

Based on (1) and considering  $\omega_K = 0$  the induction motor's SRF model using statoric and rotor currents as state variables is obtained:

$$\begin{aligned} \frac{di_s}{dt} &= \frac{u_s}{\sigma \cdot L_s} - \frac{i_s R_1}{\sigma \cdot L_s} + \beta \cdot R_2 \cdot i_{rS} - j \cdot \omega \cdot \beta (L_r \cdot i_{rS} + L_m \cdot i_s) \\ \frac{di_{rS}}{dt} &= -\beta \cdot u_s + \beta \cdot R_1 \cdot i_s - \frac{i_{rS} R_2}{\sigma \cdot L_r} + j \cdot \omega \left( \frac{i_{rS}}{\sigma} + \beta \cdot L_s \cdot i_s \right) \\ \frac{d\omega}{dt} &= \mu \cdot L_m \cdot \Im m(i_s \cdot i_{rS}^*) - M_r \cdot \frac{p}{J} - \frac{F \cdot \omega}{J} \end{aligned} \quad (2)$$

where:

$$\begin{aligned} \alpha &= \frac{R_2}{L_r}, \beta = \frac{L_m}{\sigma \cdot L_s \cdot L_r}, \sigma = 1 - \frac{L_m^2}{L_s \cdot L_r}, \mu = \frac{3}{2} \frac{p^2}{J} \\ \gamma &= \frac{L_m^2 \cdot R_2}{\sigma \cdot L_s \cdot L_r^2} + \frac{R_1}{\sigma \cdot L_s} \end{aligned} \quad (3)$$

$i_{rS}$  – the rotor current space vector expressed in the SRF.

Equations (2) and (3) may be decomposed over the both axis of the SRF ( $\alpha, \beta$ ) and rewritten as a matrix system:

$$\begin{cases} \dot{x} = A \cdot x + B \cdot u \\ y = C \cdot x \end{cases} \quad (4)$$

where:

$x = [i_{sa} \ i_{sb} \ i_{ra} \ i_{rb}]^T$  – state variables vector,  
 $u = [u_{sa} \ u_{sb}]^T$  – input signals vector,  
 $y$  – output signals vector,

$$A = \begin{bmatrix} -\frac{1}{\sigma \cdot T_s} & \frac{1-\sigma}{\sigma} \omega & \frac{L_m}{\sigma \cdot L_s \cdot T_r} & \frac{L_m}{\sigma \cdot L_s} \omega \\ -\frac{1-\sigma}{\sigma} \omega & -\frac{1}{\sigma \cdot T_s} & -\frac{L_m}{\sigma \cdot L_s} \omega & \frac{L_m}{\sigma \cdot L_s \cdot T_r} \\ \frac{L_m}{\sigma \cdot L_r \cdot T_s} & -\frac{L_m}{\sigma \cdot L_r} \omega & -\frac{1}{\sigma \cdot T_r} & -\frac{1}{\sigma} \omega \\ \frac{L_m}{\sigma \cdot L_r} & \frac{L_m}{\sigma \cdot L_r \cdot T_s} & \frac{1}{\sigma} \omega & -\frac{1}{\sigma \cdot T_r} \end{bmatrix}$$

– state variables coefficients matrix,

$$B = \begin{bmatrix} \frac{1}{\sigma \cdot L_s} & 0 \\ 0 & \frac{1}{\sigma \cdot L_s} \\ -\frac{L_m}{\sigma \cdot L_s \cdot L_r} & 0 \\ 0 & -\frac{L_m}{\sigma \cdot L_s \cdot L_r} \end{bmatrix} \quad \text{– input signals}$$

coefficients matrix,

C – output signals coefficients matrix.

The electromagnetic torque is defined by:

$$m = \frac{3}{2} p \cdot L_m \cdot (i_{ra} \cdot i_{sb} - i_{rb} \cdot i_{sa}) \quad (5)$$

## 2. THE INDUCTION MOTOR'S MATLAB-SIMULINK MODEL

Equations (4) and (5) define the induction motor's MATLAB-SIMULINK model.

It has as input signals the supplying voltages and the load torque. The load torque is invariant with the speed. The voltages are transformed in the SRF using the Clarke equations:

$$\begin{cases} u_{sa} = u_A \\ u_{sb} = \left(1/\sqrt{3}\right) \cdot (u_B - u_C) \end{cases} \quad (6)$$

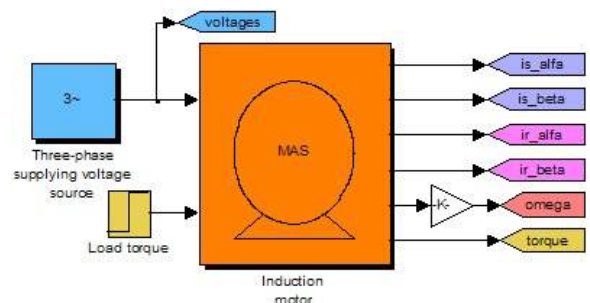


Figure 2. The induction motor's SIMULINK diagram.

The induction motor's parameters  $R_1$ ,  $R_2$ ,  $L_{\sigma 1}$ ,  $L_{\sigma 2}$ ,  $J$ ,  $p$  and  $F$  are known. The output signals are: the statoric and rotoric currents components over the

SRF axes  $i_{s\alpha}$ ,  $i_{s\beta}$ ,  $i_{r\alpha}$ ,  $i_{r\beta}$ , the rotor's mechanical speed  $n = 60 \cdot \omega / (2 \cdot \pi \cdot p)$  expressed in rpm, and the electromagnetic torque  $m$ .

Figure 2 depicts the model's SIMULINK diagram. The MAS block (figure 2) is detailed in figure 3. The S-function block (figure 3) is the MATLAB transcription of (4) and (5). The S-function code [2] is shown in figure 4.

As figure 3 shows the mutual inductance  $L_m$  is a local variable of the MAS block. Its value varies with the magnetizing current  $i_m$ . An analytical expression of the non-linear characteristic  $L_m = f(i_m)$  is given by [3]:

$$L_m = \begin{cases} L_{m0} & i_m \leq i_{m0} \\ \frac{L_{m0}}{1 + \alpha \cdot L_{m0} \cdot i_m \cdot \left( \frac{1}{i_{m0}} - \frac{1}{i_m} \right)^2} & i_m > i_{m0} \end{cases} \quad (7)$$

where  $\alpha$ ,  $L_{m0}$  and  $i_{m0}$  are analytically obtained by imposing specific coordinates ( $L_m$ ,  $i_m$ ) to the characteristic.

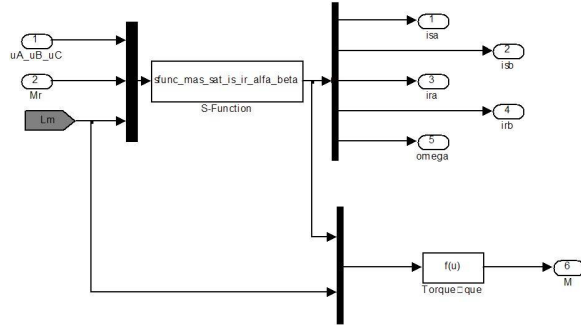


Figure 3. The MAS block from figure 2.

The magnetizing current from (7) is defined by:

$$i_m = \sqrt{(i_{s\alpha} + i_{r\alpha})^2 + (i_{s\beta} + i_{r\beta})^2} \quad (8)$$

In this way the core's saturation effect is taken into account. If the mutual inductance  $L_m$  is considered constant then the core's saturation effect is neglected.

The model described above was used for the study of the no-load and under-load starting regimes. A comparison was performed to see if the core's saturation effect has an influence over the dynamic behavior of the motor.

```
function [sys, x0] = sfunc_mas_sat_is_ir_alpha_beta(t,x,u,flag,Rs,Rr,Ls,Lr,p,F,Jm,Lls,Llr)
if flag==0,
    sys = [5,0,5,5,0,0];
    % SYS(1) = Number of continuous states.
    % SYS(2) = Number of discrete states.
    % SYS(3) = Number of outputs.
    % SYS(4) = Number of inputs.
    x0=[0;0;0;0;0;0];
elseif flag==1,
    us_alpha = u(1);
    us_beta = (u(2)-u(3))/sqrt(3);
    Lm=Lls+Llr;
    Ls=Lls+Llr;
    Lr=Llr+Llr;
    sgm=1-Lm^2/(Ls*Lr);
    Ts=Ls/Rs;
    Tr=Lr/Rr;
    a11 = -1/(sgm*Ts);
    a12 = Lm^2/(sgm*Ls*Lr)*x(5);
    a13 = Lm/(sgm*Ls*Tr);
    a14 = Lm/(sgm*Ls)*x(5);
    a21 = -a12;
    a22 = a11;
    a23 = -a14;
    a24 = a13;
    a31 = Lm/(sgm*Lr*Ts);
    a32 = -Lm/(sgm*Lr)*x(5);
    a33 = -1/(sgm*Tr);
    a34 = -1/(sgm)*x(5);
    a41 = -a32;
    a42 = a31;
    a43 = -a34;
    a44 = a33;
    b11 = 1/(sgm*Ls);
    b12 = 0;
    b21 = 0;
    b22 = 1/(sgm*Ls);
    b31 = -Lm/(sgm*Ls*Lr);
    b32 = 0;
    b41 = 0;
    b42 = b31;
    sys(1)=a11*x(1)+a12*x(2)+a13*x(3)+a14*x(4)+b11*us_alpha+b12*us_beta;
    sys(2)=a21*x(1)+a22*x(2)+a23*x(3)+a24*x(4)+b21*us_alpha+b22*us_beta;
    sys(3)=a31*x(1)+a32*x(2)+a33*x(3)+a34*x(4)+b31*us_alpha+b32*us_beta;
    sys(4)=a41*x(1)+a42*x(2)+a43*x(3)+a44*x(4)+b41*us_alpha+b42*us_beta;
    m=3/2*p*Lm*(x(3)*x(2)-x(4)*x(1));
    sys(5)= p/Jm*(m-u(4)-F*x(5)/p);
elseif flag==3,
    sys(1)=x(1);
    sys(2)=x(2);
    sys(3)=x(3);
    sys(4)=x(4);
    sys(5)=x(5); %turtatia rotorului [rad/s]
else
    sys=[];
end
```

Figure 4. The S-function code.

### 3. THE NO-LOAD STARTING REGIME

Figure 5 shows the *mas\_param.m* file where the user defines the MATLAB-SIMULINK model's parameter values.

```
File Edit Text Go Tools Des
1 Rs=...;
2 Rr=...;
3 Lm0=...;
4 Lls=...;
5 Llr=...;
6 Ls=Lls+Lm0;
7 Lr=Llr+Lm0;
8 Jm=...;
9 F=...;
10 p=...;
```

Figure 5. The *mas\_param.m* file.

The induction motor is characterized by the following rated data:

- $P_n = 4 \text{ kW}$  ;
- $n_n = 1455 \text{ rpm}$  ;
- $U_n = 400 \text{ V}$  (wye connection);
- $I_n = 4.3 \text{ A}$  ;
- $\cos \varphi_n = 0.86$  ;
- $f_n = 50 \text{ Hz}$  .
- $R_s = R_1 = 3.914 \Omega$  ;
- $R_r = R_2 = 2.71 \Omega$  ;

$$L_{ls} = L_{\sigma 1} = 3.58 \cdot 10^{-2} \text{ H} ;$$

$$L_{lr} = L_{\sigma 2} = 5.86 \cdot 10^{-2} \text{ H} ;$$

$$J = 0.0084 \text{ kgm}^2 ;$$

$$F = 0.005 \text{ N} \cdot \text{m} \cdot \text{s/rad} .$$

The non-linear characteristic  $L_m = f(i_m)$  is shown in figure 6.

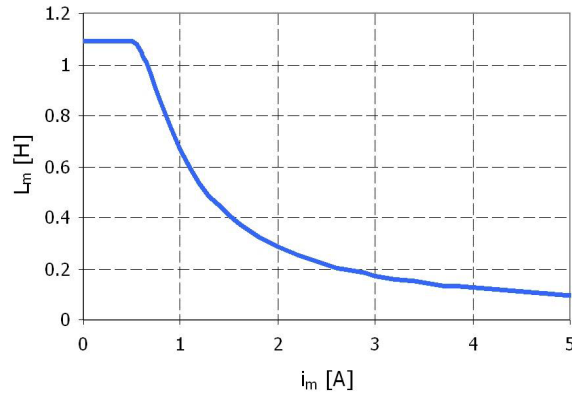


Figure 6. The non-linear characteristic  $L_m = f(i_m)$ .

The above characteristic is described by (7) with the following parameters:

$$i_{m0} = 1.096 \text{ A} ;$$

$$L_{m0} = 1.09 \text{ H} ;$$

$$\alpha = 0.55 .$$

During the dynamic regime the leakage inductances and the resistances are supposed constant. The load torque is generated only by the windage losses described by the parameter  $F$ . The supplying voltages are those presented in figure 7.

Based on the simulations the following quantities time variations are presented: the instantaneous statoric current (figure 8), the RMS statoric current (figure 9), the electromagnetic torque (figure 10), the instantaneous rotoric current (figure 11), the mutual inductance (figure 12) and the motor's speed (figure 13).

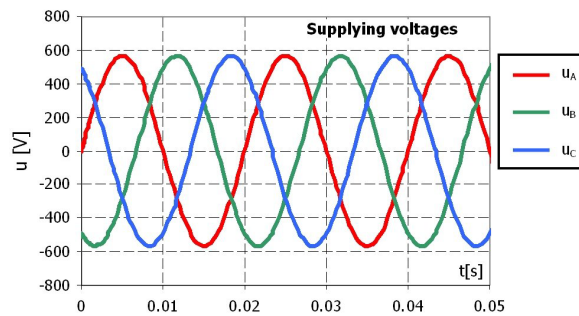


Figure 7. The supplying voltages.

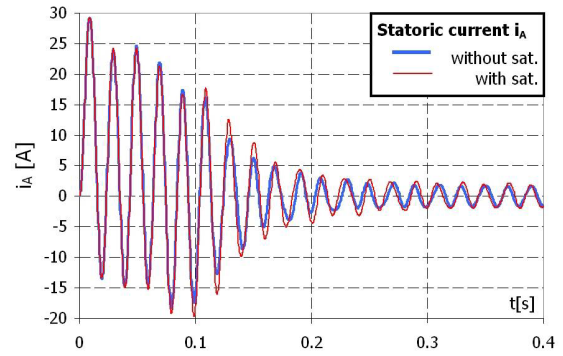


Figure 8. The instantaneous statoric current.

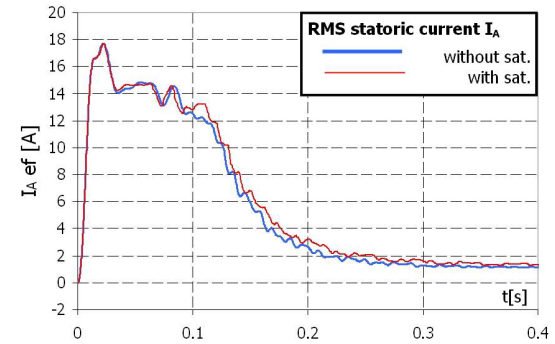


Figure 9. The RMS statoric current.

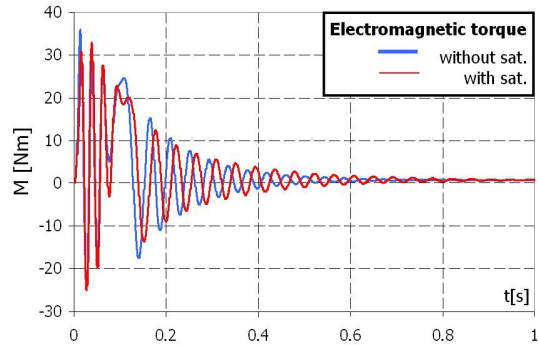


Figure 10. The electromagnetic torque.

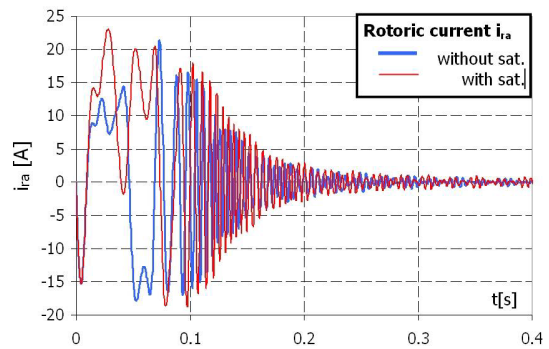


Figure 11. The instantaneous rotoric current.

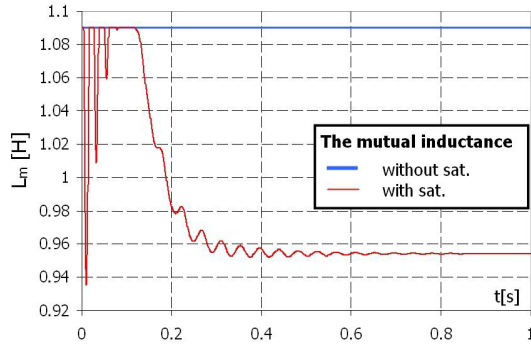


Figure 12. The mutual inductance.

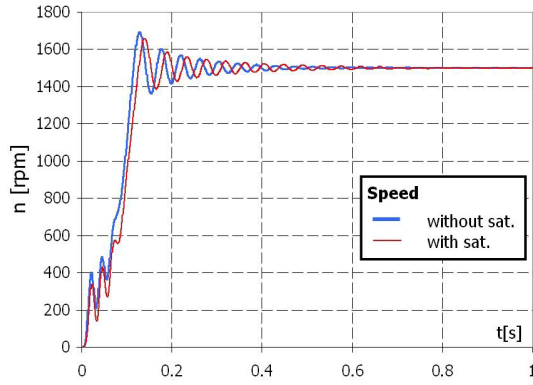


Figure 13. The motor's speed.

#### 4. THE UNDER-LOAD STARTING REGIME

For the study of this regime the same MATLAB model as it has been described in section 2 was used. The motor's parameters and rated data are those described in section 3.

The used non-linear characteristic  $L_m = f(i_m)$  is the one shown in figure 6. The supplying voltages are those presented in figure 7.

In this case the resistant torque is formed by the load torque ( $m_r = 7 \text{ Nm}$ ) and by the windage torque (described by the parameter  $F$ ).

The following quantities time variations are the same as those presented in section 3: the instantaneous statoric current (figure 14), the RMS statoric current (figure 15), the electromagnetic torque (figure 16), the instantaneous rotor current (figure 17), the mutual inductance (figure 18) and the motor's speed (figure 19).

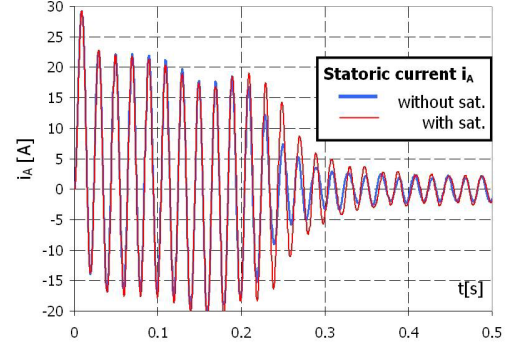


Figure 14. The instantaneous statoric current.

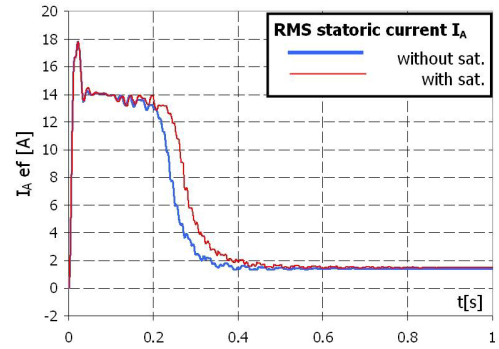


Figure 15. The RMS statoric current.

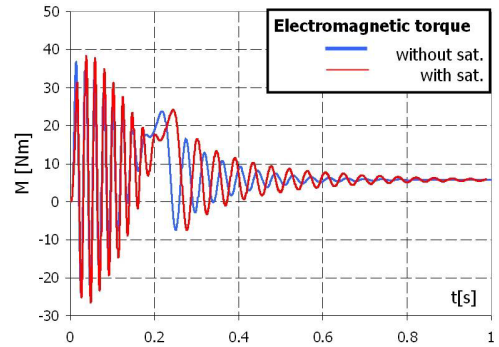


Figure 16. The electromagnetic torque.

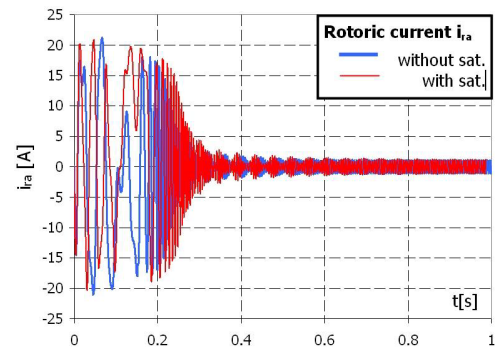


Figure 17. The instantaneous rotor current.



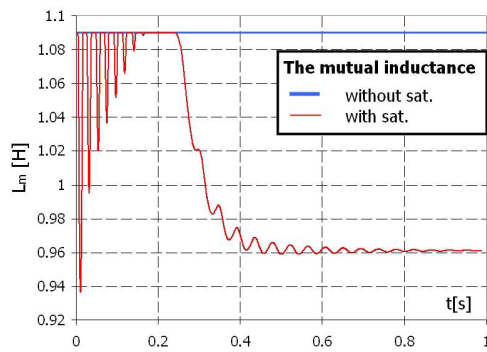


Figure 18. The mutual inductance.

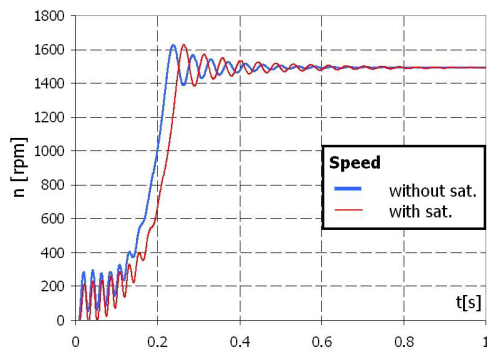


Figure 19. The motor's speed.

#### 4. CONCLUSIONS

An overview of a wound induction motor, with short-circuited rotor, simulation model, developed in the MATLAB-SIMULINK environment, is performed. Based on this simulation model, a study for the no-load and under-load starting regimes may be carried out during the early stages of the motor's design.

The main contribution of this work has been the development of the S-function file (as a transcription of (4) and (5) – see figure 4), which is the MATLAB SRF mathematical model based on the statoric  $i_s$  and rotoric  $i_{rS}$  currents, and of the induction motor's SIMULINK diagram (see figure 2).

For the no-load starting regime some interesting phenomena may be observed. Thus, when the core's

saturation effect is considered, the values of the currents are increased. This effect may be observed on the instantaneous values (figures 8 and 11) and the RMS values (figure 9). The increased values of the currents lead to increased values of the electromagnetic torque, therefore the saturation effect should be considered in all computer simulations.

Another phenomenon is that the instantaneous values of the statoric currents are less damped when the saturation effect is considered.

Although there are only small differences between the two cases, the one "with saturation" has a slower acceleration (figure 13).

For the under-load starting regime the phenomena are similar to those obtained for the no-load starting regime.

Because of the motor's load the currents reach higher values, a higher driving torque being needed.

The currents values are larger if the core's saturation effect is considered but the difference between "with saturation" and "without saturation" values is greater in this case.

As one may expect the starting regime lasts longer in this case (about two times the no-load starting regime). Likewise the no-load starting regime the motor has a slower acceleration if the saturation effect is considered but in this case the difference between "with saturation" and "without saturation" values is more important (figure 19).

All the above prove that the developed simulation model supplies realistic results. The great advantage of this model is that the user has to buy only the MATLAB- SYMULINK environment there being no reason for buying additional toolboxes.

#### References

- [1] C. Ghiță, *Convertoare electromecanice – Volumul 1*, Editura ICPE, București, 1998.
- [2] <http://www.mathworks.com>.
- [3] R. K. Ursem, P. Vadstrup, *Parameter Identification of Induction Motors using Stochastic Optimization Algorithms*, Applied Soft Computing, 13<sup>th</sup> August 2003.