# METROLOGICAL CARTESIAN COORDINATES IMPEDANCE SIMULATORS 

Vitalie NASTAS<br>Technical University of Moldova, Chişhinău, e-mail: vitalienastas@gmail.com


#### Abstract

The paper contains the procedure of formally - structural synthesis and analysis of the metrological Cartesian coordinates impedance simulators. The simulators are characterized by high accuracy of the reproduced impedance and possess the possibility of separate control of the impedance components. The devices are intended for applying as impedance measure in automatic resonance impedance meters and are oriented to control by microcontroller or by another computer.


Keywords: impedance measurement, Cartesian coordinates impedance simulator.

## 1. INTRODUCTION

A modern practical implementation of the resonance method [1] is based on application of impedance simulator as the impedance standard. They ensure the creation of impedance with different character of components: active, reactive or complex [2]. The impedance simulator executes the function of the reference elements (measure) and there are imposed to it some requirements bound with metrological support of measurements. Among them:
Low error and high stability of reproduced impedances;
Possibility of any character impedance reproduction and the separate control of the impedance components;
The known and warranted systematic error;
Digital control.
Application of Cartesian - coordinates impedance simulators (C-MSI) for these purposes present a large practical interest, because in this case the considerable simplification of measurement algorithm and of it practical implementation is possible.
To C-MSI, apart from defined above requirements, the following supplementary requirements can be formulated:
Separate regulation of the active and reactive components of the reproduced impedance;
Necessary resolution and exactitudes of the impedance components regulation;
Assurance of control bands for the active component in the limits $-\boldsymbol{R}_{\min } \div+\boldsymbol{R}_{\text {max }}$ and for the reactive component in the limits
$-j X_{\text {min }} \div+$

The absolute stability of the circuits at the variation of the reproduced impedance components and of the external impedances in necessary limits.

Realization of impedance simulators answering to the aforecited requirements is possible on the basis of operational amplifiers (OA) with negative and positive feedbacks. Due to properties of modern OA, the parameters of reproduced impedances are determined with high exactitude only by feedbacks and do not depend on OA characteristics. The nonideality of OA properties results in an error of reproduced impedances which magnitude can be determinate.

## 2. IMPEDANCE SIMULATORS

As it is known [3], two basic types of impedance simulators are in essence possible: the current controlled (I - MSI) and the voltage controlled ( U - MSI).
Both types of MSI presents the practical interest, since I-MSI save stability down to a condition of noload operation, and U-MSI - down to a condition of a short-circuit [3]. These properties do by their suitable for application as impedance measures accordingly in the series (I - MSI) and in the parallel ( U - MSI) resonance measuring circuit [1].

For elaboration of the impedance simulators circuits the method of formal synthesis based on the conversion algorithm of magnitudes in the synthesized device is used [5].

### 2.1. The Current Controlled Impedance Simulator (I-MSI)

The current $\mathbf{I}_{\mathbf{i}}$, flowing past through the reproduced impedance is used as initial entering quantity for I-MSI at synthesis of its structure. The impedance $\mathbf{Z}_{\mathbf{i}}$, reproduced by the simulator, may be represented as:

$$
\begin{equation*}
\mathbf{Z}_{\mathrm{i}}=\mathrm{R}_{\mathrm{i}}+\mathrm{j} \mathrm{X}_{\mathrm{i}}, \tag{1}
\end{equation*}
$$

where $R_{i}$ - the active component of synthesized impedance, $X_{i}-$ its reactive component. The components $\mathrm{R}_{\mathrm{i}}, \mathrm{X}_{\mathrm{i}}$ must possess the possibility of independent control.

For synthesis the simulator circuit, the block diagram of the information conversion algorithm represented in the fig. 1.a is used.
The current $\mathbf{I}_{\mathbf{i}}$ is converted into the voltage $\mathbf{U}_{\mathbf{1}}$, used for creation the voltage drops on the active $\left(\mathbf{U}_{\mathbf{R}}\right)$ and on the reactive $\left(\mathbf{U}_{\mathbf{x}}\right)$ components of the reproduced impedance $\mathbf{Z}_{\mathbf{i}}$. The turn of the voltage $\mathbf{U}_{\mathbf{1}}$ phase on the angle $90^{\circ}$ with consequent regulation of its magnitude at the factor $\mathrm{N}_{\mathrm{X}}$ for creation the voltage
$\mathbf{U}_{\mathbf{X}}$ are used. Only the regulation of magnitude $\mathbf{U}_{\mathbf{1}}$ on factor $N_{R}$ for creation $U_{R}$ is applied.
The voltages $\mathbf{U}_{\mathbf{R}}$ and $\mathbf{U}_{\mathbf{X}}$ are summarized, forming the voltage $\mathbf{U}_{\mathbf{i}}$, which, in conjunction with the current $\mathbf{I}_{\mathbf{i}}$, form the reproduced impedance $\mathbf{Z}_{\mathrm{i}}$.
Presented above algorithm of information conversion is realized in the block - diagram of the impedance simulator represented in the fig. 1.b.


Fig. 1. The conversion algorithm (a) and the structure (b) of I-MSI

The current - voltage converter IUC is applied for conversion of the current $\mathbf{I}_{i}$ in voltage $\mathbf{U}_{1}$ :

$$
\begin{equation*}
\mathbf{U}_{\mathbf{1}}=\mathbf{I}_{\mathbf{i}} \cdot \mathrm{R}_{\mathrm{C}}-\mathbf{U}_{\mathbf{i}} \tag{2}
\end{equation*}
$$

where $\mathrm{R}_{\mathrm{C}}$ - the conversion factor of the converter $\mathrm{I} / \mathrm{U}$. To obtain algorithmically correct dependence between the current $\mathbf{I}_{\mathbf{i}}$ and the voltage $\mathbf{U}_{\mathbf{1}}$ by elimination of effect of a stray feedback [5], the differential amplifier DA is applied. The voltage on it output:

$$
\begin{equation*}
\mathbf{U}_{\mathbf{1}}{ }^{1}=\mathbf{I}_{\mathbf{i}} \cdot \mathrm{R}_{\mathrm{C}}-\mathbf{U}_{\mathbf{i}}+\mathbf{U}_{\mathbf{i}}=\mathbf{I}_{\mathbf{i}} \cdot \mathrm{R}_{\mathrm{C}} \tag{3}
\end{equation*}
$$

For creation of phase shift $90^{\circ}$ the phasor F , and for regulation of voltages - the programmable amplifiers PA1, PA2 are used. Formed with these elements the voltages $\mathbf{U}_{\mathbf{R}}, \mathbf{U}_{\mathbf{X}}$ are equal respectively:

$$
\begin{gather*}
\mathbf{U}_{R}=N_{R} \cdot \mathbf{U}_{\mathbf{1}}{ }^{1}=N_{R} \cdot R_{C} \cdot \mathbf{I}_{\mathbf{i}}  \tag{4}\\
\mathbf{U}_{\mathbf{X}}=\mathrm{N}_{\mathrm{X}} \cdot \mathbf{U}_{\mathbf{1}}{ }^{1} \cdot \mathrm{j} \sin 90^{\circ}=\mathrm{j} \mathrm{~N}_{\mathrm{X}} \cdot \mathrm{R}_{\mathrm{C}} \cdot \mathbf{I}_{\mathbf{i}} \tag{5}
\end{gather*}
$$

The summer amplifier SA sum the voltages $\mathbf{U}_{\mathbf{R}}, \mathbf{U}_{\mathbf{X}}$ and forming the voltage $\mathbf{U}_{\mathbf{i}}$ applied to the input of the simulator:

$$
\begin{gather*}
\mathbf{U}_{\mathbf{i}}=\mathbf{U}_{\mathbf{R}}+\mathbf{U}_{\mathbf{X}}=\mathrm{N}_{\mathrm{R}} \cdot \mathrm{R}_{\mathrm{C}} \cdot \mathbf{I}_{\mathbf{i}}+\mathrm{j} \mathrm{~N}_{\mathrm{X}} \cdot \mathrm{R}_{\mathrm{C}} \cdot \mathbf{I}_{\mathbf{i}}= \\
=\mathrm{R}_{\mathrm{C}}\left(\mathrm{~N}_{\mathrm{R}}+\mathrm{j} \mathrm{~N}_{\mathrm{X}}\right) \mathbf{I}_{\mathbf{i}} \tag{6}
\end{gather*}
$$

The impedance $\mathbf{Z}_{\mathbf{i}}$ reproduced by the simulator on its entering poles is determined:

$$
\begin{equation*}
\mathbf{Z}_{\mathbf{i}}=\mathbf{U}_{\mathbf{i}} / \mathbf{I}_{\mathbf{i}}=\mathrm{R}_{\mathrm{C}}\left(\mathrm{~N}_{\mathrm{R}}+\mathrm{j} \mathrm{~N}_{\mathrm{X}}\right) \tag{7}
\end{equation*}
$$

As follows from (7), the reproduced impedance $\mathbf{Z}_{\mathbf{i}}$ is represented in the Cartesian coordinates and allows
realizing separate control of its active and reactive components by change the gain factors $\mathrm{N}_{\mathrm{R}}, \mathrm{N}_{\mathrm{X}}$ of the programmable amplifiers PA1, PA2. From (7) also follows (Fig. 2) that the character of the reproduced impedance depend only on the band of variation of $N_{R}$ and $N_{X}$. If the band of $N_{R}$ is located in the field of positive values and the band of $\mathrm{N}_{\mathrm{X}}$ - in the domain ( $-\mathrm{N}_{0} \div+\mathrm{N}_{0}$ ), the reproduced impedance can have the character of a resistance in a combination with inductive or capacitive component. The case when the both $\mathrm{N}_{\mathrm{R}}$ and $\mathrm{N}_{\mathrm{X}}$ have a range of change ( $\mathrm{N}_{0} \div$ $+\mathrm{N}_{0}$ ) is more interesting. As follows from (7), the area of regulation of $\mathbf{Z}_{i}$ character in this case covers all complex plane; i. e. $\mathbf{Z}_{\mathbf{i}}$ can have the character of a different combination of the positive or negative


Fig. 2. The various character of simulated impedance
resistance with the capacitive or inductive impedance component (Fig. 2). This makes it possible to provide the reproduction of the impedances $\mathbf{Z}_{\mathbf{i} 1}, \mathbf{Z}_{\mathbf{i} 2}, \mathbf{Z}_{\mathbf{i}}, \mathbf{Z}_{\mathbf{i 4}}$ with any nature and values of components only by control the gain factors of the amplifiers PA1, PA2.

### 2.2. The Voltage Controlled Impedance

Simulator (U-MSI)
Synthesis of the voltage controlled Cartesian coordinate impedance simulator ( U - MSI) can be carried out similarly to considered above synthesis of I - MSI. The conversion algorithm of information in

this simulator, accepted as an initial condition for synthesis of $\mathrm{U}-\mathrm{MSI}$ is presented in the fig. 3.

Fig. 3. The conversion algorithm of information in the U-MSI


Fig. 4. The block - diagram of the synthesized

$$
\mathrm{U}-\mathrm{MSI}
$$

The procedure of synthesis U - MSI is differed from the procedure of synthesis I - MSI examined above in terms of fact that as the initial input value of imitator is used the voltage Ui on the reproduced impedance Zi . The block diagram of U - MSI structure is represented in the fig. 4.
From the input voltage Ui, under the influence of factors NR, NX are formed the intermediate voltages UR and UX for the active and reactive components of the reproduced impedance:

$$
\begin{gather*}
\mathbf{U}_{\mathbf{R}}=\mathrm{N}_{\mathrm{R}} \cdot \mathbf{U}_{\mathbf{i}},  \tag{8}\\
\mathbf{U}_{\mathbf{X}}=\mathrm{N}_{\mathbf{X}} \cdot \mathbf{U}_{\mathbf{i}} \cdot \mathrm{j} \sin 90^{\circ}=\mathrm{j} \mathrm{~N}_{\mathrm{X}} \cdot \mathbf{U}_{\mathbf{i}} \tag{9}
\end{gather*}
$$

Further, by summing up of the voltages $\mathrm{U}_{\mathrm{R}}$ and $\mathbf{U}_{\mathbf{X}}$ the voltage $\mathbf{U}_{\mathbf{Z}}$ is obtained, which, with the aid of the voltage - current converter DU/I, is converted into the current $\mathbf{I}_{\mathbf{i}}$ pass through the reproduced impedance:

$$
\begin{gather*}
\mathbf{U}_{\mathbf{Z}}=\mathbf{U}_{\mathbf{R}}+\mathbf{U}_{\mathbf{X}}=\mathrm{N}_{\mathrm{R}} \cdot \mathbf{U}_{\mathbf{i}}+j \mathrm{~N}_{\mathrm{X}} \cdot \mathbf{U}_{\mathbf{i}}= \\
=\left(\mathrm{N}_{\mathrm{R}}+\mathrm{j} \mathrm{~N}_{\mathrm{X}}\right) \mathbf{U}_{\mathbf{i}}  \tag{10}\\
\mathbf{I}_{\mathbf{i}}=\mathrm{G}_{\mathrm{C}} \cdot \mathbf{U}_{\mathbf{Z}}=\mathrm{G}_{\mathrm{C}}\left(\mathrm{~N}_{\mathrm{R}}+\mathrm{j} \mathrm{~N}_{\mathrm{X}}\right) \mathbf{U}_{\mathbf{i}} \tag{11}
\end{gather*}
$$

where $G_{C}$ - the conversion factor of voltage - current converter. Since in this case is carried out the conversion of form "voltage --> current", it is convenient to examine instead of the reproduced impedance $\mathrm{Z}_{\mathrm{i}}$ the reproduced admittance $\mathrm{Y}_{\mathrm{i}}$. Taking into account (11), $\mathrm{Y}_{\mathrm{i}}$ can be represented:

$$
\begin{equation*}
\mathbf{Y}_{\mathbf{i}}=\mathbf{I}_{\mathbf{i}} / \mathbf{U}_{\mathbf{i}}=\mathrm{G}_{\mathrm{C}}\left(\mathrm{~N}_{\mathrm{R}}+\mathrm{j} \mathrm{~N}_{\mathrm{X}}\right) \equiv \mathrm{G}_{\mathrm{i}}+\mathrm{j} \mathrm{~B}_{\mathrm{i}} \tag{12}
\end{equation*}
$$

The expression (12) is analogous to (7), considering the duality of the utilized values, what is the consequence of the principle of duality. On (12) it follows that the regulation of active $\left(\mathrm{G}_{\mathrm{i}}\right)$ and reactive $\left(\mathrm{B}_{\mathrm{i}}\right)$ components of the simulated admittance $\mathbf{Y}_{\mathbf{i}}$ can be executed separately, by regulating respectively the values $\mathrm{N}_{\mathrm{R}}$ and $\mathrm{N}_{\mathrm{X}}$. Also, as in the preceding case, ensuring the guarantee of the $\mathrm{N}_{\mathrm{R}}$ and $\mathrm{N}_{\mathrm{X}}$ regulation domain ( $-\mathrm{N}_{0} \div+\mathrm{N}_{0}$ ), the reproduced admittance can have any nature, obtained as a result of combination by positive or negative active component $\mathrm{G}_{\text {in }}$ with the positive or negative reactive component $\mathrm{B}_{\text {in }}$.

### 2.3. Stability of Impedance Simulators

The problem of the MSI stability is rather important and requires a complex analysis. As in the MSI circuits there is a combined variable negative and positive feedbacks, it is necessary to consider three types of stability:
The stability on a direct current,
The stability on high frequencies band,
The functional stability.
The stability on a direct current can be interrupt because of appearance of trigger effect [4] at excess of depth of a positive feedback above negative at the direct current. To ensure absolute stability on a direct current it is necessary to ensure a negative character of the common feedback at the direct current at variation of the circuit parameters and of the exterior resistance in all operating range. Obviously, for the considered circuits this condition will be respected when the summary transfer factor on a direct current through the common feedback loop $\mathrm{K}_{0 \text { summ }}$ is negative. It is achieved by combination of inverting and noninverting connection on a direct current of all circuit units.
The stability on the high frequencies band is ensured by correct frequency correction of OA characteristic in all circuit units, considering the MSI structure as a many-stage amplifier with common feedback [4].
The functional stability of MSI represents a separate question. On the condition of functional stability exert influence the following factors:
The type of MSI;
The values of active and reactive components of the simulated impedance;
The character and the value of the equivalent external impedance connected to terminals of the MSI.
Thus, the condition of functional stability should be determined for each type of MSI separately, in view of control bands of components of simulated impedance and parameters of the measuring circuit in which it is applied.
For estimation of stability conditions, we shall take advantage of Nyquist criterion in application to the circuits containing operational amplifiers [4]. As it is known, the circuits with feedbacks ensure the stability if the critical point $(-1,+\mathrm{j} 0)$ is located to the left of the hodograph of the transfer characteristic on the loop of the circuit feedback at frequency change from $\mathrm{f}=0$ up to $\mathrm{f}-->\infty$. Thus, for estimate the stability conditions for the circuits according to Nyquist criterion, it is necessary to determine its loop gain factor $\boldsymbol{\beta} \mathbf{A}$ and to examinate it in neighborhood of critical point $(-1,+j 0)$ in coordinates $\operatorname{Re}(\boldsymbol{\beta} \mathbf{A})$, $\operatorname{Im}(\boldsymbol{\beta} \mathbf{A})$. Obviously, the condition of absolute stability of the circuit can be defined [4]:

$$
\begin{equation*}
\operatorname{Re}(\boldsymbol{\beta} \mathbf{A})>-1 \tag{13}
\end{equation*}
$$

where $\boldsymbol{\beta}$ - the feedback unit transmission factor, $\mathbf{A}$ - the basic unit transmission factor.

The analysis of functional stability must be carried out taking into account the concrete type of the impedance simulator ( I - MSI or U - MSI) and of the external impedances, as this is made in [5].

## 3. CONCLUSIONS

1. The creation of the impedance imitators for simulation the impedances with any character, which possess the possibility of the separate control of components in the Cartesian coordinates is possible on the base of operational amplifiers with combined negative and positive feedbacks. They can be used as the reference elements in the impedancemeters.
2. In accordance with the classical theory of the impedance simulators, there are two types of Cartesian - coordinates impedance simulators: with current control and with voltage control. The first of them preserves stability up to the no-load regime and can be used in the impedancemeters on the base of the series resonance circuit, the second type are stable up to the regime of short circuit and can be used in the impedancemeters on the base of parallel resonance circuit.
3. Application of the offered impedance simulators will allow simplifying essentially the impedancemeters designs and its equilibration algorithms, having kept high accuracy of measurement and having excluded the necessity of application of controllable reactive elements as impedance measures.

## References

V. Nastas, M. Scînteianu, "The impedance measurement by method of simulated resonance" Proceedings of the $8^{\text {th }}$ International Conference OPTIM 2002, vol.3, pp. 683-688, Braşov, 2002
V. Nastas, , "Impedance simulators with variable character for metrology" Moldavian Journal of the Physical Sciences, Vol. 1, No. 3, 2002, pp. 28 - 31, Academy of Sciences of Republic Moldova, Chisinau, 2002
F. Bening, „Negative viderstande in elektronischen schaltungen", VEB VERLAG TECHNIC, Berlin, 1971
J. Dostal "Operational amplifiers", ELSEVIER SPC, New York, 1981.
V. Nastas, "Polar - Coordinates Impedance Simulators and Polar - Coordinates Impedance Meter" Proceedings of the $9^{\text {th }}$ International Conference OPTIM 2004, vol.4, pp. 89-96, Braşov, 2004

