



ANALYSIS OF METHODS FOR ALLOCATING TRANSMISSION ACTIVE LOSSES IN POWER GRIDS

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Abstract – In this paper an analysis of existent methods for allocation of active losses at the transmission and distribution level of the network is proposed. The results of calculations of active power loss allocation are presented for test power network. The advantages and lacks of each method are given.

Keywords: Allocation transmission active losses, tracing power flow, electrical network, generating bus, loading bus.

1. INTRODUCTION

Until recently in most countries of the world power industry was under control of the state, and represented, as the organizational structure, vertically integrated companies which united functions of generation, transmission, distribution and selling of the electric energy.

There are over the past decade in the majority developed countries serious changes in electricpower industry: deregulation, privatization of power units and adoption of the competitive market. The principal causes which have induced the governments of the majority countries to start reforming of electric-power industry were low efficiency and lack of investments for development of power sector.

The results which were got after carrying out of competitive re-structuring of electric power industry worldwide, have confirmed the majority of forecasts. Reforming has caused a number of positive tendencies: growth of economic efficiency of electric power industry; energy prices reduction as a result of effective competition; increase of operating efficiency the generating stations and cost reduction; consolidation of enterprises; increase of investment attraction of power companies and significant inflow of investment resources.

As a result of reforming electric power industry and formation of the electric power competitive market the new problems appeared [1]:

Definition of electrical energy price for each consumers in consideration of transmission power losses;

Definition of the responsibility of generators and consumers for power losses in an electric network;

Definition of a payment for transit power;

Definition of the tariff for reactive power;

Definition of the nodal prices;

Choice the suppliers of the electrical energy;

Other problems.

For the decision of these problems it is necessary to solve one of important, actual and disputable issue of allocation the active power losses between energy markets participants. In recent years the many publications with various methods of power losses allocation have appeared. Analyzed existent methods of allocation power losses they have been grouped as follows:

Allocation of power losses using additional conditions;

Allocation of power losses by using idea of topological methods of tracing active power flows; Z-bus loss allocation;

Allocation of power losses using equivalent scheme of electrical network;

Allocation of power losses using tracing power flows.

2. ALLOCATION OF ACTIVE POWER LOSSES USING ADDITIONAL CONDITIONS

In article [2] the methods of allocation of active power losses in branches of a network between consumers are presented, using additional conditions of losses allocation:

Uniform losses allocation;

Losses allocation on the basis of demand;

Losses allocation on the basis of demand squared.

Calculation starts from root generating nodes or node of the first circle of the graph. Using one of three conditions of losses allocation, the author allocates losses in lines, connecting nodes of the first and second circle, to nodes of the second circle. Similarly the losses are allocated to third, fourth and following nodes.

Having allocated active power losses between nodes, the calculation of allocation power losses between the loads connected to these nodes is carried out.

In work [3] author uses another approach for allocation of active power losses in branches of a network between consumers, using additional conditions for allocation.

Active power flow in lines $l P_l$, can be considered as the sum of power of consumers supplying directly from this line.

$$\boldsymbol{P}_{lC1} + \boldsymbol{P}_{lC2} + \ldots + \boldsymbol{P}_{lCn}, \qquad (1)$$

where, n – number of independent nodes;

 P_{lCi} – a component of active power in branch *l* caused by load C_i .

As the losses in line $l(\Delta P_i)$ depend on a square of power flow in the line, the following expression can be written down for active power:

$$\Delta P_{I} = \frac{\left(P_{IC1} + P_{IC2} + \dots + P_{ICn}\right)^{2} R_{I}}{U_{I}^{2}} = \frac{\left(\sum_{i=1}^{n} P_{ICi}^{2} + \sum_{i=1}^{n} \sum_{j=1}^{n} P_{ICi} P_{ICj}\right) R_{I}}{U_{I}^{2}} \cdot (2)$$

From expression (2) it is visible, that complexity of allocation of losses consists in distribution of the common multiplier $\sum_{i=1}^{n} \sum_{j=1}^{n} P_{ci} P_{cj}$. In particular case,

for two consumers, this multiplier can be written down as $2P_{\mu_1}P_{\mu_2}$.

Author allocates the component $\sum_{i=1}^{n} \sum_{j=1 \atop j \neq i}^{n} P_{Ci} P_{Cj}$ some

methods, using the next conditions of allocation:

Proportional allocation. The component of losses is distributed among loads in proportion to active power of these loads.

Quadratic allocation; The component of losses is distributed among loads in proportion to square of active power of these loads.

Geometric allocation. The component of losses is distributed among loads subject to logarithm of active power of these loads.

3. ALLOCATION OF POWER LOSSES USING IDEA OF TOPOLOGICAL METHODS OF TRACING ACTIVE POWER FLOWS

This algorithm [4] allows allocating active power losses among generating and loading buses. Meanwhile the half of network losses is allocated to generating buses but another one is allocated to loading buses.

In order to determine active power losses allocated to loading buses it is necessary to make the distributive matrices $[A_u]$ (upstream matrix):

$$\begin{bmatrix} \boldsymbol{A}_{u} \end{bmatrix}_{i,k} = \begin{cases} 1 & \text{for } i = k \\ -\boldsymbol{c}_{ki} = -|\boldsymbol{P}_{ki}|/\boldsymbol{P}_{k} & \text{for } k \in \alpha_{i}^{(u)} \\ 0 & \text{otherwise} \end{cases}$$
(3)

where $\alpha_i^{(u)}$ is the set of nodes directly supplying node *i*.

Then, the next equation can be written down:

$$\begin{bmatrix} \boldsymbol{A}_{u} \end{bmatrix} \cdot \begin{bmatrix} \Delta \boldsymbol{P}^{(u)} \end{bmatrix} = \begin{bmatrix} \Delta \boldsymbol{P}_{\Sigma}^{(u)} \end{bmatrix}; \qquad (4)$$

where $\left[\Delta \mathbf{P}_{\Sigma}^{(u)}\right]$ is the matrix of the sum of the actual transmission losses in all the lines supplying node *i*;

 $\Delta \mathbf{P}_i^{(u)}$ - the upstream loss allocated to the receiving node *i*.

The losses allocated to the loading bus k can be determined with:

$$\Delta \boldsymbol{P}_{Ck} = \Delta \boldsymbol{P}_{k}^{(u)} \cdot \frac{\boldsymbol{P}_{Ck}}{\boldsymbol{P}_{k}} \cdot \frac{1}{2}$$
(5)

Similarly the active transmission power losses can be allotted to generating buses using the distributive matrices $[A_d]$ (downstream matrix).

4. Z-BUS LOSSES ALLOCATION

This method [5] allows allocating active losses among generators and loads of network, called Z – *bus losses allocation*.

The total network losses $[\Delta \underline{S}]$ can be defined with the next expression,

$$\left[\Delta \underline{S}\right] = 3 \cdot \left[\underline{\underline{I}}_{I}\right]_{I} \cdot \left[\underline{Z}_{I}\right] \cdot \left[\underline{I}_{I}\right], \tag{6}$$

where, $[\underline{I}_l]$ – matrix of complex current in line *l*, Z_l - the impedance of line *l*.

Matrix $[\underline{I}_{l}]$ can be represented as,

$$[\underline{I}_{I}] = \frac{1}{\sqrt{3}} \cdot [\underline{Z}_{I}]^{-1} \cdot [\underline{U}_{I}].$$
(7)

where, $[\underline{U}_I]$ – matrix of complex bus voltages. In turn,

$$\left[\underline{U}_{I}\right] = \left[\underline{M}\right]_{t} \cdot \left[\underline{U}_{\Delta}\right]. \tag{8}$$

where, [M] – incidence matrix. Then,

$$[\underline{I}_{I}] = \frac{1}{\sqrt{3}} \cdot [\underline{Z}_{I}]^{-1} \cdot [\underline{M}]_{I} \cdot [\underline{U}_{\Delta}], \qquad (9)$$

or,

$$\left[\underline{\underline{I}}_{I}\right]_{t} = \frac{1}{\sqrt{3}} \cdot \left[\underline{\underline{U}}_{\Delta}\right]_{t} \cdot \left[\underline{M}\right] \cdot \left[\underline{\underline{Z}}_{I}\right]_{t}^{-1}.$$
 (10)

Let us substitute expressions (9) and (10) in (6),

$$\left[\Delta \underline{S}\right] = \left[\underline{\dot{U}}_{\Delta}\right]_{t} \cdot \left[\underline{\dot{Y}}_{n}\right]_{t} \cdot \left[\underline{U}_{\Delta}\right].$$
(11)

In turn, \underline{U}_{Λ} can be written as,

$$\left[\underline{U}_{\Delta}\right] = \left[\underline{Z}_{n}\right] \cdot \left[\underline{J}_{n}\right], \qquad (12)$$

or,

$$\left[\underline{\underline{U}}_{\Delta}\right]_{t} = \left[\underline{\underline{J}}_{n}\right]_{t} \cdot \left[\underline{\underline{Z}}_{n}\right]_{t}$$
(13)

where, $[\underline{Z}_n]$ – impedance matrix, $[\underline{J}_n]$ - complex matrix of bus current injections. Let us substitute expressions (12) and (13) in (11), then the next equation can be written,

$$\left[\Delta \underline{S}\right] = \left[\underline{\underline{I}}_{n}\right]_{t} \cdot \left[\underline{Z}_{n}\right] \cdot \left[\underline{I}_{n}\right]. \tag{14}$$

But,

$$\left[\underline{\dot{J}}_{n}\right]_{t} = \left[J_{n}'\right]_{t} - j\left[J_{n}''\right]_{t}; \qquad (15)$$

$$\left[\underline{J}_{n}\right] = \left[J'_{n} + jJ''_{n}\right]; \qquad (16)$$

$$\left[\underline{Z}_{n}\right] = \left[R_{n} + jX_{n}\right]. \tag{17}$$

Take into consideration (15), (16) and (17) the next equation can be written,

$$\begin{bmatrix} \Delta \boldsymbol{P} + \boldsymbol{j} \Delta \boldsymbol{Q} \end{bmatrix} = \left\| \begin{bmatrix} \boldsymbol{J}'_n \end{bmatrix}_t - \boldsymbol{j} \begin{bmatrix} \boldsymbol{J}''_n \end{bmatrix}_t \right\| \cdot \\ \cdot \begin{bmatrix} \boldsymbol{R}_n + \boldsymbol{j} \boldsymbol{X}_n \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{J}'_n + \boldsymbol{j} \boldsymbol{J}''_n \end{bmatrix}$$
(18)

Then, the active power losses will be equal,

$$\begin{bmatrix} \Delta \boldsymbol{P} \end{bmatrix} = \begin{bmatrix} \boldsymbol{J}'_n \end{bmatrix}_d \cdot \begin{bmatrix} \boldsymbol{R}_n \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{J}'_n \end{bmatrix} + \begin{bmatrix} \boldsymbol{J}''_n \end{bmatrix}_d \cdot \begin{bmatrix} \boldsymbol{R}_n \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{J}''_n \end{bmatrix} - \begin{bmatrix} \boldsymbol{J}'_n \end{bmatrix}_d \cdot \begin{bmatrix} \boldsymbol{X}_n \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{J}''_n \end{bmatrix} + \begin{bmatrix} \boldsymbol{J}''_n \end{bmatrix}_d \cdot \begin{bmatrix} \boldsymbol{X}_n \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{J}'_n \end{bmatrix}$$
(19)

Author shown [4] (Appendix A) that,

$$-\left[\boldsymbol{J}_{n}'\right]_{d}\cdot\left[\boldsymbol{X}_{n}\right]\cdot\left[\boldsymbol{J}_{n}''\right]+\left[\boldsymbol{J}_{n}''\right]_{d}\cdot\left[\boldsymbol{X}_{n}\right]\cdot\left[\boldsymbol{J}_{n}'\right]=0.$$
(20)

Thus, author uses only the first two sums of equation (19) to allocate active power losses.

It is necessary to note, that the expression (20) is executed in those cases if are given only the longitudinal branches of equivalent elements of the power networks.

5. ALLOCATION OF POWER LOSSES BY USING EQUIVALENT SCHEMA OF ELECTRICAL NETWORK

Essence of the method consists in the following. The initial equivalent of electrical network leads to one of equivalent network by gradual transformation. One of them has one generating node, and another consists of one loading node (fig. 1).

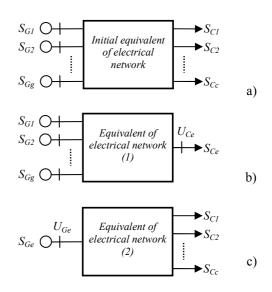


Fig. 1: Equivalent of electrical network.

 S_G , S_C – generating and loading power respectively. g – number of generating network buses, k – number of loading network buses.

Compositing of equivalent schemes is implemented with a method of equivalent REI - Dimo [6]. The first equivalent scheme is used for distribution of power losses in an electric network among loading buses, and the second among generating ones.

For initial power network the balance equation can be written down as:

$$\sum_{j=1}^{ng} \underline{I}_{Gj} = \sum_{i=1}^{nc} \underline{I}_{Ci} + \underline{I}_p , \qquad (21)$$

or,

$$\underline{I}_{Ge} = \underline{I}_{Ce} + \underline{I}_{p} = \underline{I}_{Ce} \cdot \left(1 + \frac{\underline{I}_{p}}{\underline{I}_{Ce}}\right) = \underline{R} \cdot \underline{I}_{Ce} , \quad (22)$$

where, \underline{I}_{Gi} , \underline{I}_{Ci} - the currents in generating buses *j* and loading buses *i* respectively;

 \underline{I}_{p} - sum of currents in transversal branches.

The power in a single generating bus is defined from expression:

$$\underline{S}_{Gg} = \sum_{j=1}^{ng} \underline{S}_{Gj} = \underline{U}_{Ge} \cdot \underline{\dot{I}}_{Ge} = \underline{U}_{Ge} \cdot \underline{\dot{R}} \cdot \underline{\dot{I}}_{Ce} =$$

$$= \underline{U}_{Ge} \cdot \underline{\dot{R}} \cdot \underline{\dot{I}}_{Ce} = \underline{U}_{Ge} \cdot \cdot \underline{\dot{I}}_{Ce} = \underline{U}_{Ge} \cdot \sum_{i=1}^{nc} \underline{\dot{I}}_{Ci}$$
(23)

From expression (23) it is result that $\underline{I}_{Ge} = \underline{I}_{Ce}$

 $\left(\sum_{j=1}^{ng} I_{jg} = \sum_{j=1}^{nc} I_{ci}\right), \text{ and then the scheme (fig. 1, a) can}$ be represented as (fig. 2).

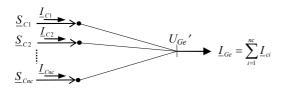


Fig. 2: The equivalent radial scheme with a one generator bus, it is used for allocation power losses among loading busses.

Analyzing the given equivalent scheme it is possible to notice, that power losses of the electric network, caused by a current of each loads, depend only on losses in one branch, which connects the given loading bus to the generalized generating bus with a voltage \underline{U}_{Ge} , namely:

$$\Delta \underline{\mathbf{S}}_{G} = \left(\underline{U}_{G} - \underline{U}_{Ge}'\right) \cdot \underline{\dot{\mathbf{I}}}_{Gi} . \tag{24}$$

If to address to an equivalent presented on fig. 1.b, then with similar reasoning the next equation can be obtained:

$$\Delta \underline{S}_{Gj} = \left(\underline{U}_{Gj} - \underline{U}_{Ce}'\right) \cdot \underline{\dot{I}}_{Gj}.$$
(25)

The expression (24) is used for allocation of power losses among loading buses, but the expression (25) is used for loss allocation among generating buses.

6. ALLOCATION OF POWER LOSSES BY USING TRACING POWER FLOWS

The essence of this method in detail expounds in [7, 8].

Algorithm of active power loss allocation consists of the following stages:

- Definition of power losses due to the power transfer from *j*-th power plant to *k*-th load;
- Definition of power losses in *l*-th line due to transmission from *j*-th power plant;
- Definition of power losses in *l*-th line due to the supply of *k*-th load.

6.1. Definition of power losses due to the power transfer from *j*-th power plant to *k*-th load

For definition of power losses due to the power transfer from *j*-th power plant to k-th load it is necessary to use the results of solution of tracing power flows [7, 8].

Knowing the power transmitted from *j*-th generator to *k*-th load and the amount of power received by *k*-th load from *j*-th generator we can find the power losses. For this purpose let's write the following expression:

$$\left[\Delta \boldsymbol{P}_{jk}\right] = \left[\boldsymbol{P}_{jk}\right] - \left[\boldsymbol{P}_{kj}\right]_{t}, \qquad (26)$$

where: $\left[\Delta \mathbf{P}_{jk}\right]$ - is a matrix of order $(n \times n)$. Each element *jk* of this matrix is the amount of power loss due to the power transfer from *j*-th power plant to *k*-th load.

But $[\mathbf{P}_{ik}]$ and $[\mathbf{P}_{ki}]$ can be obtained from [9-10]:

$$\begin{bmatrix} \boldsymbol{P}_{jk} \end{bmatrix} = \begin{bmatrix} \boldsymbol{P}_{G} \end{bmatrix}_{d} \cdot \begin{bmatrix} \boldsymbol{P} \end{bmatrix}_{d}^{-1} \cdot \begin{bmatrix} \boldsymbol{A}' \end{bmatrix}^{-1} \cdot \begin{bmatrix} \boldsymbol{P}_{C} \end{bmatrix}_{d}, \quad (27)$$
$$\begin{bmatrix} \boldsymbol{P}_{kj} \end{bmatrix} = \begin{bmatrix} \boldsymbol{P}_{C} \end{bmatrix}_{d} \cdot \begin{bmatrix} \boldsymbol{P} \end{bmatrix}_{d}^{-1} \cdot \begin{bmatrix} \boldsymbol{A}'' \end{bmatrix}^{-1} \cdot \begin{bmatrix} \boldsymbol{P}_{G} \end{bmatrix}_{d}. \quad (28)$$

Where $[P_G]$ and $[P_c]$ are a column matrices of generating and loading powers in all nodes respectively; [P] is column matrix of total nodal powers [7, 8]. In turn matrices [A'] and [A''] are equal:

$$\begin{bmatrix} \boldsymbol{A}' \end{bmatrix} = \left\| \begin{bmatrix} \boldsymbol{u} \end{bmatrix} + \begin{bmatrix} \boldsymbol{M}_{\Sigma}^{+} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{P}_{l} \end{bmatrix}_{d} \cdot \begin{bmatrix} \boldsymbol{M}_{\Sigma}^{-} \end{bmatrix}_{l} \cdot \begin{bmatrix} \boldsymbol{P} \end{bmatrix}_{d}^{-1} \right\|; (29)$$

$$\begin{bmatrix} \boldsymbol{A}'' \end{bmatrix} = \left\| \begin{bmatrix} \boldsymbol{u} \end{bmatrix} + \begin{bmatrix} \boldsymbol{M}_{\Sigma}^{-} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{P}_{l}'' \end{bmatrix}_{d} \cdot \begin{bmatrix} \boldsymbol{M}_{\Sigma}^{+} \end{bmatrix}_{l} \cdot \begin{bmatrix} \boldsymbol{P} \end{bmatrix}_{d}^{-1} \right\|; (30)$$

where: $[\boldsymbol{u}]$ – is a unitary matrix; $[\boldsymbol{M}_{\Sigma^+}]$ - is the matrix of branches between all the nodes of network, elements of which are "0" or "+1"; $[\boldsymbol{M}_{\Sigma^-}]$ - the same matrix, but containing as its elements "0" or "-1"; $[\boldsymbol{P}_l']$ and $[\boldsymbol{P}_l'']$ are column matrices of power flows in begin and end of each branch respectively.

6.2. Definition of power losses in *l*-th line due to transmission from *j*-th power plant

Having determined powers received at the beginning $\begin{bmatrix} P_{ij} \end{bmatrix}$ and end $\begin{bmatrix} P_{ij} \end{bmatrix}$ *l*-th line from *j*-th power plant [7, 8]:

$$\begin{bmatrix} \boldsymbol{P}_{ij}^{'} \end{bmatrix} = \begin{bmatrix} \boldsymbol{P}_{i}^{'} \end{bmatrix}_{d} \cdot \begin{bmatrix} \boldsymbol{M}_{\Sigma}^{+} \end{bmatrix}_{t} \cdot \begin{bmatrix} \boldsymbol{P} \end{bmatrix}_{d}^{-1} \cdot \begin{bmatrix} \boldsymbol{A}^{''} \end{bmatrix}^{-1} \cdot \begin{bmatrix} \boldsymbol{P}_{G} \end{bmatrix}_{d}, (31)$$

$$\begin{bmatrix} \boldsymbol{P}_{ij}^{''} \end{bmatrix} = \begin{bmatrix} \boldsymbol{P}_{i}^{''} \end{bmatrix}_{d} \cdot \begin{bmatrix} \boldsymbol{M}_{\Sigma}^{+} \end{bmatrix}_{t} \cdot \begin{bmatrix} \boldsymbol{P} \end{bmatrix}_{d}^{-1} \cdot \begin{bmatrix} \boldsymbol{A}^{''} \end{bmatrix}^{-1} \cdot \begin{bmatrix} \boldsymbol{P}_{G} \end{bmatrix}_{d}, (32)$$

we can define the power losses in this line caused by *j*-th power plant using the following expression:

$$\left[\Delta \boldsymbol{P}_{ij}\right] = \left[\boldsymbol{P}_{ij}^{\prime}\right] - \left[\boldsymbol{P}_{ij}^{\prime\prime}\right], \qquad (33)$$

where: $\left[\Delta P_{ij}\right]$ - is a matrix of order (*l*×*n*). Each element *lj* of this matrix shows the amount of power losses in *l*-th line, caused by *j*-th generator.

6.3. Definition of power losses in *l*-th line due to the supply of *k*-th load

Knowing the power at the beginning $\begin{bmatrix} P_{lk}' \end{bmatrix}$ and end $\begin{bmatrix} P_{lk}'' \end{bmatrix}$ *l*-th line, caused by *k*-th load [7, 8],

$$\begin{bmatrix} \boldsymbol{P}_{lk}' \end{bmatrix} = -\begin{bmatrix} \boldsymbol{P}_{l}' \end{bmatrix}_{d} \cdot \begin{bmatrix} \boldsymbol{M}_{\Sigma}^{-} \end{bmatrix}_{t} \cdot \begin{bmatrix} \boldsymbol{P} \end{bmatrix}_{d}^{-1} \cdot \begin{bmatrix} \boldsymbol{A}' \end{bmatrix}^{-1} \cdot \begin{bmatrix} \boldsymbol{P}_{C} \end{bmatrix}_{d}, \quad (34)$$

$$\begin{bmatrix} \boldsymbol{P}_{lk}^{"} \end{bmatrix} = -\begin{bmatrix} \boldsymbol{P}_{l}^{"} \end{bmatrix}_{d} \cdot \begin{bmatrix} \boldsymbol{M}_{\Sigma}^{-} \end{bmatrix}_{t} \cdot \begin{bmatrix} \boldsymbol{P} \end{bmatrix}_{d}^{-1} \cdot \begin{bmatrix} \boldsymbol{A}' \end{bmatrix}^{-1} \cdot \begin{bmatrix} \boldsymbol{P}_{C} \end{bmatrix}_{d}, \quad (35)$$

we can define the power losses in *l*-th line, due to *k*-th load, using the following expression:

$$\left[\Delta \boldsymbol{P}_{lk}\right] = \left[\boldsymbol{P}_{lk}'\right] - \left[\boldsymbol{P}_{lk}''\right], \qquad (36)$$

where: $[\Delta P_{lk}]$ - is a matrix with order $(l \times n)$. Each element lk of this matrix shows the amount of power losses in *l*-th line due to the supply of *k*-th load. The analogical method loss allocation is represented in paper [9, 10]. But in these articles the others approach of tracing power flows are used.

7. CASE STUDY

The examined loss allocation methods have been tested on a set of networks of varying sizes and types. In this paper the results of calculation active power loss allocation by all examined methods are presented Table 1. The calculation was made for 13-bus network Fig.3. From results of calculation it is visible that the power loss allocation is depended on selected method of allocation. Therefore the losses must be allocated true and objectively.

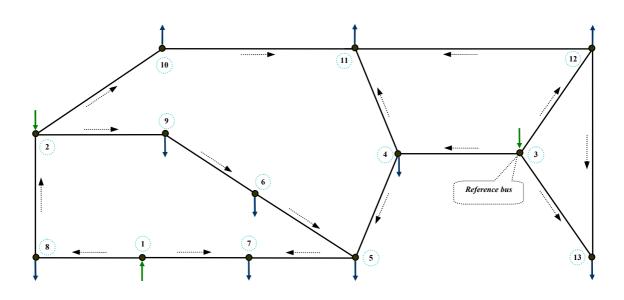


Fig. 3: Example system.

| | Allocation of power losses using additional conditions | | | | | | ing of | | e cal | to ent ·k | SS | ing |
|------------------------|---|---|---|-------------------------|----------------------|----------------------|--|-----------------------|--|---|---|---|
| method | Uniform loss allocation | Loss allocation on the basis of demand | Loss allocation on the basis of demand squared | Proportional allocation | Quadratic allocation | Geometric allocation | Allocation of power losses using idea of topological methods of tracing active power flows | Z-bus loss allocation | generating buses we generating buses using equivalent schema of electrical | Allocation of power losses to loading buses using equivalent schema of electrical network | Incremental transmission loss allocation | Allocation of power losses using tracing power flows |
| G ₁ | 0 | 0 | 0 | 0 | 0 | 0 | 6.4658 | -0.407 | 5.692 | 0 | -1.396 | 0 |
| G_2 | 0 | 0 | 0 | 0 | 0 | 0 | 2.0836 | 8.791 | 5.448 | 0 | -0.93 | 0 |
| G_3 | 0 | 0 | 0 | 0 | 0 | 0 | 3.0356 | 0 | 12.354 | 0 | 25.591 | 0 |
| <i>C</i> ₄ | 0.720 7 | 0.74 | 0.610 8 | 0.686 7 | 0.610 8 | 0.726 | 0.3703 | 2.719 | 0 | 2.413 | 0 | 0.74 |
| C ₅ | 1.657 7 | 2.582 4 | 2.993 6 | 2.792 6 | 2.993 6 | 2.683 6 | 1.2916 | 3.007 | 0 | 3.527 | 0 | 2.582 |
| <i>C</i> ₆ | 1.052 7 | 1.812 1 | 2.12 | 2.026 9 | 2.12 | 1.924 | 0.9059 | 0.475 | 0 | 1.111 | 0 | 1.811 |
| C ₇ | 14.24 2 | 12.99 8 | 12.66 1 | 12.73 8 | 12.66 1 | 12.85 3 | 6.4998 | 8.27 | 0 | 12.553 | 0 | 13.001 |
| <i>C</i> ₈ | 0.202 | 0.398 6 | 0.403 9 | 0.403 8 | 0.403 9 | 0.403 6 | 0.1993 | 0.052 | 0 | 0.397 | 0 | 0.398 |
| <i>C</i> ₉ | 1.336 3 | 1.231 3 | 1.172 5 | 1.201 | 1.172 5 | 1.217 1 | 0.6155 | -0.466 | 0 | 0.202 | 0 | 1.23 |
| <i>C</i> ₁₀ | 0.466 8 | 0.774 6 | 0.815 3 | 0.812 4 | 0.815 3 | 0.799 8 | 0.3871 | -0.925 | 0 | -0.265 | 0 | 0.774 |
| C ₁₁ | 1.972 2 | 1.153 8 | 0.830 3 | 0.993 5 | 0.830 3 | 1.081 8 | 0.577 | 0.564 | 0 | 0.945 | 0 | 1.154 |
| <i>C</i> ₁₂ | 0.398 7 | 0.669 7 | 0.825 9 | 0.740 6 | 0.825 9 | 0.709 2 | 0.3348 | 0.73 | 0 | 1.06 | 0 | 0.669 |
| <i>C</i> ₁₃ | 1.445 7 | 1.133 5 | 1.061 8 | 1.101 5 | 1.061 8 | 1.098 3 | 0.5673 | 0.689 | 0 | 1.555 | 0 | 1.135 |
| sum | 23.497 | | | | | | | | | | | |

Table 1: Bus real power loss allocation.

8. CONCLUSIONS

The method of losses allocation using results of solution of problem of tracing power flows has the next advantages as compared with others known methods:

- The power loss allocation only on basis of results of tracing power flows is performed and the other principles of distribution are not used;
- This method allows allocating power losses among all participants of energy market. The payment of losses can be entrusted as only to loads as only to suppliers or to either in consistent proportion;
- Determination of share of losses in line *l* caused by power station *j* or consumer *k* allows to determine responsible suppliers or consumers for

transmission losses and to determine their costs for every participant of energy market.

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