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POWER FLOW NON-LINEAR SYSTEM OF EQUATIONS WITH SVD REGULARIZATION NEWTON METHOD

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Abstract **– The paper illustrates a potential numerical method to be applied when dealing ill-conditioned systems of equations. Partial regularization of the affected sub matrices for a partitioned global matrix appears to be less time consuming and to add stability to the solution. For the numerical method analyzed, the case study refers to a power engineering problem.**

Keywords: *power flow, ill-conditioned problem, partial regularization, stability.*

1. INTRODUCTION

The study of electrical engineering problems may lead to the solving of numerical linear or non-linear equation systems. Usually, these case studies do not raise instabilities in the solutions. But when the problems make use of an inverse formulation, or simply by the conventional way, there are situations when the coefficients matrices of the systems are very ill-conditioned, or include ill-conditioned sub matrices [1].

The foreseen effect on the latter mentioned cases may be reflected on the instability of the solutions, when a certain perturbation occurs and, in our opinion, also on a higher computational effort.

Our concern is to improve the condition number of the global matrix or of the affected sub matrices and thus minimize or eliminate the stability weakness and simultaneously reduce the global computational work.

We consider an electric power engineering problem of power flow determination. If the numerical solving approaches Newton's classical method for non-linear systems of equations, on each iteration the resulted linear system of corrections contains two ill-conditioned sub matrices. These matrices are supposed to affect the optimal result, both with instability and an increased number of iterations.

2. PROBLEM DESCRIPTION

Let us consider a power system with a known configuration and network parameters. The generated and consumed powers, both active and reactive, are available on each bus of the network. We also admit that the last bus of the power system is the slack bus (with a known voltage amplitude and phase). The request is to find the amplitudes and voltage phases for the other buses, the equilibrium active and reactive powers and the power flow in the network, including the losses on the whole power system and on each element. Figure 1 presents the configuration of the power system analyzed. Numerical details of this exemplification will later complete the application [2].

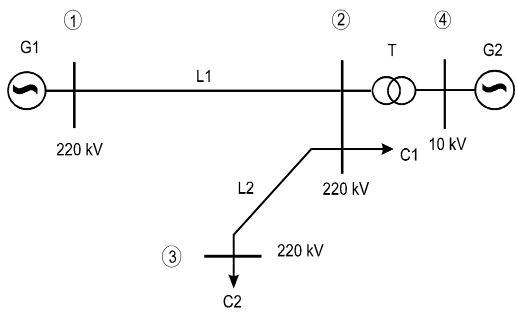

Fig. 1. Power system configuration.

The problem requires a steady state analyze of the power flow in the system. The corresponding mathematical model consists in a system of non-linear equations, with complex coefficients, which may be separated in real and imaginary parts of the following types:

$$f_{ip} = U_i^2 \cdot G_{ii} + \sum_{\substack{j=1 \\ j\neq i}}^{n} U_i \cdot U_j \left[G_{ij} \cdot \cos(\delta_i - \delta_j) + B_{ij} \cdot \sin(\delta_i - \delta_j) \right] - P_i = 0 \tag{1}$$

$$f_{iq} = -U_i^2 \cdot B_{ii} + \sum_{\substack{j=1 \\ j\neq i}}^{n} U_i \cdot U_j \left[G_{ij} \cdot \sin(\delta_i - \delta_j) + B_{ij} \cdot \cos(\delta_i - \delta_j) \right] - Q_i = 0; \quad i = \overline{1;n}. \tag{2}$$

In the above equations, the admittance matrix elements are expressed in rectangular coordinates and the voltages in polar coordinates:

$$\underline{Y}_{ij} = G_{ii} + j \cdot B_{ij} \tag{3}$$

$$\underline{U}_i = U_i \cdot e^{j \cdot \delta_i} \tag{4}$$

The generated powers have the expressions:

$$\underline{S}_i = P_i + j \cdot Q_i \tag{5}$$

The unknown variables, of the non-linear system of equations given by relations (1) and (2), are the voltages in amplitude and phase: $\underline{U}_i \quad i = \overline{1; n-1}$.

In the usual version of Newton's numerical method of solving the non-linear system of equations a partitioned linear system formulation is adopted, with the corrections as unknowns, as the following relations show:

$$\begin{bmatrix} J_{p\delta} & J_{pu} \\ J_{q\delta} & J_{qu} \end{bmatrix} \cdot \begin{bmatrix} h_\delta \\ h_u \end{bmatrix} = -\begin{bmatrix} f_p \\ f_q \end{bmatrix} \tag{6}$$

$$x^k = x^{k-1} + h^{k-1} \tag{7}$$

The x variable represents the amplitudes and phases of the unknown voltages.

3. PARTIAL REGULARIZATION

In this stage, the example taken into consideration proves that the coefficient matrix J, as it is partitioned, contains some very ill-conditioned sub matrices, on the secondary diagonal $J_{q\delta}$; J_{pu}.

The usual treatment in such cases, may not take account of the interior ill-conditioned matrices, and precede the solving process on each iteration with a worse convergence rate. We assume that, in this case, the final result may be affected if a perturbation occurs, e.g. a sudden power absorption in one of the buses or a short-circuit; due to the ill-posed character of the sub matrices.

Another alternative in handling the linear system of corrections is to neglect those two ill-conditioned sub matrices and perform the calculus in this hypothesis. It appears to be a lower cost computation, but even if the physical model of the application allows this, we may lose information.

The method we suggest aims to improve the condition number of the interior sub matrices and then run the numerical solving of the linear system of corrections.

To better understand the numerical behavior of this claimed procedure let us focus on the first iteration resulted in a linear equation system. First of all, for the configuration given in figure 1, we define the available powers and voltage:

P_1	P_2	P_3	P_4	U_4
0.75	-0.50	-1.15	0.93	1.1
Q_1	Q_2	Q_3	Q_4	δ_4
0.20	-0.20	-0.45	0.44	0

Table 1, Numerical per unit values of the parameters

The active power – voltage and reactive power – phase coefficient matrices, both have an ill-conditioned number:

$$J_{pu} = \begin{pmatrix} 4.65 & -4.65 & 0 \\ -4.65 & 6.98 & -2.33 \\ 0 & -2.33 & 2.33 \end{pmatrix}$$

$$J_{q\delta} = \begin{pmatrix} -4.65 & 4.65 & 0 \\ 4.65 & -6.98 & 2.33 \\ 0 & 2.33 & -2.33 \end{pmatrix}$$

The condition number is evaluated with the relation:

$$k(J) = \frac{\sigma_{max}}{\sigma_{min}} \tag{8}$$

$$k(J_{pu}) = k(J_{q\delta}) = 7.38 \cdot 10^{16}.$$

The minimum and maximum singular values of a matrix J may easy be evaluated with a numerical method [2].

An established way of improving the condition number of a coefficient matrix and thus reaching physically achievable and stable solutions is to apply the singular values decomposition as bellow [3]:

$$J = U \cdot \Sigma \cdot V^T \; ; \; J \cdot h = f \tag{9}$$

The matrices U and V are orthogonal and Σ is a diagonal matrix containing the singular values of J.

In equation (9) if we drop those singular values in the diagonal matrix Σ, that are lower than an imposed limit, and replace them with the unit, we will reach am improved value of the condition number.

The easiest way of performing a partial regularization in equation (6) is to simply decompose the affected sub matrices, and replace all singular values that are smaller than the imposed limit:

$$\begin{bmatrix} J_{p\delta} & SVD_{pu} \\ SVD_{q\delta} & J_{qu} \end{bmatrix} \cdot \begin{bmatrix} h_\delta \\ h_u \end{bmatrix} = -\begin{bmatrix} f_p \\ f_q \end{bmatrix} \tag{10}$$

If the ill-condition characteristic is extended to the global coefficients matrix, the regularization procedure, relation (10) may very well be applied.

By employing one of these alternatives, the condition number of the sub matrices will radically improve its value [1]. Therefore, we also assume an improvement in the stability and a better rate of convergence to an error imposed solution. Further investigations should strongly confirm the hypothesis assumed here. The partial regularization has to be applied on each iteration.

4. CONCLUSIONS

A new approach in solving linear ill-conditioned systems of equations has been proposed. This consists in a partial regularization procedure, to which are subjected the interior sub matrices that may cause instability in the solution and higher computational effort.

Future reports will provide solid numerical results for the validation of current proposal.

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References

[1] Micu D. D., Ceclan A., Micu D., Simion E., *Synthesis Method of an Inductive Sensor using Tikhonov Regularization Procedure*, OIPE'06, Sorrento, 2006, pp. 177 – 178.

[2] Kilyeni St, Numerical methods in power engineering, 3rd edition. (in romanian), Ed. Orizonturi Universitare, Timisoara, 2004.

[3] Tikhonov A. N., Arsenin V. I., Methodes de resolution de problemes mal poses, Edition MIR, Moscou, 1974.

[4] Fan Zhang, Cheng, C.S., A modified Newton method for radial distribution system power flow analysis, IEEE Transactions on Power Systems, Volume 12, Issue 1, Feb. 1997 pp. 389 – 397.

[5] Geng Yang, Chunming Rong, Performance analysis of partition algorithms for parallel solution of nonlinear systems of equations, Proceedings of the Fourth International Conference on Parallel and Distributed Computing, Applications and Technologies, 2003. PDCAT'2003, 27-29 Aug. 2003, pp. 684 – 687.