6TH INTERNATIONAL CONFERENCE ON ELECTROMECHANICAL AND POWER SYSTEMS
October 4-6, 2007 - Chişinău, Rep.Moldova

NUMERICAL METHOD FOR A MERIDIAN PLANE OF SYMMETRY OF AN ELECTROSTATIC MODEL

Micu Dan, Micu D. Dan, Andrei Ceclan and Emil Simion

Technical University of Cluj-Napoca, Electrotechnical Department
Dan.Micu@et.utcluj.ro

Abstract **– The paper deals with a non-trivial electrostatic model which may be regarded as a reference for certain numerical testing calculus. The model comprises two spheres which intersect under an angle (π/n), with *n* a natural number, and R_1, R_2 şi *n* as parameters. The analytical expression and numerical values of the electrical potential created by the two spheres are developed, in relate with the mentioned parameters.**

Keywords: *electrostatic model, symmetry, electrical charge.*

1. INTRODUCTION

The electrical potential formula is deduced using the image charges and the geometrical inversions method in the real plane. The true electric point charges q_i are situated at the locations Q_i, which are at the d_i distances from the point O in a homogeneous dielectric. The potential established by these charges in the P_j points which are at the R_j distances from the point O is V_j. The point O is picked as a inversion pole and p^2 as the power of inversion. In the Q_i' points, which are the reverse of Q_i points, the values of the charges are: [1]

$$q_i' = \frac{p}{d_i} q_i \tag{1}$$

The potential founded in P_j' points by the q_i' charges is:

$$V_j' = \frac{R_j}{p} V_j \tag{2}$$

2. THE IMAGE CHARGES FOR A SYSTEM OF TWO CONDUCTING SPHERES

We consider the conductor spheres of O_1 and O_2 centre and radius R_1 and R_2 respectively, which make a φ angle. This means that in the cross section of fig. 1, $\angle O_1PO_2 = \pi - \varphi$. We take as known values for R_1, R_2, φ and the total charge q_t of a conductor sized by the two spheres. We can assume exactly the same that instead of q_t charge is known the V potential of conductor. We propose to find the position and the value of the image charges function of q_t (or function of V). For this purpose an inversion of P pole and $p^2=4R_2^2$ power is made. The sphere of R_2 radius is transformed in the plane π_2, $PO_2 \perp \pi_2$, that is tangent to the sphere in T_2. The sphere of radius R_1 is transformed into the plane π_1 so that $\pi_1 \perp PO_1$ and:

$$PT_1 = \frac{4R_2^2}{2R_1} \tag{3}$$

The circle resulted from the intersection of the two spheres have PQ as diameter which is next transformed into the straight line $\pi_1 \cap \pi_2$ which is perpendicular to the drawing plane in O.

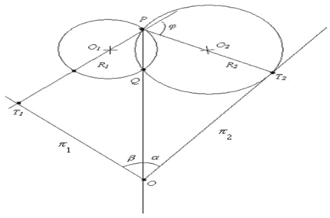

Fig. 1. The two conductor spheres

The sphere surface was equipotential, with V potential, but the surface of planes π_1 and π_2 is not equipotential. If we would put a charge in the P point:

$$q = -4\pi\varepsilon p V = -4\pi\varepsilon 2R_2 V \tag{4}$$

so the surface π_1 and π_2 goes to zero potential.

It results that in the absence of the q charge the potential of π_1 and π_2 surface is the same with the potential established by the electrical images of the q charge related the π_1 and π_2 planes. This is a known problem presented in figure 2. The number of image charges is *2n-1*. These charges are in the points A_1, A_2, .. A_{2n-1} which are situated on the circle of OP radius.

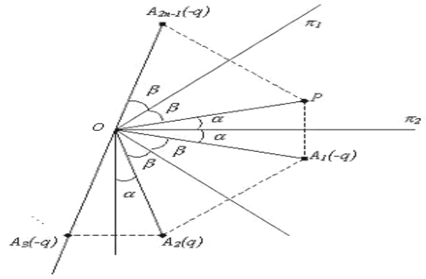

Fig. 2 – The electrical images of the q charge related the π_1 and π_2 plans

The initial situation is obtained through a new inversion of the same pole and same power. The charges from $A_1, A_2, ..., A_{2n-1}$ are transformed into an images charges $q_1', q_2', ..., q'_{2n-1}$ situated in $A'_1, A'_2, ..., A'_{2n-1}$. The point $A_1, A_2, ..., A_{2n-1}$ are situated in the circle with *O* centre and *OP* radius, so the reverses points $A'_1, A'_2, ..., A'_{2n-1}$ are situated in the straight line which is perpendicular from *PO*. The reverse point of A_1 is exactly O_2 (because $PA_1 \cdot PO_2=4R_2 \cdot R_2=4R_2^2$) so the images charges are situated at the O_1O_2 straight line.

3. CALCULUS OF THE VALUES AND OF THE POSITION CHARGE IMAGES

We use the notation $\lambda=R_2/R_1$ and

$$c = \sqrt{\lambda^2 + 2\lambda \cos\varphi + 1} \tag{5}$$

and with figure 2:

$$PA_{2m-1} = \frac{4R_2}{\sin\varphi} \cdot c \cdot \sin(m\varphi - \beta) \tag{6}$$

$$PA_{2m} = \frac{4R_2}{\sin\varphi} \cdot c \cdot \sin(m\varphi) \tag{7}$$

The position of image charges on O_1O_2 is given by:

$$PA'_{2m-1} = \frac{R_2 \sin\varphi}{c} \frac{1}{\sin(m\varphi - \beta)},$$

$m=1, 2, ..., n$ (8)

The position of even images charges is given by:

$$PA'_{2m} = \frac{R_2 \sin\varphi}{c} \frac{1}{\sin(m\varphi)}, \quad m=1, 2, ..., n-1 \tag{9}$$

The system after the two inversions is identically with the initial system, and after the substitution of the plane charges (zero potential plane) with the image charges, the total charge is not changed [2].

We note that:

$$S = \sum_{m=1}^{n} \frac{1}{\sin(m\varphi - \beta)} - \sum_{m=1}^{n-1} \frac{1}{\sin(m\varphi)} \tag{10}$$

And it results:

$$q = -q_t \frac{2c}{S \sin\varphi} \tag{11}$$

The final form deduced from the value of image charges is:

$$q'_{2m-1} = q_t \frac{1}{S \sin(m\varphi - \beta)}; \quad q'_{2m} = -q_t \frac{1}{S \sin(m\varphi)} \tag{12}$$

The V potential of the spheres can be calculated from (4), and (11):

$$V = -\frac{q}{8\pi\varepsilon R_2} = q_t \frac{c}{4\pi\varepsilon R_2 S \sin\varphi} \tag{13}$$

or directly with the image charges.

4. NUMERICAL CALCULUS FORMULA

First we choose the Cartesian system of coordinates *xO'y*, taking into consideration the meridian plane symmetry.

Then we set up the *O'x* axis, so as to comprise the O_1 and O_2 points and the axis *O'y* so as to raft across the *P* and *Q* points.

Let us first calculate:

$$x_{2m-1} = PA'_{2m-1} \sin[90 - (m\varphi - \beta)] = \frac{R_2 \sin(\varphi) ctg(m\varphi - \beta)}{c}$$

$$x_{2m} = PA'_{2m} \cos m\varphi = \frac{R_2 \sin\varphi}{c} ctg(m\varphi) \tag{14}$$

Then we choose the pair point (x,y) on which we want to evaluate the potential

$$(x - R_2 \cos\alpha)^2 + y^2 - R_2^2 \geq 0;$$

$$(x - R_1 \cos\beta)^2 + y^2 - R_1^2 \geq 0 \tag{15}$$

In case of equality in (14) V(x,y)=V. And yields the general expression of the electrical potential:

$$V(x,y) = \frac{VR_2 \sin\varphi}{c} \left[\sum_{m=1}^{n} \frac{1}{\sin(m\varphi - \beta)\sqrt{(x - x_{2m-1})^2 + y^2}} - \sum_{m=1}^{n-1} \frac{1}{\sin(m\varphi)\sqrt{(x - x_{2m})^2 + y^2}} \right] \tag{16}$$

A formula which can be adapted for any particular case in the context. Using this formula (13) the capacitance of the spheres system can easily follow. [3].

5. NUMERICAL TESTING AND VALIDATION

We took as an example certain numerical values of the spheres and of the initial potential: $R_1=0.5$ m; $R_2=1$ m; $n= 25$; $V_0= 100V$; $\varphi=\pi/n = 7.2$ deg. The proposed numerical model acts as expected, that is the potential decreases along the *Oy* axis starting from the initial value, as figure 3 shows:

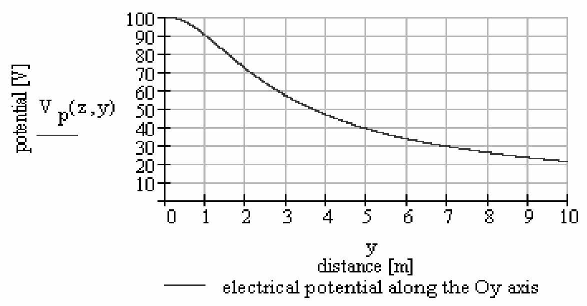

Fig. 3 – Potential behaviour along the *Oy* axis

6. CONCLUSIONS

The main objective of this paper is to present and test an electrostatic analytical and numerical model for electrostatic potential evaluation. The model depends on 3 parameters and can be adapted on certain numerical calculus needs. The proposed model can be valuable for non-destructive evaluation simulations, like [4], or in biomedical applications [5].

References

[1] D. Micu, R. Marschalko, “Electrostatica”, Ed. Mediamira, Cluj-Napoca, 1997.

[2] J. Van Bladel, “Electromagnetic Fields”, Appl.Sci.Res.Sect. B, pp. 267-270, 1967.

[3] D. Micu, “Applications of the geometrical inversion in electrostatics”, Revue Roumaine des Science Techniques,Tome49, 2004, pp.283-294.

[4] M. Morozov, et all, “Numerical Models of Volumetric Insulating Cracks in Eddy-Current Testing With Experimental Validation”, IEEE, Trans. Magn., Vol. 42, No. 5, May 2006, pp. 1568 – 1576.

[5] H. Braurer, et all, “Evaluation of Inverse Field Solutions with Biomedical Applications”, COMPEL, Vol. 20, No. 3, 2001, pp. 665 – 675.