ON THE ELECTROMAGNETIC WAVE PROPAGATION IN A BI-ISOTROPIC CHIRAL MEDIUM

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Abstract **– This paper describes the propagation of an electromagnetic plane wave incident on a chiral medium. By particularization of the Maxwell's equations for a chiral bi-isotropic medium in terms of the tangential electric and magnetic fields we can demonstrate the electromagnetic propagation in a homogeneous chiral medium or in a stratified chiral slab.**

Keywords: *electromagnetic wave, chiral medium*

1. INTRODUCTION

A bi-isotropic chiral medium is a macroscopically continuous medium composed of microscopic chiral objects that are uniformly distributed and randomly oriented. The chiral object is the three-dimensional object that has a specific handedness: that mirror image cannot be made to coincide with the original object by means of translations and rotations. When a linearly polarized electromagnetic wave is normally incident on a chiral slab, two propagations modes with different phase velocities are generate in the medium. After propagation through the slab the polarization of the transmitted field is rotated with respect to the polarization of the incident field [1]. This approach has been applied to stratified reciprocal media in the case of normally and obliquely incident waves in the frequency domain [2], [3],[4]. In this presentation we study the propagation of an electromagnetic plane wave incident on a general bi-isotropic chiral medium.

2. WAVE PROPAGATION IN BI-ISOTROPIC CHIRAL MEDIA

2.1. The Maxwell equations in frequency domain

In the macroscopic level, the description of bi-isotropic medium is contained in the material parameters of the constitutive relations, written in this look in the following form [5]:

$$D = \varepsilon \cdot E + (\chi - j \cdot k)\sqrt{\mu_0 \cdot \varepsilon_0}H \qquad (1)$$

$$B = \mu \cdot H + (\chi + j \cdot k)\sqrt{\mu_0 \cdot \varepsilon_0}E \qquad (2)$$

The dielectric response of the material is contained in the permittivity ε and the permeability μ is the corresponding magnetic parameter. The essence of bi-isotropic medium are the magnetoelectric material parameters χ and k, which are dimensionless in the equations (1),(2) as the free-space constant $\sqrt{\mu_0 \cdot \varepsilon_0}$ is separated. The imaginary unit j emphasizes the frequency domain character of the equations, and comes from the time-harmonic dependence *exp(jωt).* The chirality parameter k measures the degree of the handedness of the material and the parameter χ describes the magnetoelectric effect.

For a lossless medium and then the wave vector of the incident wave W_i is parallel of the xz- plane and makes an angle θ with the z axe (Figure 1), the fields are y- independent.

In this case, in a Cartesian coordinate system, the Maxwel's equations are:

$$\nabla \times E = -j\omega B - M \qquad (3)$$

$$\nabla \times H = j\omega D + J \qquad (4)$$

where E,B,D,H, are the electromagnetic field vectors and J, M the electric and magnetic current source vectors.

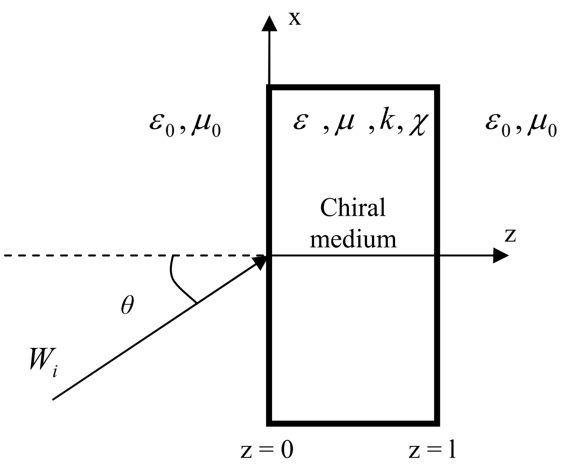

Figure 1: The wave direction

Inserting the equations (1), (2) in the equations (3), (4), we obtain:

$$\nabla \times E = -j\omega[\mu H + (\chi + j \cdot k)\sqrt{\mu_0 \cdot \varepsilon_0} E] - M \quad (5)$$

$$\nabla \times H = j\omega[\varepsilon E + (\chi - j \cdot k)\sqrt{\mu_0 \cdot \varepsilon_0} H] + J \quad (6)$$

2.2. The wavefield decomposition

If we consider the wave an electromagnetic plane wave one, with the x and t dependence $exp[j(\omega t\text{-}hx)]$, where $h = \sin n_w \theta$ is the propagation constant (n_w and θ are the wave number and the incident angle of the incident wave) the first two Maxwell's equations are:

$$\nabla \times E = -j\omega B \quad (7)$$

$$\nabla \times H = j\omega D \quad (8)$$

respectively:

$$\nabla \times E = -j\omega[\mu H + (\chi + j \cdot k)\sqrt{\mu_0 \cdot \varepsilon_0} E] \quad (9)$$

$$\nabla \times H = j\omega[\varepsilon E + (\chi - j \cdot k)\sqrt{\mu_0 \cdot \varepsilon_0} H] \quad (10)$$

Note that since the incident plane is assumed to be the xz-plane and using the last two Maxwell's equations:

$$\nabla \times E = 0 \quad (11)$$

$$\nabla \times H = 0 \quad (12)$$

on can express the third component of the electromagnetic fields in terms of the xy components of the fields:

$$\begin{bmatrix} E_3 \\ H_3 \end{bmatrix} = M_1 \begin{bmatrix} E_1 \\ E_2 \\ H_1 \\ H_2 \end{bmatrix} \quad (13)$$

where M_1 is a 2x4 matrix.

The first two Maxwell's equations can be rewritten in the following form:

$$\partial_z \begin{bmatrix} E_1 \\ E_2 \\ H_1 \\ H_2 \end{bmatrix} = M_2 \begin{bmatrix} E_1 \\ E_2 \\ H_1 \\ H_2 \end{bmatrix} \quad (14)$$

where M_2 is a 4x4 matrix. It expresses the Maxwell's equations for a chiral bi-isotropic medium in terms of the tangential electric and magnetic fields.

2.2.1. The wavefield vectors in bi-isotropic chiral medium

It is most convenient to describe the electric and magnetic field vectors E and H with two other field quantities, the wave fields, E^+, H^+ (propagating in the positive z direction) and E^-, H^- (propagating in the negative z direction), which make up the total fields (Figure 2.) as:

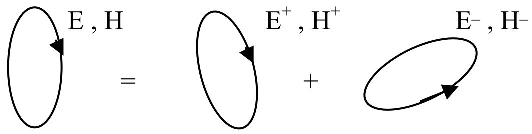

Figure 2: The wavwfield decomposition

$$E = E^+ + E^- \quad (15)$$

$$H = H^+ + H^- \quad (16)$$

$\nabla \times H = j\omega D$

If the two wavefields see the bi-isotropic chiral medium as a echivalent isotropic media with the parameters ε^+, μ^+ and ε^-, μ^-, the medium parameters and the electric and magnetic field vectors must satisfy special conditions:

$$D^+ = \varepsilon E^+ + (\chi - j \cdot k)\sqrt{\mu_0 \cdot \varepsilon_0} H^+ = \varepsilon^+ E^+ \quad (17)$$

$$B^+ = \mu H^+ + (\chi + j \cdot k)\sqrt{\mu_0 \cdot \varepsilon_0} E^+ = \mu^+ H^+ \quad (18)$$

$$D^- = \varepsilon E^- + (\chi - j \cdot k)\sqrt{\mu_0 \cdot \varepsilon_0} H = \varepsilon^- E^- \quad (19)$$

$$B^- = \mu H^- + (\chi + j \cdot k)\sqrt{\mu_0 \cdot \varepsilon_0} E^- = \mu^- E^- \quad (20)$$

with:

$$(\varepsilon - \varepsilon^+)(\mu - \mu^+) - (\chi^2 - k^2)\mu_0 \cdot \varepsilon_0 = 0 \quad (21)$$

$$(\varepsilon - \varepsilon^-)(\mu - \mu^-) - (\chi^2 - k^2)\mu_0 \cdot \varepsilon_0 = 0 \quad (22)$$

Also, the wave field vectors can be written in the form:

$$E^+ = -j\eta^+ H^+ \quad (23)$$

$$E^- = -j\eta^- H^- \quad (24)$$

with the wave impedance parameters:

$$\eta^+ = j\frac{(\chi - j \cdot k)\sqrt{\mu_0 \cdot \varepsilon_0}}{\varepsilon^+ - \varepsilon} = \frac{\mu^+ - \mu}{(\chi + j \cdot k)\sqrt{\mu_0 \cdot \varepsilon_0}} \quad (25)$$

$$\eta^- = -j\frac{(\chi - j \cdot k)\sqrt{\mu_0 \cdot \varepsilon_0}}{\varepsilon^- - \varepsilon} = \frac{\mu^- - \mu}{(\chi + j \cdot k)\sqrt{\mu_0 \cdot \varepsilon_0}} \quad (26)$$

2.2.2. The chirality parameter

The two wave fields E^+, H^+ and E^- , H^- are indepemdent, they do not couple in a bi-isotropic medium. In this case the Maxwell equations are:

$$\nabla \times E^+ + j\omega\mu^+ H^+ = 0 \quad (27)$$

$$\nabla \times E- + j\omega\mu - H- = 0 \quad (28)$$

$$\nabla \times H^{+} - j\omega\varepsilon^{+}E^{+} = 0 \tag{29}$$

$$\nabla \times H- - j\omega\varepsilon - E^{-} = 0 \tag{30}$$

Inserting (23), (24) in (29), (30) respectively, gives us:

$$\begin{aligned} &\nabla \times H^{+} - j\omega\varepsilon^{+}E^{+} = \\ &-\frac{1}{j\eta^{+}}(\nabla \times E^{+} + j\omega\varepsilon^{+}\eta^{+^2}H^{+}) = 0 \end{aligned} \tag{31}$$

$$\begin{aligned} &\nabla \times H- - j\omega\varepsilon - E- = \\ &\frac{1}{j\eta^{-}}(\nabla \times E^{-} + j\omega\varepsilon - \eta -^{2} H^{-}) = 0] \end{aligned} \tag{32}$$

which should coincide with (27), (28) respectively.

This leads to the relations:

$$\eta^{+} = \sqrt{\frac{\mu^{+}}{\varepsilon^{+}}} \tag{33}$$

$$\eta^{-} = \sqrt{\frac{\mu^{-}}{\varepsilon-}} \tag{34}$$

Substituting from (25), (26) and ε^{+}, ε^{-}, from (21), (22) in term of μ^{+} , μ^{-}, in (33), (34), we have the folowing algebric equations for $\mu^{+}$and μ^{-}:

$$\eta^{+^2} = \frac{(\mu^{+} - \mu)^2}{\mu_0 \cdot \varepsilon_0(\chi + j \cdot k)^2} = \frac{\mu^{+}}{\varepsilon + \dfrac{(\chi^2 - k^2)\mu_0 \cdot \varepsilon_0}{\mu^{+} - \mu}} \tag{35}$$

$$\eta^{-^2} = \frac{(\mu^{-} - \mu)^2}{\mu_0 \cdot \varepsilon_0(\chi + j \cdot k)^2} = \frac{\mu^{-}}{\varepsilon + \dfrac{(\chi^2 - k^2)\mu_0 \cdot \varepsilon_0}{\mu^{-} - \mu}} \tag{36}$$

They are of the second order and can be written as a single equation:

$$\begin{aligned} &(\eta^{\pm} - \mu)^2 + 2\eta(\chi + j \cdot k)\chi_r\sqrt{\mu_0 \cdot \varepsilon_0}(\eta^{+} - \mu) - \\ &-\eta^2(\chi^2 - k^2)\mu_0 \cdot \varepsilon_0 = 0 \end{aligned} \tag{35}$$

there:

$$\chi_r = \frac{\sqrt{\mu_0 \cdot \varepsilon_0}}{2\sqrt{\mu \cdot \varepsilon}} \tag{36}$$

If we note:

$$\chi_r = \sin\gamma \tag{37}$$

and chosing the convention:

$$\begin{aligned} &\eta^{+} = \mu - j\eta(\chi + j \cdot k)\sqrt{\mu_0 \cdot \varepsilon_0}e^{-j\gamma} = \\ &= \mu(\cos\gamma + \chi_r)e^{-j\gamma} \end{aligned} \tag{38}$$

$$\begin{aligned} &\eta^{-} = \mu + j\eta(\chi + j \cdot k)\sqrt{\mu_0 \cdot \varepsilon_0}e^{j\gamma} = \\ &= \mu(\cos\gamma + \chi_r)e^{j\gamma} \end{aligned} \tag{39}$$

respectively:

$$\varepsilon^{+} = \varepsilon(\cos\gamma + \chi_r)e^{j\gamma} \tag{40}$$

$$\varepsilon^{-} = \varepsilon(\cos\gamma - \chi_r)e^{-j\gamma} \tag{41}$$

the two impedance parameters are the form:

$$\eta^{+} = \sqrt{\frac{\mu^{+}}{\varepsilon^{+}}} = \eta e^{-j\gamma} \tag{42}$$

$$\eta^{-} = \sqrt{\frac{\mu^{+}}{\varepsilon^{+}}} = \eta e^{j\gamma} \tag{43}$$

and the wave numbers:

$$w_n{}^{+} = \omega\sqrt{\mu^{+}\varepsilon^{+}} = k(\cos\gamma + k_r) \tag{44}$$

$$w_n{}^{-} = \omega\sqrt{\mu^{-}\varepsilon^{-}} = k(\cos\gamma - k_r) \tag{45}$$

Using (42) and (43) we can write:

$$\eta^{+}\mu^{-} = \mu^2 \tag{46}$$

$$\eta^{+} + \eta^{-} = 2\eta\cos\gamma \tag{47}$$

From them equations are seen the following consequences:

- the wave numbers are not real for lossless medium;
- the wave impedance as well as the parameters $\varepsilon^{\pm}$, $\mu^{\pm}$, are complex for nonzero values of parameter χ;
- The impedance parameter $\eta^{\pm}$ do not depend on the chirality parameter k, or v transforms η^{+} to η^{-} and conversely;
- The wave numbers $w_n{}^{\pm}$ depend on the both chirality parameters, change of sign of χ does not afect them but transforms k^{+} to $k^{-}$and conversely.

Because we can write:

$$w_n{}^{+} - w_n{}^{-} = 2kk_r \tag{48}$$

$$w_n{}^{+} - w_n{}^{-} = 2k\cos\gamma \tag{49}$$

it is seen that the difference in wave numbers depends on the chirality of the bi-isotropic medium while the arithmetic means of the wave numbers depend solely on the χ parameter.

- For lossless medium, the wave numbers $w_n{}^{\pm}$ are both positive if the parameters satisfy the conditions:

$$\cos\gamma > |k_r| \qquad (50)$$

$$\chi_r^{\,2} + k_r^{\,2} < 1 \qquad (51)$$

This can be seen as a certain limiting condition for the relative chiral parameters which the nature of the medium is radically changed.

3. CONCLUSIONS

By particularization of these equations, we can demonstrate the electromagnetic wave propagation in a homogeneous bi-isotropic chiral medium or in a stratified chiral slab. In all this cases the total electromagnetic field will be decomposed into the down-going modes (propagating in the positive z direction) and up- going (propagating in the negative z direction). Hence, we can calculate a reflection and a transmission coefficient matrix, and finally the tangential and the transmitted E and H fields. Finaly we can estimate the chiral parameters then the nature of the medium is changed.

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