MODELING THE SEDIMENTATION PROCESS IN A WASTEWATER TREATMENT SYSTEM

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***Abstract* – In this paper we focus on the theory of the modeling, applied on the sedimentation process in a wastewater treatment system. Every system – mechanical, electrical, biological or whatever – can be accurately described by a mathematical model. The models can be applied in practice because the computers allow us to solve it in a numerically way.**

Keywords: *mathematical model, sedimentation process, wastewater treatment system.*

1. INTRODUCTION

Ever since Isaac Newton published his fundamental work *Mathematical Principles of Natural Philosophy* in 1687 where the fundamental laws of force and motion were formulated, the conclusion within the scientific community has been: *Nature has laws, and we can find them*. The importance of this statement cannot be overestimated. It implies that every system – mechanical, electrical, biological or whatever – can be accurately described by a mathematical model.

The models can today also be applied in practice as the computers allow us to numerically solve process models of such complexity that could hardly be imagined a couple of decades ago. In an ideal world, process modeling would be a trivial task. Models would be constructed in a simple manner yet in every way reproduce the true process behavior.

The word 'model' has a wide spectrum of interpretations: mental model, linguistic model, visual model, physical model and mathematical model. In this work we will restrict ourselves to mathematical models, that is, models within a mathematical framework where equations of various types are defined to relate inputs, outputs and characteristics of a system.

Primarily, mathematical models are an excellent method of conceptualizing knowledge about a process and to convey it to other people. Models are also useful for formulating hypotheses and for incorporating new ideas that can later be verified (or discarded) in reality. An accurate model of a process allows us to predict the process behavior for different conditions and thereby we can optimize and control a process for a specific purpose of our choice. Finally, models serve as an excellent tool for many educational purposes.

2. GENERAL MODELING STRATEGY

We need a general strategy for model building. In overview, the modeling of any system occurs in five rather distinct steps, as illustrated in Figure 1.

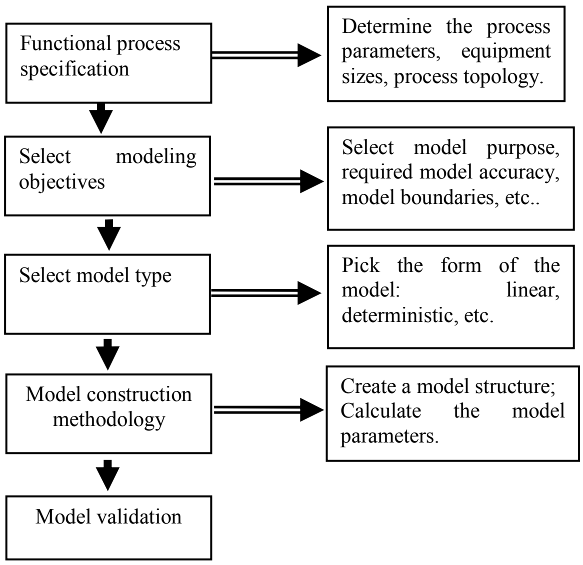

Figure1: Modeling process

Step one is to delineate the system being modeled as a functional specification. A quantitative understanding of the structure and parameters describing the process is required. Typically for wastewater applications this functional specification may include such information as equipment type and size, flow-sheet layout, environment variables, nominal operating conditions.

The modeling objectives are decided and then the desired model type selected. A model building strategy is then followed to arrive at the appropriate model for the desired application.

Any given process may have different 'appropriate' models. The chosen appropriate model will depend on its objectives. These *a priori* decisions about the model must be made before the model construction

can begin. Some of the more relevant objectives concern model *purpose*, *system boundaries*, *time constraints* and *accuracy*.
A wide variety of models are possible, each of which may be suitable for a different application. A clear statement of the model intention is needed as a first step in setting the model objectives. This entails listing all the relevant process variables and the accuracy to which they must be modeled.
Within the field of wastewater treatment we can define a number of general purposes for mathematical models (also applicable to many other fields). These are listed below.

• *Design* – models allow the exploration of the impact of changing system parameters and development of plants designed to meet the desired process objectives at minimal cost.
• *Research* – models serve as a tool to develop and test hypotheses and thereby gaining new knowledge about the processes.
• *Process control* – models allow for the development of new control strategies by investigating the system response to a wide range of inputs without endangering the actual plant.
• *Forecasting* – models are used to predict future plant performance when exposed to foreseen input changes and provide a framework for testing appropriate counteractions.
• *Performance analysis* – models allow for analysis of total plant performance over time when compared with laws and regulations and what the impact of new effluent requirements on plant design and operational costs will be.
• *Education* – models provide students with a tool to actively explore new ideas and improve the learning process as well as allowing plant operators training facilities and thereby increasing their ability to handle unforeseen situations.

2.1. Modeling the Sedimentation Process

In this paper we focus on the mathematical modeling of a sedimentation process.
In the cleaned waste water process, the solid suspension has to be separated from the treated water to produce clear final effluent. This solid-liquid separation process is usually achieved by gravity sedimentation in traditional vat with decantation.
Common one-dimensional models are based on the flux theory. The force that makes the sedimentation of the particles in the liquid possible, originates from gravity and the density differences between the particles and the liquid.
It is assumed that in clarifiers the profiles of horizontal velocities are uniform and that horizontal gradients in concentration are negligible. Consequently, only the processes in the vertical dimension are modeled. The resulting idealized settling cylinder is treated as a continuous flow reactor.

2.1.1. Simple modeling of a vat with decantation with horizontal flow

The principle of a vat with decantation with horizontal flow consists in making circulate, at speed constant, $\vec{v_h}$, a current of water containing the particles of different masses in a device which one can model in the way of figure 3.

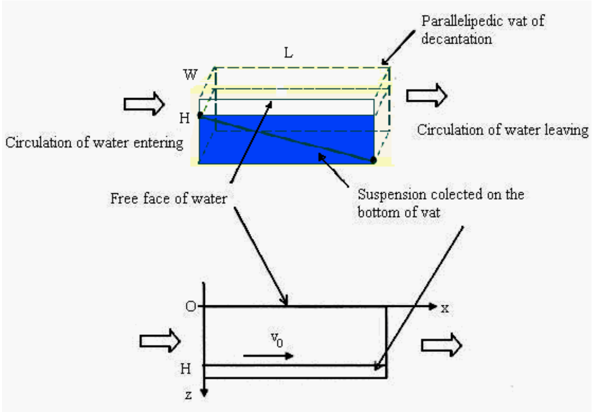

Figure 3: Vat of decantation schema

The vat of decantation has width W, height H, length L and for a particle, the horizontal speed is: v_h=Q/(WH), where Q is the cross flow, the vertical speed is: v_h and the particles forms a deposit on the bottom of the basin during its passage named time of retention ($t_r = \frac{SH}{Q}$).
According to the characteristics of the particles, these last will fall at the bottom from the vat in different places.
A suspended particle arriving on the surface at the entry of the decanter elutriates with a constant speed v_h. The decantation is finished when the particle settled on the foundation raft and the duration of fall is equal to *H/v*. The possibility for the particles of reaching the bottom of the work is obviously possible only if $t > H/v$, or $v_h > Q/S$. The term *Q/S* is called speed of Hazen, it is often expressed in cubic meter hour per square meter ($m^3/h/m^2$) or measures per hour (m/h).

2.1.2. Study of the fall of a particle in a viscous liquid

To model the sedimentation process we are interested in the movement of a particle initially on the surface of water, the coast z = 0 and penetrating in the vat in x = 0.
To study the fall of a particle in a viscous liquid we prepare a homogeneous mixture made up of a liquid

of density ρ_l and of solid particles of the spherical shape of ray R, of density ρ_s and of mass m.

We deposit, at the time t = 0 s, a fine layer (which one neglects the thickness) of this mixture homogeneous on the surface of the container containing the same liquid, at the pure state, as it preceding mixture.

From this moment, the particles, which were initially supposes at rest, move vertically towards the bottom of the container. It is supposed that the speed limit is sufficiently low and we have a free sedimentation.

Free sedimentation is the sedimentation of heterogeneous systems in witch there aren't interactions between particles. To study the movement of the particle, we place oneself in a one - dimensional reference, mark the axis Oz vertical and directed to the bottom, of origin O on the level of the free face of the liquid.

To establish the speed of the sedimentation it is considered the particle of the solid phase. On this particle action the forces (figure 4): F_r: force of frictions; F_a: pushed of Archimedes and G: weight of the particle.

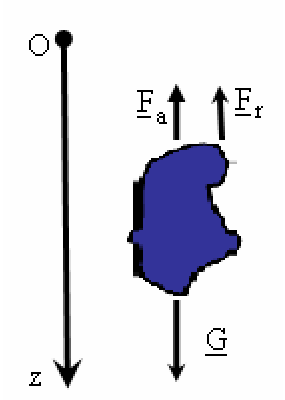

Figure 4: Forces which are action on the particle

In the terrestrial reference frame, supposed Galilean, the 2nd law of Newton applied to the particle is expressed by:

$$\vec{G} - \vec{F_a} - \vec{F_r} = m\vec{a}. \qquad (1)$$

Let us project (1) on axis OZ and separated the variables:

$$mg - \rho_l Vg - \xi A \frac{v^2}{2}\rho_l = \frac{mdv}{dt}, \qquad (2)$$

$$F_r = \xi A \frac{v^2}{2}\rho_l$$

$$\frac{dv}{dt} = g\left(1 - \frac{\rho_1 . V}{\rho_S . V}\right) - \xi A \frac{v^2}{2m}\rho_l. \qquad (3)$$

The notations are those from the literature: V is the volume of the drop or the volume of liquid moves ($m = \rho_S V$), A is the free surface of the fluid; v is the speed and ξ is the coefficient of friction.

Coefficient of friction, ξ, is determinate according with the criteria of Reynolds and it knows that this dependence is modifying for two values of Reynolds number: Re=2 and Re=500.

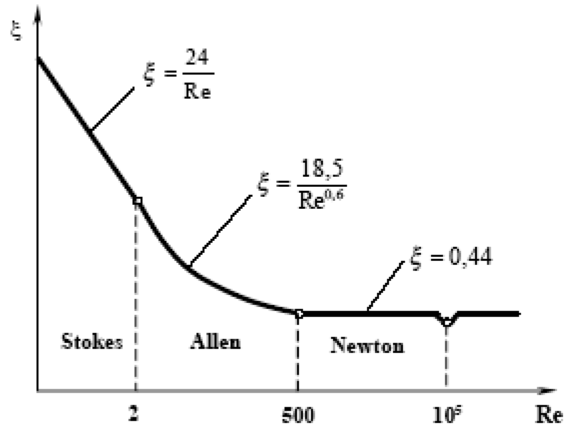

Figure 5: Dependence $\xi(Re)$

The relative differential equation at the speed of the particle is:

$$\frac{dv}{dt} + \xi A \frac{v^2}{2m}\rho_l = g\left(\frac{\rho_S - \rho_l}{\rho_S}\right). \qquad (4)$$

When speed limits v_l is reached, its value remains constant: $\frac{dv}{dt} = 0$.

Once a particle is entrained in a fluid it begins to sink again under gravitational forces with the speed of free sedimentation. The speed of free sedimentation is:

$$v_0 = \sqrt{\frac{2mg(\rho_s - \rho_l)}{\xi A \rho_l \rho_s}} \qquad (5)$$

For the spherical particle ($m = \frac{4\pi\rho_s R^3}{3}$) the speed of free sedimentation becomes:

$$v_0^2 = \frac{8Rg(\rho_s - \rho_l)}{3\xi\rho_l} \qquad (6).$$

The speed of free sedimentation could be obtaining from the criteria equations, which are given the relations between Reynolds number (Re) and Arhimede number (Ar). The steps, in this case are:

- Squared the relation (6):

$$v_0^2 = \frac{8Rg(\rho_s - \rho_l)}{3\xi\rho_l}$$

- From the expression of Reynolds result:

$$\mathrm{Re}^2 = \frac{4v_0^2\rho_l^2R^2}{\eta_c^2};$$

- Speed expression become: $v_0^2 = \dfrac{\mathrm{Re}^2\,\eta_c^2}{4R^2\rho_c^2}$;

- After calculus results: $\mathrm{Re}^2\,\xi = \dfrac{32R^3g(\rho_s - \rho_l)}{3\eta_c^2}$

-The expression of Arhimede criteria has the expression: $Ar = \dfrac{8R^3g(\rho_s - \rho_l)\rho_l}{\eta_c^2}$;

- The last expression for Reynolds number is:

$$\mathrm{Re}^2\,\xi = \frac{4}{3}Ar. \qquad (7)$$

So, for the critical values of Reynolds number, Re=2 and Re=500, there are the following critical values of Arhimede's criteria's: Re=2, Ar=36 and Re=500, Ar=84000.
Te following step is to establish the criteria equations for the three areas of sedimentation:
- Stokes areas: $\mathrm{Re} \le 2, Ar \le 36 \Rightarrow Ar = 18\,\mathrm{Re}$;
- Allen areas:

2 < Re < 500,36 < Ar < 84000ÛË

$$Ar = \frac{3}{4}Re^2\frac{18.5}{Re^{0.6}}, Re = \left(\frac{Ar}{13.9}\right)^{\frac{1}{1.4}};$$

- Newton areas:

Re ˇÝ500, Ar ˇÝ84000ÛË

$$Ar = \frac{3}{4}Re^2 \quad 0.44, Re = 1.71\sqrt{Ar}.$$

The way in which is used the criteria equations is following the steps:
- calculate the value of Arhimede criteria and is establishing the sedimentation areas;
- function of sedimentation areas, is choosing the equations, and is calculating the expression of Reynolds and then the speed of free sedimentation.
To find the diameter of the particles which are sediment with a known speed is used the criteria's of Liascenko.
The distance which the particle travels depends on the drag force of the fluid and the settling velocity of the particle. The sedimentation speed is calculated using Stoke's Law, which can be considered as the sum of the gravitational pull downward versus the drag force of the fluid pushing upward.

2.1.3. Numerical example

For the numerical application, considers $\rho_l = 1{,}0 \times 10^3$ kg.m^{-3} and the spherical particles with R = 2,0 × 10^{-6} m, $\rho_S = 1{,}5 \times 10^3$ kg.m^{-3} and m = 5,0 ×10^{-14} kg.
Using the calculus numeric in Matlab, we can represent the variations of the speed of the particle in time, figure 5.

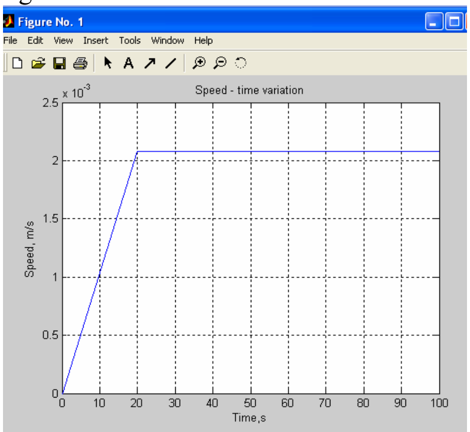

Figure 5: Speed variation in time for a spherical particle.

4. CONCLUSIONS

The application of one-dimensional models gives a reasonable approximation of the sedimentation process. Furthermore, application of these models does not require much computation capacity. However, in real case there exist several phenomena that cannot be reflected in 1-D models. Nevertheless, one-dimensional models are widely used and accepted in computer simulation of wastewater treatment plants nowadays.

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