# THE ANALYTICAL EVALUATION OF THE CONNECTING TIME FOR THE AC ELECTROMAGNETIC COMMUTATION APPARATUSES WITH CONTACTS

Radu BURLICA, Eugen HNATIUC, Bogdan HNATIUC

Technical university "Gh. Asachi" Iasi, Romania, e-mai : rburlica@ee.tuiasi.ro

Abstract – Electromagnetic devices often do the command of the circuit breakers. The minimum response time of these devices is an essential condition for an efficient circuit connection. The most used devices for the circuit breakers command are the AC electromagnets, due to their fast response time. In this paper the disconnection response time have been studied, taking into account their response time dependency and the possibility to control the transient regime of these electromagnets and the duration of the connecting-disconnecting cycle by taking into account the stray magnetic field. The conclusions of this study can lead to the possibility of an optimal design of the AC electromagnets used for circuit breakers command.

*Keywords: ac electromagnets, dynamic regime, electrical apparatuses.* 

# **1. INTRODUCTION**

The paper proposes a model for the disconnecting time evaluation for AC commutation electromagnetic apparatuses with electrical contacts (EAC), which permit the mathematical analysis of the EAC dynamic regime.

The principles of the analytical mechanics permit to reduce the mass of a mechanism and the forces acting on it's elements, to an equivalent element (in this case the electromagnet armature) characterized by the "reduced mass", on which is acting the "reduced force". Using the specific notions of the cinematic and dynamic analysis [1], finally it is possible to assimilate the EAC with an electromechanical energy conversion system with two-freedom degree, which can be described by non-linear differential equations. Thus, the all EAC ensemble became equivalent with an electromagnet, which must act the "reduced mass" and the "reduced force" of the mechanism.

Due to the fact that the differential equations system has not analytical solution, we propose a numerical method to approximate the solution, for which have been demonstrated it's existence and it's oneness.

#### 2. THE MATHEMATICAL MODEL

Starting from the Lagrange energetically equations applied to the equivalent electromechanical system for the EAC, on can determine the differential equations system which describes mathematically the EAC behavior during the dynamic regime [2]. The system variables will be: the electromagnet armature displacement x(t) and the magnetic flux  $\phi(t)$ .

The equivalent EAC electromechanical system is presented in Fig. 1.



Fig. 1. The equivalent electromechanical system for the EAC.

For an accurate modeling of the EAC behavior during the transient regime it is necessary to specified, in the mathematical system, the contribution on the electromagnet functioning of the straying magnetic field, also the influence of the bending of the magnetic field in the air gap vicinity. Thus, we will introduce in the differential equations a straying coefficient  $k_{\sigma}$ , define like the ratio between the total magnetic flux and the flux through the electromagnet air gap.

$$\boldsymbol{k}_{\sigma} = \frac{\boldsymbol{\varphi}}{\boldsymbol{\phi}_{u}} = 1 + \boldsymbol{\sigma}(\boldsymbol{d}_{0} - \boldsymbol{x}) \tag{1}$$

where:  $\phi$  is the total magnetic field,  $\phi_u$  is the magnetic flux through the electromagnet air gap,  $\sigma$  is the magnetic straying field coefficient.

Taking into account the notation  $\Omega_0 = k / m$ , the electromechanical system, presented in Fig. 1, is described by the following system of differential equations:

$$\begin{cases} \frac{d^{2}x(t)}{dt^{2}} + \Omega_{0}(x(t)+b) = \frac{\phi^{2}}{2\mu_{0}Sm(1+\sigma(d_{0}-x(t)))^{2}} \\ \frac{d\phi(t)}{dt} + \frac{R}{\mu_{0}SN^{2}}(d_{0}-x(t)+a)\phi(t) = \frac{U(t)}{N} \end{cases}$$
(2)

with the initial conditions:

$$\frac{dx(t_0)}{dt} = 0, \quad \phi(t_0) = \phi_0$$
  

$$U(t) = U_m \sin(\omega t + \varphi)$$
  
where: 
$$\frac{dx(t_0)}{dt}$$
 is initial speed of the mobile

armature,  $\phi_0$  is initial magnetic flux from which the mobile armature movement starts, U(t) = 0 (in the case of the disconnecting regime for AC electromagnet), x(t) is the armature displacement,  $\phi(t)$  is the magnetic flux, m is the reduced mass of the mechanism, k is the elastic constant of the antagonist reduced force, b is a constant which represent the initial antagonist force, S is the transversal aria of the magnetic core, N is the turns number of the electromagnet coil, R is the electromagnet coil resistance, a is a constant which represent the contribution of the magnetic core permeability to the electromagnet inductance,  $d_0$  is the maximum value of the electromagnet air gap,  $\mu_0$ is the air magnetic permeability ( $\mu_0 = 4\pi 10^{-7}$  H/m), and  $\varphi$  is the connecting angle, which represents in equations the voltage the electromagnet coil is supplied, considering the power supply voltage sinusoidal.

The straying magnetic field coefficient can be determined starting from the equivalent scheme of the electromagnet magnetic circuit with the relation (1), [5].

Due to the fact that the system of equation (2) has not an analytical solution it is necessary to apply an approximate numerical method to solve the system based on the finite difference method, [4], [5]. The method permits to substitute the derivative expressions, in equation system (2), with relations as the following:

$$\frac{d^2 x(t)}{dt^2} = \frac{x_{i+2} - 2x_{i+1} + x_i}{h^2}$$

$$\frac{d\phi}{dt} = \frac{\phi_{i+1} - \phi_i}{h}$$
(3)

where: h is the time interval between two consecutive points of calculus;  $x_i$  and  $\phi_i$  are the armature displacement and the magnetic flux values at the time ih.

The differential equation system became:  

$$\begin{cases}
x_{i+2} = \left[\frac{\phi_i^2}{\mu_0 Sm} - \Omega_0(x_i + b)\right] h^2 + 2x_{i+1} - x_i \\
\phi_{i+1} = \frac{\sqrt{2}U \sin(\omega t + \phi)}{N} h - \frac{2R}{\mu_0 SN^2} \phi_i(d_0 - x_i + a) h + \phi_i
\end{cases}$$
(4)

The displacement of the mobile armature of the electromagnet in time, x(t), for the connecting regime, depends on the commutation angle  $\varphi$ . In Figures 2-6 are presented the evolution of the armature displacement x of the electromagnet for different commutation angles.



Fig.2 The armature displacement for  $\varphi = 0$ 



Fig. 3 The armature displacement for  $\varphi = \pi / 6$ 



Fig.4 The armature displacement for  $\varphi = \pi / 4$ 



Fig. The armature displacement for  $\varphi = \pi / 2$ 



Fig.5 The armature displacement for  $\varphi = 2\pi/3$ 



Fig.6 The armature displacement for  $\varphi = 5\pi/6$ In Fig. 7 is presented the evolution of the connecting time with the commutation angle.



Fig.7 The evolution of the connecting time with the commutation angle

On can emphasize a minimum for the connection time for a commutation angle between  $\varphi = \pi/6$  and  $\varphi = \pi/4$ .

In the case of disconnecting transient regime the differential equation system (4) became:

$$\begin{cases} x_{i+2} = \left[\frac{\phi_i^2}{2\mu_0 Sm(1+\sigma(d_0-x_i))^2} - \Omega_0(x_i+b)\right]h^2 + \\ +2x_{i+1} - x_i \\ \phi_{i+1} = \frac{0}{N}h - \frac{R}{\mu_0 SN^2}\phi_i(d_0-x_i+a)h + \phi_i \end{cases}$$
(5)

with the initial conditions:  $x_1 = x_0$ ,  $\phi_1 = \phi_0$ .

In the case of a mechanical characteristic with in the contacts touching point, it is necessary to solve two differential equation systems, of the same type as system (4), which mathematically describe the two parts of the mechanical characteristic, characterized by the elastic coefficients  $k_1$  and  $k_2$ , respectively.

In order to validate the mathematical model proposed for the EAC have been solve the differential equations system (4) for a AC contactor's connecting regime.

# **3. EXPERIMENTAL RESULTS**

The mathematically model analyzed describes the dynamic behavior during the disconnecting transient regime of a AC contactor characterized by the following parameters: U=220 V;  $d_0$ =8 mm; R=110  $\Omega$ ; S=0.0003 m<sup>2</sup>; m=0.3 Kg; N=2700 turns; k=1200; a=0.0003

In the Fig. 8 are represented the curves for the electromagnet armature displacement with time.



Fig. 8. The armature displacement. a) considering the stray magnetic field; b) without considering the stray magnetic field

### **5.** CONCLUSIONS

The mathematically results obtained from the numerical solving of the differential equation system, show the possibility to approximate the disconnecting time for AC EAC mobile assemble during the transient regime in a domain of errors smaller then 10%. The numerical analysis for the mentioned AC contactor shows a smaller disconnecting time when the influence of the stray magnetic field is taking into account. This conclusion leads to possibility to reduce the disconnecting time for AC commutation apparatuses by increasing the dispersion magnetic field in the air gap zone.

In the case of connecting regime of the electromagnet a minimum for the connecting time can be emphasize.

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