EQUIVALENT CIRCUITS OF LOW POWER SYNCHRONOUS MOTORS AT FREQUENCY COMMAND

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***Abstract* – The equivalent scheme of low power permanent magnet synchronous motors is established in this paper. There is emphasized the influence of the command quantities, of the excitation degree, of the electrical parameters value and of the load magnitude on the components of the current taken by the synchronous motors. Thus, several families of curves are plotted for emphasizing these influences.**

***Keywords*:** *synchronous motors, permanent magnets, equivalent circuits.*

1. INTRODUCTION

The electrical driving systems in alternating current, which are adjustable and rated at low power, have developed, being fitted out both with asynchronous motors and reluctance synchronous motors or permanent magnet (PM) synchronous motors. The utilization of the fractional power synchronous motors ensures the synchronous motion of some mechanisms, without using special synchronizing equipments.

Further on the low power permanent magnet synchronous motors are taken into account. The paper aims to establish and to justify the equivalent circuit of these motors, which is a basic element for their characteristics computation, when operating at variable frequency. The obtained results allows the obtaining of the general equivalent circuits for fractional power motors without commutator, useful for the computation of the characteristics for all types of motors controlled in frequency.

In order to establish the equivalent circuit, it is important to choose the parameter characterizing the motor load. The internal angle, θ, is usually chosen as parameter of load; this angle is defined as being the angle between the phasor of the terminal voltage and the phasor of the e.m.f. induced by the inductor field.

2. EQUIVALENT CIRCUIT HAVING THE INTERNAL ANGLE θ AS PARAMETER

The following voltages equation written in complex for one phase of the armature winding results by considering that the motor is supplied by a voltage having the root-mean-square U and the frequency f:

$$\underline{U} = -\underline{U}_{e0} + j\omega L_{ad}\underline{I}_d + j\omega L_{aq}\underline{I}_q + jX_\sigma \underline{I} + R\underline{I}, \quad (1)$$

with

$$\omega = 2\pi f, \quad (2)$$

and the other quantities have known meaning [1, 3, 4]. In accordance with this one, the phasors diagram depicted in the figure 1 can be plotted and the following relations are obtained by means of the projections on the axes d, q:

$$U\cos\theta = 2\pi f L_d I_d + U_{e0} + RI_q;$$
$$U\sin\theta = 2\pi f L_q I_q - RI \quad (3.a,b)$$

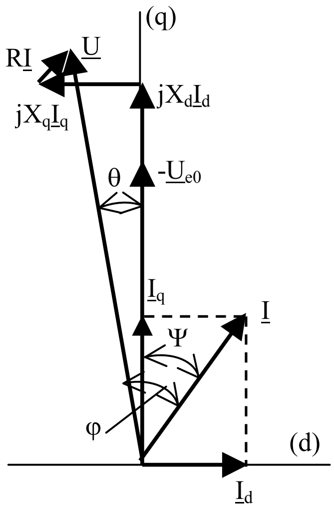


Figure 1. Phasors diagram for the synchronous motor

The influence of the armature reaction phenomenon and the magnetic core losses are neglected further on.

By noting with: U_N, f_N, U_{e0N} the rated supplying voltage, the rated frequency and the e.m.f. induced by the inductor field at rated speed and with

$$\alpha_u = U / U_N \quad (4)$$

the signal factor for voltage,

$$\alpha_f = f / f_N \quad (5)$$

the signal factor for frequency and

$$k_e = U_{e0N} / U_N \qquad (6)$$

the excitation factor, the equations system (3) is written in the form:

$$\alpha_u U_N \cos\theta - \alpha_f k_e U_N = \alpha_f X_d I_d + R I_q ;$$
$$\alpha_u U_N \sin\theta = \alpha_f X_q I_q - R I_d , \qquad (7.a, b)$$

where X_d and X_q are the synchronous reactances of the motor at rated frequency.

The root-mean-square values of the components for the current taken from mains are obtained by solving the system (7).

$$I_q = \frac{U_N}{X_d} \cdot \frac{\alpha_u k_r \cos\theta + \alpha_u \alpha_f \sin\theta - \alpha_f k_e k_r}{k_r^2 + k_x \alpha_f^2} ;$$

$$I_d = \frac{U_N}{X_d} \cdot \frac{\alpha_u \alpha_f k_x \cos\theta - \alpha_u k_r \sin\theta - \alpha_f^2 k_e k_x}{k_r^2 + k_x \alpha_f^2}$$

(8.a,b)

in which:

$$k_r = \frac{R}{X_d}, \qquad k_x = \frac{X_q}{X_d}. \qquad (9.a,b)$$

The phasor components of the current are obtained by setting the supplying voltage phasor along the real axis:

$$\underline{I}_q = I_q e^{-j\theta} ;$$

$$\underline{I}_d = I_d e^{-j(\frac{\pi}{2}+\theta)} , \qquad (10.a,b)$$

and

$$\underline{I} = \underline{I}_d + \underline{I}_q . \qquad (11)$$

The restricted expression of the current taken from mains is obtained by taking into account the relations (9) and (10).

$$\underline{I} = \underline{I}_0 + \underline{I}_e + \underline{I}_{dq} , \qquad (12)$$

where:

$$\underline{I}_0 = \frac{\alpha_u}{\alpha_f} \cdot \frac{U_N}{X_d} \cdot \frac{\alpha_f (1 - k_e \alpha_f / \alpha_u)}{k_r^2 + k_x \alpha_f^2} (k_r - j k_x \alpha_f) \qquad (13)$$

is the current taken by the motor when the load angle, θ, is equal to zero,

$$\underline{I}_e = \frac{\alpha_u}{\alpha_f} \cdot \frac{U_N}{X_d} \cdot \frac{k_e \alpha_f^2 \sin\theta}{\alpha_u (k_r^2 + k_x \alpha_f^2)} \times$$
$$\times \left[\left(k_x \alpha_f + k_r tg \frac{\theta}{2} \right) - j \left(k_x \alpha_f tg \frac{\theta}{2} - k_r \right) \right] \qquad (14)$$

is the component corresponding to the interaction torque between the inductor field and the induced field and

$$\underline{I}_{dq} = \frac{\alpha_u}{\alpha_f} \cdot \frac{U_N}{X_d} \cdot \frac{\alpha_f^4 (1 - k_x) \sin 2\theta}{(k_r^2 + k_x \alpha_f^2)^2} (1 - j tg\theta) \qquad (15)$$

is the component corresponding to the reluctance torque.

The equivalent circuit for low power synchronous motors can be plotted (fig. 2) on the basis of the relations (12)$\div$(15), at frequency command, the internal angle, θ, being the load parameter.

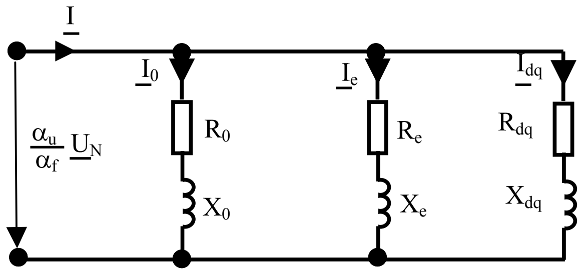


Figure 2. Equivalent circuit for low power synchronous motors, with frequency command.

The electrical parameters of the circuit are computed with the relations:

$$\underline{Z}_0 = R_0 + jX_0 = X_d \frac{\alpha_u (k_r^2 + k_x \alpha_f^2)}{\alpha_f (k_r^2 + k_x^2 \alpha_f^2)(\alpha_u - k_e \alpha_f)} (k_r + j k_x \alpha_f) ;$$

$$\underline{Z}_e = R_e + jX_e = X_d \cdot \frac{(k_r^2 + k_x \alpha_f^2)}{\alpha_f^4 (1 - k_x) tg\theta} (1 + j tg\theta) ;$$

$$\underline{Z}_{dq} = R_{dq} + jX_{dq} = X_d \cdot \frac{\alpha_u}{\alpha_f} \cdot \frac{(k_r^2 + k_x \alpha_f^2)}{2 k_e \alpha_f tg \frac{\theta}{2} (k_r^2 + k_x^2 \alpha_f^2)} \times$$
$$\times [(k_r tg \frac{\theta}{2} + k_x \alpha_f) + j (k_x \alpha_f tg \frac{\theta}{2} - k_r)] . \qquad (16.a,b,c)$$

As for the equivalent circuit depicted in the figure 2 the following observations can be made:

- the equivalent circuit impedances are complex functions of the command quantities (α_f and α_u);
- the magnitude and the sense of the current $\underline{I}_0$ depend on the excitation degree, by means of k_e; the motor is over-excited for $k_e > \frac{\alpha_u}{\alpha_f}$ and under-excited for $k_e < \frac{\alpha_u}{\alpha_f}$, respectively;
- the value of the current $\underline{I}_0$ does not depend on the load angle (when the armature reaction phenomenon influence is neglected), but the values of the

components $\underline{I}_e$ and $\underline{I}_{dq}$ are complex functions of this angle;

- the component $\underline{I}_{dq}$ does not depend on the excitation degree and for sunken poles synchronous motors ($k_x = 1$) , I_{dq} = 0;

- for $\theta = 0$, the limit for passing from motor regime to the generator regime is not obtained, because the electromagnetic power becomes in this case.

$$P_{M0} = P_1 - p_{j1} = mI_0^2 (R_0 - R) , \qquad (17)$$

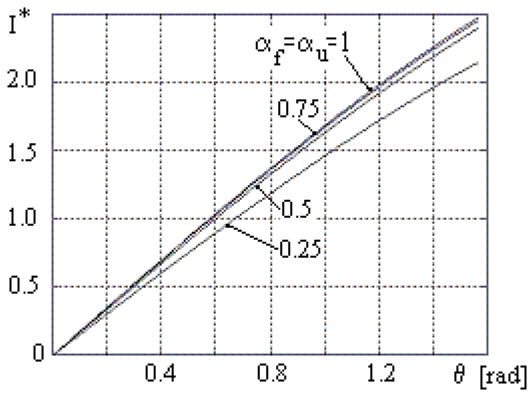
and its value is function both of the phase resistance, R, and of the signal factors α_u and α_f .

3. COMPUTATION RESULTS

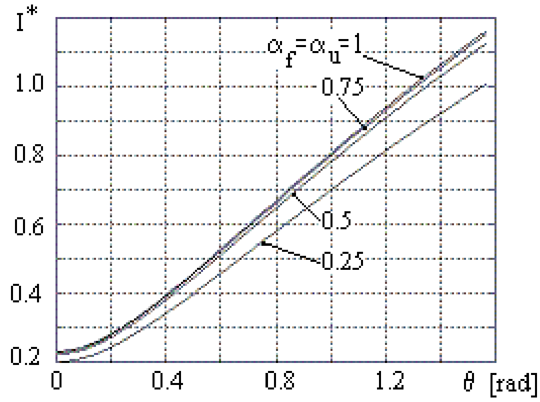
The results obtained by computation with the help of the relations presented before are presented further on; the results correspond to a three phase permanent magnet synchronous motor [2], rated at: $U_{Nf} = 220$ V, $I_{Nf} = 0{,}55$ A, $2p = 8$ poles, $f_{1N} = 50$ Hz, $X_d = 226\,\Omega$, $X_{\sigma 1} = 74\,\Omega$, $R_1 = 53\,\Omega$. The results are expressed in per unit (p.u.), noted with superior index * and the base quantities have been adopted:

$$\Omega_b = \Omega_1 = \frac{2\pi f_1}{p}; \quad M_b = \frac{mU_{Nf} I_{Nf}}{\Omega_b}; \quad Z_b = \frac{U_{Nf}}{I_{Nf}}. \qquad (18)$$

The influence of different quantities on the current taken by the synchronous motor is presented in figures 1 – 4.

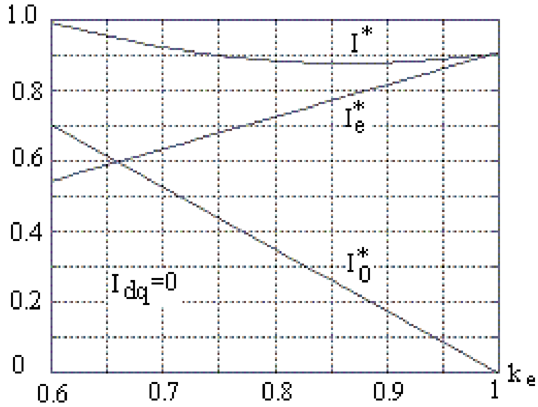


a)

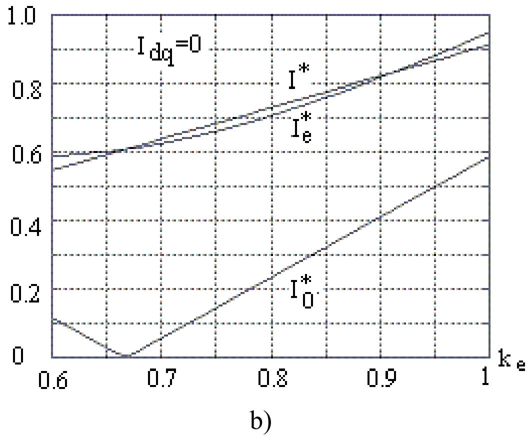


b)

Figure 1. Current taken by the synchronous motor function of the internal angle, for $k_x = 1$, $k_r = 0{,}15$, $\alpha_u = \alpha_f$ in the cases: a) $k_e = 1$; b) $k_e = 0{,}75$.



a)



b)

Figure 2. Influence of the excitation degree on the components of the current taken by the synchronous motor, for $k_x = 1$, $k_r = 0{,}15$, $\theta = 30^0$, in the case: a) $\alpha_u = \alpha_f = 1$; b) $\alpha_u = 1$; $\alpha_f = 1{,}5$.

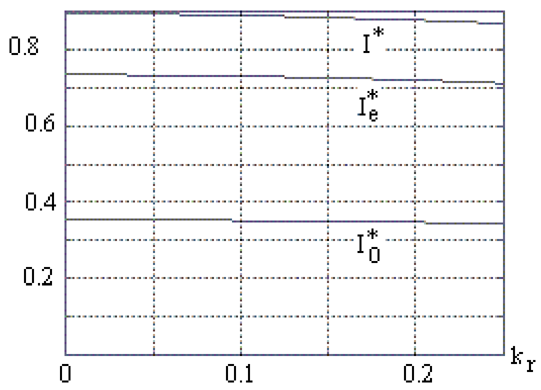


Figure 3. Influence of the armature winding resistance on the components of the current taken by the synchronous motor, for $k_x = 1$, $k_e = 0{,}8$, $\alpha_u = \alpha_f = 1$ and $\theta = 30^0$.

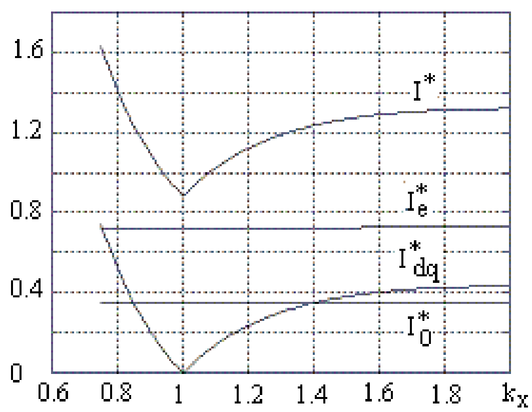


Figure 4. Influence of the rotor magnetic asymmetry on the components of the current taken by the synchronous motor, for $k_r = 0{,}15$, $k_e = 0{,}8$, $\alpha_u = \alpha_f = 1$ and $\theta = 30^0$.

4. CONCLUSIONS

The equivalent circuits allow the characteristics computation for different types of synchronous motors and the obtaining of the current diagram.

For the permanent magnet synchronous motors, $k_e = ct$ and for the reluctance synchronous motors $k_x \neq 1$.

When the iron losses are considered it is imposed to introduce in the branch of the current I_0 a supplementary equivalent resistance, R_m, determined by these losses magnitude.

The following conclusions can be emphasized from the analysis of the curves presented before:

- the synchronous motor excitation degree has an important influence on the components of the taken current and a lower influence on the resultant current;
- the current component corresponding to the reluctance torque modifies in large limits when the rotor magnetic asymmetry degree is modified;
- the minimum value of the magnetization current depends both on the excitation degree and on the rotor magnetic asymmetry.

5. REFERENCES

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