ROTOR PERMEANCES OF SOME TYPE OF RELUCTANCE MOTORS

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Abstract: **– One type of reluctance motors, with ferromagnetic insertion in rotor looks as in fig. 1a, [1]. In the paper, using appropriate conformal mappings, approximate formulas for two domains are derived and verified with finite element method.**

Keywords: *Reluctance motor, permeance, conformal mapping.*

1. INTERNAL RELUCTANCE OF 4-POLES ROTOR

We will approximate the left border AB of ferromagnetic insertion with a hyperbolic arc (fig.2). Let consider the domain bordered by the semi-axis OC and the hyperbola AB (fig. 2a),

$$x^2 - y^2 = a^2 \qquad (1)$$

The analytical function

$$w = z^2 ; \quad z = r\, e^{i\frac{\pi}{4}} \Rightarrow w = i\, r^2$$
$$z_{\mathrm{B}} = r\, e^{i\theta} \quad \Rightarrow \quad w_{\mathrm{B}} = r^2\, e^{2i\theta} \qquad (2)$$

maps the shaded domain from fig. 2a into the corresponding domain from fig. 2 b, [3].

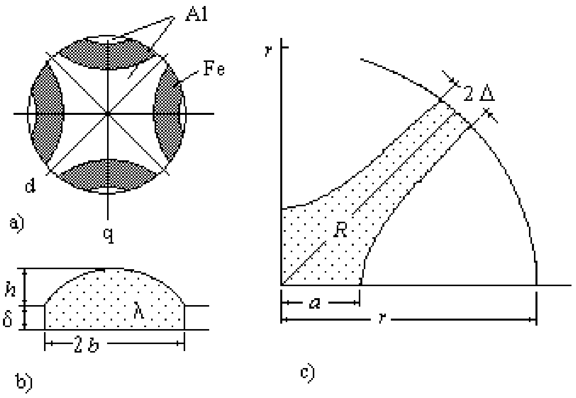


Fig. 1 Rotor cross-section

The coordinates of points B and B' (fig. 2) will be:

$$\begin{cases} x^2 + y^2 = r^2 \\ x^2 - y^2 = a^2 \end{cases} \Rightarrow x_0 = \sqrt{\frac{r^2 + a^2}{2}} \; ; \; y_0 = \sqrt{\frac{r^2 - a^2}{2}} \qquad (3)$$

$$v_{\mathrm{B}'} = r^2 \sin(2\theta); \quad \tan(\theta) = \sqrt{\frac{r^2 - a^2}{r^2 + a^2}}$$

The shaded domain in w-plane (fig. 2) can be considered a rectangle with the medium height $r^2\,(1 + \sin 2\theta)/2$.

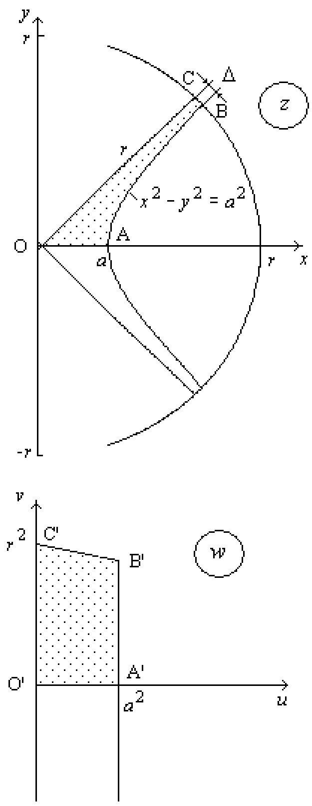


Fig. 2 Domain of internal reluctance

For enough large *r/a*, the magnetic field lines are almost straight and the geometric reluctance of shaded domain in *z*-plane (for 1 m length) can be approximated with:

$$R_1 \cong \left(\frac{a}{r}\right)^2 \frac{2}{1 + \sin(2\theta)} \qquad (4)$$

This reluctance depends essentially on the small gap BC = Δ. This is why we are looking for an expression in terms of Δ/*r*, where *r* is the rotor radius.

The length of the segment BC will be:

$$\Delta^2 = \left(\frac{r}{\sqrt{2}} - x_0\right)^2 + \left(\frac{r}{\sqrt{2}} - y_0\right)^2 \Rightarrow$$

$$\sqrt{1+\frac{a^2}{r^2}} + \sqrt{1-\frac{a^2}{r^2}} = 2 - \frac{\Delta^2}{r^2} \qquad (5)$$

Solving the equation (5) for a, we obtain:

$$\frac{a}{r} = \sqrt[4]{4\left(\frac{\Delta}{r}\right)^2 - 5\left(\frac{\Delta}{r}\right)^4 + 2\left(\frac{\Delta}{r}\right)^6 - \frac{1}{4}\left(\frac{\Delta}{r}\right)^8} \qquad (6)$$

$$\frac{\Delta}{r} < 2\sin\frac{\pi}{8} \cong 0.765$$

For $4 < \Delta / r < 0.765$, the hyperbolic arc is almost coincident with a circular one, with the radius a.

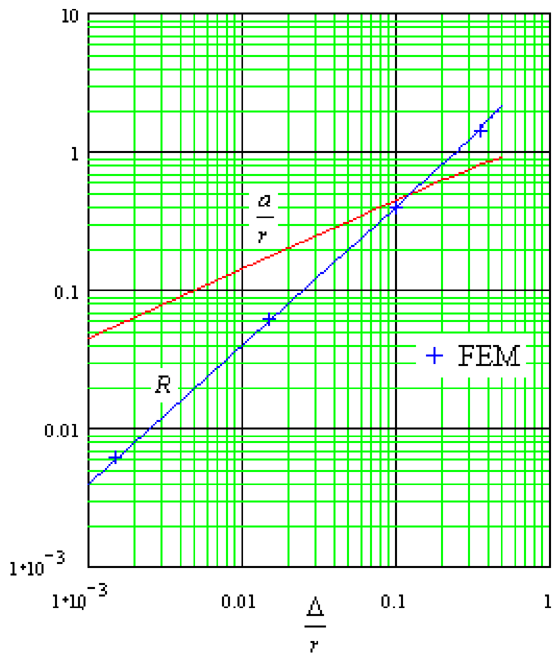


Fig. 3 Internal geometric reluctance (7)

Applying a well-known double-angle trigonometric relation to equation (4), we obtain the formula for internal transversal (geometric) reluctance of 1m length 4-poles rotor:

$$R \cong \frac{4\left(\frac{a}{r}\right)^2}{1+\sqrt{1-\left(\frac{a}{r}\right)^4}} \approx \frac{2\left(\frac{a}{r}\right)^2}{1-\frac{1}{4}\left(\frac{a}{r}\right)^4} \qquad (7)$$

The values of internal reluctance and of a/r are given in fig. 3 as functions of Δ/r. An almost linear dependence in logarithmic scale can be observed.
For $\Delta/r < 0.1$, $R \approx 4\,\Delta / r$.

2. RELUCTANCE OF CIRCULAR BORDER, LUNATE CROSS-SECTION, GROOVE

We will consider the rotor surface plane and the domain of interest as in fig. 4a. This domain can be transformed in horizontal tape using the analytical function [3]

$$w = \ln\frac{z-c}{z+c}; \quad z = -c\coth\frac{w}{2} \qquad (8)$$
$$z = x + i\,y; \quad w = u + i\,v$$

Considering the equivalent (taking into consideration the stator dentation) air gap δ we have:

$$c = \rho\sin\alpha; \quad H = h + \delta = \rho(1-\cos\alpha) = c\tan\psi$$

$$\beta = \arcsin\frac{2\,b\,h}{b^2 + h^2} \qquad (9)$$

Applying the height theorem in the right triangles

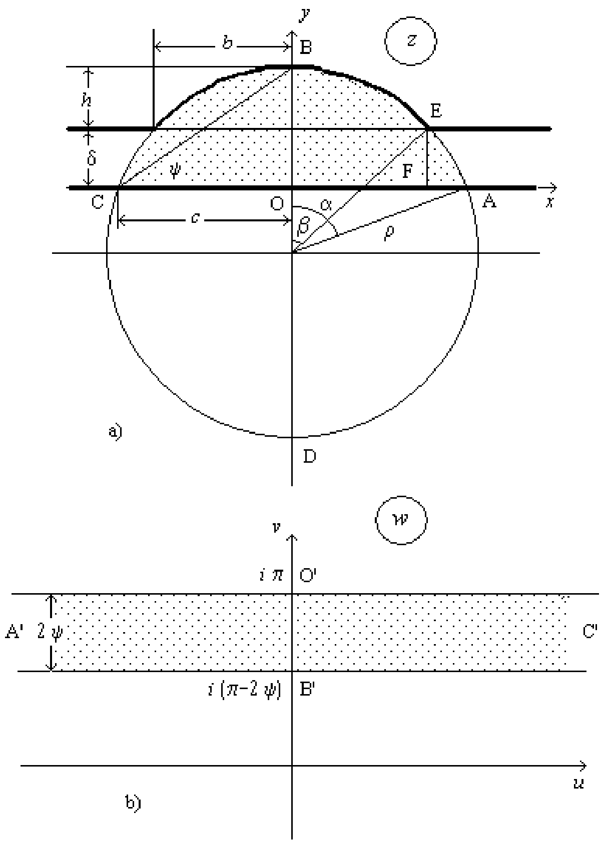


Fig. 4 Domain for “lunate” permeance

ABD and EBD, we can write:

$$\begin{cases} b^2 = h\,(2\,\rho - h) \\ c^2 = (h+\delta)(2\,\rho - h - \delta) \end{cases} \qquad (10)$$

The radius of the circle in fig. 4 will be:

$$\rho = \frac{b^2 + h^2}{2h} = \frac{c^2 + (h+\delta)^2}{2(h+\delta)} \qquad (10')$$

From last equation it results:

$$\frac{b^2}{h} - \frac{c^2}{h+\delta} = \delta \qquad (11)$$

The expression for c as function of groove dimensions is:

$$c = h\sqrt{\left(\frac{b^2}{h^2} - \frac{\delta}{h}\right)\left(1 + \frac{\delta}{h}\right)} \qquad (12)$$

The reluctance between OF and arc BE can be considered approximately equal to the reluctance of the flux path bordered by the flux lines OB and the flux line passing through the middle of the segment EF (fig. 4). The middle of this segment has the following coordinates, depending on the angle ψ or on the relative dimensions of the groove cross-section and of the gap:

$$z_m = \begin{cases} b + i\delta/2 & \text{for } h+\delta < \rho \\ c + i\delta/2 & \text{for } \rho < h+\delta < 2\rho \end{cases} \qquad (13)$$

In w-plane, the abscissa of the corresponding point will be:

$$u = \mathrm{Re}\left(\ln\frac{z_m - c}{z_m + c}\right) = \begin{cases} \ln\sqrt{\dfrac{4(c-b)^2 + \delta^2}{4(c+b)^2 + \delta^2}}, & \psi < 45° \\ -\ln\sqrt{16\left(\dfrac{c}{\delta}\right)^2 + 1}, & 45° < \psi < 90° \end{cases}$$

The geometric permeance of a half of groove will be:

$$\frac{\lambda}{2} = \frac{-u}{2\psi} \qquad (15)$$

Taking into account (9), the geometric permeance of an 1m length groove can be evaluate with the formulas:

$$\lambda = \frac{\ln\sqrt{\dfrac{4(c+b)^2 + \delta^2}{4(c-b)^2 + \delta^2}}}{\arctan\dfrac{h+\delta}{c}} \quad \text{for } \delta h < \frac{b^2 - h^2}{2} \qquad (16)$$

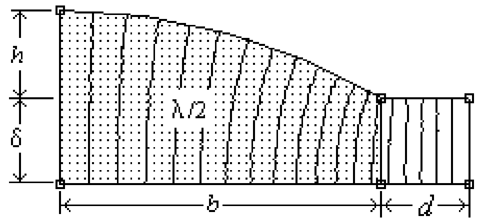
$$\lambda = \frac{\ln\sqrt{1 + 16\left(\dfrac{c}{\delta}\right)^2}}{\arctan\dfrac{h+\delta}{c}} \quad \text{for } \frac{b^2 - h^2}{2} < \delta h < b^2 \qquad (17)$$

where c is given by (12).

For very small gaps, the following simplified formula could be used:

$$\lambda_1 = \frac{\ln\dfrac{b+c}{b-c}}{\arctan\dfrac{h+\delta}{c}} \qquad (18)$$

For $b/h < 0.6$, the lunate permeance could be completely neglected and more accurate results could be obtained probably using the formulas for constriction permeances [2], [4].



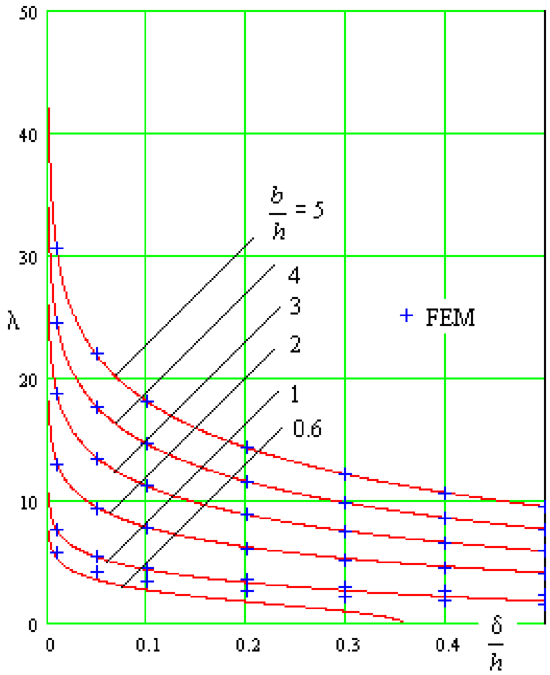


Fig. 5 " Lunate" geometric permeance (16)

The values of the permeance λ of the groove given by (15) as function of the gap, base width over groove depth - parameter, are shown in fig. 5.

The permeance of the gap per pole is two times the permeance of whole represented in fig. 5 domain and can be calculated as:

$$\lambda_\delta = \lambda + \frac{2d}{\delta} \qquad (19)$$

Neglecting the saturation the l_i length rotor transversal reluctance will be:

$$R_q \approx \frac{1}{l_i}\left(\frac{2}{\lambda_\delta} + \frac{R}{2}\right) \qquad (20)$$

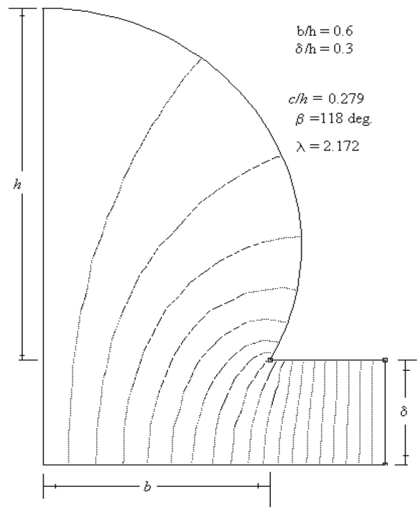


Fig. 6a Flux lines for *b/h* < 1 groove

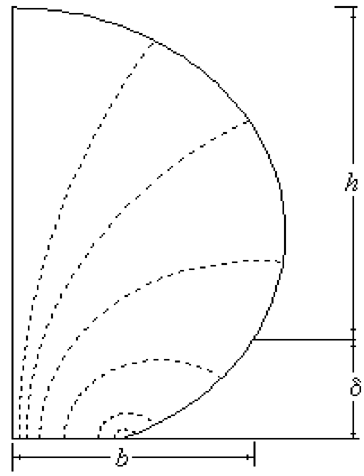


Fig. 6b Flux lines in the model for *b/h* < 1

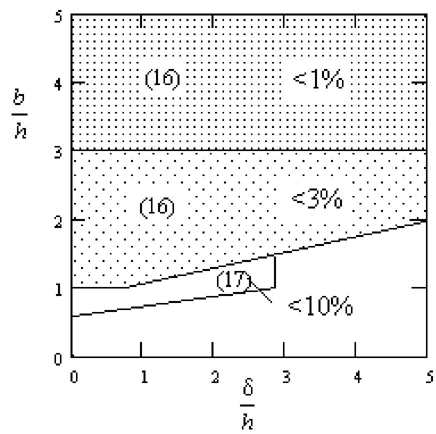


Fig. 7 Domain of application of (16) and (17) and error levels

3. FORMULAS VERIFICATION

To verify formulas (7), (16) and (17) for internal and lunate reluctance the FEMM program was used.

In the case of internal reluctance cross-section the arc BC (fig. 2) corresponds to the angle arcsin(Δ/r).

The hyperbolic arc AB was modeled with one circular arc with radius a and 6 segments (fig. 8). The first 3 segments had the length 1/12 and the next 3 - ¼ from the total length. The arc angle α was determined from the condition of $\varepsilon = r/1000$ largest deviation from hyperbola (largest distance between the circular and hyperbolic lines).

The equations are:

$$\begin{aligned}(2a-x)^2+y^2&=(a+\varepsilon)^2\\ x^2-y^2&=a^2\end{aligned} \tag{21}$$

It results for the angle α:

$$\alpha = \arccos\left[1-\sqrt{\frac{\varepsilon}{a}+\frac{1}{2}\left(\frac{\varepsilon}{a}\right)^2}\right] \approx \arccos\left[1-\sqrt{\frac{\varepsilon}{a}}\right] \tag{22}$$

In fig. 9 are given the values of the angle α versus ε/a.

The FEM calculated values are shown in fig. 3 with (+), the relative deviations being smaller than 3 %.

The formulas (16) and (17) was verified using the Bela FEM package. The results for λ are shown in fig. 5.

Formula (16) gives errors smaller than 3% for b/h and h/δ larger than 2. For $b/h > 3$ and $h/\delta > 2$ the relative errors are even smaller than 1% (fig. 7).

Formula (17) gives too large errors, because of quite different field pattern and shape of magnetic flux lines in right bottom part of cross-section, in the vicinity of the gap (fig. 6), combined with low value of the permeance, and cannot be recommended.

4. CONCLUSIONS

The obtained formulas (7), (12), (15), (16), as well as fig 3 and 5, can be used for more accurate reluctance motor parameters evaluation.

References

[1] Ф. М. Юферов, Электрические машины автоматических устройств, Высшая школа, М., 1976

[2] H. C. Roters, *Electromagnetic devices*, John Wiley & Sons, Inc. New York, 1967.

[3] K. J. Binns, P. J. Lawrenson, *Analysis and computation of electric and magnetic field problems*, Pergamon Press, Oxford, 1963

[4] G. A. Cividjian, A. G. Cividjian, Constriction and Corner Permeances for Finite Domains, *IEEE Transaction on Magnetics,* vol. 42, Issue 12, (Dec. 2006), pp. 3825-3831