THE DYNAMIC RELATIONS INSIDE THE STRESS-STRENGTH VECTOR A MODELS FOR THE RELIABILITY PREDICTION

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Abstract **– The paper focuses on the evolution of the technical systems reliability using the inside relation the stress-strength vector. A complex system state is defined by its state vector or so called fingertip. A new approach in technical systems reliability evaluation means to include in the state vector not only the stress and strength components parameters, but also the consequences as well as defining their relationship. The stress – strength consequence relationship inside the state vector, is different to relation the stress and strength numerical values. In this paper, these approaches are sustained by logical reasoning and by the examples of the reliability prediction orientated in the proposed sense.**

Keywords**:** *stress and strength vectors, state vector, reliability, lifetime.*

1. INTRODUCTION

The study of the reliability is concerned with the calculation and prediction of the reliability for technical systems at any stage during their life.

Electric power systems are examples of systems where a very high degree of reliability is expected. Reliability indices are therefore an important aspect of system design and operation. It is widely recognized now that the proper measures of system reliability can only be described in the context of probability. Many simple power system problems can be analyzed with the help of elementary probability theory.

From a reliability point of view, a system is characterized by multiple modes of failure, and the analyses of various failure modes is a principal technique currently applied in power system studies. Each of the potential failure modes can be analyzed in terms of component behavior, which, in reliability terms, can be connected in series or parallel or combination.

2. THE CONCEPT OF STATE VECTOR

The reliability, as the probability that an element or a system accomplishes its functions for an established given time, with environment and confidence level is given by the conditions vector and the time.

The conditions vector is mainly based on the stress ($\overline{S}$) and strength ($\overline{C}$) vectors relationship. In the case of the real technical systems, the situation of a single stress vector $\overline{S}$ is corresponding to a strength vector $\overline{C}$. They are representing, for a stated moment, the so-called state vector $\overline{V}$ or the fingertip.

The probability of occurrence of an event, as failure, is a numerical measure of the chance of $\overline{S} > \overline{C}$. This measure may be obtained from measurement of the long-term frequency of the event for generally similar system, or may be simply a subjective estimation of numerical value.

Knowing the state vector components values to a certain moment we can estimate the reliability *R:*

$$R = f(\overline{S}, \overline{C}) = f(\overline{V}) \tag{1}$$

If the instantaneous probability density functions f_C and f_S of the strength and stress respectively are know, the instantaneous reliability can be obtained from:

$$R = \int_{-\infty}^{\infty} [1 - F_{Ci}(S)] f_{Si}(S) dS \tag{2}$$

where $i = 1, 2, 3, ...,n$ is the number of $\overline{S}$ and $\overline{C}$ vector components [1].

Knowing the evolution in time of $\overline{V}$ components we can estimate the *R(t)*.

$$R(t) = P[C(t) > S(t)] = P[f(\overline{V}(t))]$$

Schematically the changes inside of $\overline{S}$, $\overline{C}$ and $\overline{V}$ vectors in time can be described as in Figure 1.

So, the reliability prediction results from the following equation:

$$R(t) = 1 - \int f_{V(t)}[V(t)] dV(t) \tag{4}$$

where f_V is the joint probability density for the two vectors $\overline{V} = [\overline{S}, \overline{C}]$.

For example we will suppose that the stress and strength are normally distributed.

$$f_S(S) = \frac{1}{\sigma_S \sqrt{2\pi}} \exp\left[-\frac{1}{2} \left(\frac{S - \mu_S}{\sigma_S} \right)^2 \right]$$

and (5)

$$f_C(C)=\frac{1}{\sigma_C\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{C-\mu_C}{\sigma_C}\right)^2\right]$$

where:

$\mu_S,\ \mu_C$ = mean of stress and strength distribution;

$\sigma_S,\ \sigma_C$ = standard deviation of stress and strength distribution;

The reliability is defined by equation (4) where:

$$f_V(V)=f_{V(SC)}(S,C,\rho)=$$

$$=\frac{1}{2\pi\sigma_S\sigma_C\sqrt{1-\rho^2}}\exp\left[\frac{-\frac{1}{2}\left(h^2+k^2-2\rho hk\right)}{1-\rho^2}\right] \quad (6)$$

with $h=\frac{s-\mu_S}{\sigma_S}$, $k=\frac{c-\mu_C}{\sigma_C}$ and, very important, the state vector components relationship, that cannot be neglected $\rho(S,C)=\frac{\text{cov}[S,C]}{\sigma_S\sigma_C}$.

3. THE STATE VECTOR ($\overline{V}$) COMPONENTS RELATIONSHIP

The state vector ($\overline{V}$) components can be included in two categories: $\overline{S}$ and $\overline{C}$. In general, $\overline{C}$ is a function of material proprieties and element or structure dimensions, while $\overline{S}$ is a function of applied loads, material densities and perhaps dimension of the structure, each of which may be a random variable. Also, $\overline{C}$ and $\overline{S}$ may not be independent, so the relationship from that can be classified also from different point of views:

A1) Inside vector $\overline{S}$ relationship

A2) Inside vector $\overline{C}$ relationship

A3) Between $\overline{S}$ and $\overline{C}$ relationship

The A1 type relationship is depending on the physical stress governing laws. For example:

- the corrosion stress given by some chemical substances or salts, strongly influenced by water presence;
- the metal oxidation stress influenced by environmental temperature;
- climatic dependent stress factors; for example, non simultaneous maximum wind and existing frost, etc.

As concerns the A3 type, it is well known the proportional relationship between stress and strength but the stress effects are the intermediary factors governing it.

The saturation phenomenon, illustrating the materials strength and establishing the stress – effects domain boundary, is of a great importance. It splits this domain in the following three zones:

- *elastic zone*, specific for the lower stress defined by the non remaining effects after stress;
- *plastic zone*, defined by the remaining after-stress effects or even irreversible effects which can be taken as elements for an objective evaluation of the $\overline{S}$ and $\overline{C}$ components dependence before the system collapse;
- *failure (or collapse) zone,* which obviously means the system looses its operational function; the zone corresponds to a higher stress.

An elementary moment that involves both variable, S and C, is the covariance:

$$\text{cov}[S,C]=M\{[S-M(S)][C-M(C)]\}$$

$$\text{cov}[S,C]=M(SC)-M(S)M(C) \quad (7)$$

The correlation coefficient

$$\rho(S,C)=\frac{\text{cov}[S,C]}{\sigma_S\sigma_C} \quad (8)$$

is a measure of linear dependence between two random variable.

The reliability definition includes the time factor importance. The essence is that the strength vector components are not increasing in lifetime or in time intervals between maintenance operations. The stress laws are given time depending shapes. One of the stress factors can directly influence the strength vector components like the temperature slope which is the source of mechanical efforts leading, for example, to a stress pushing the system component from elastic to plastic zone.

The evolution in time of the stress, strength and state vector components is presented in Figure 1.

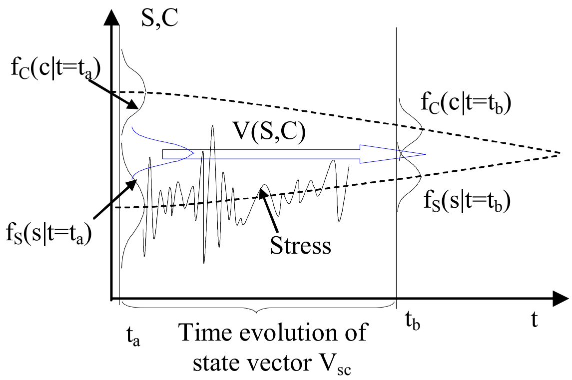

Figure 1: The evolution of the stress-strength components.

The stress and strength process vector components have already known shapes: stationary process, quasi-stationary process (linear), quasi -stationary process (linear fan) and quasi -stationary process (nonlinear).

The time factor is affecting the vectors dependence in such manner that the system wearing is proportionally accelerated by the stress.

4. ESTIMATING AND PREDICTION METHODS

As it was previously mentioned the stress vector is the source, through its effects, of the system state parameters. Monitoring the state values is not so important if we are not considering as well as their corresponding stress and the stress-strength relationship.

Monitoring the stress and state parameters relationship is consequently more efficient and objective method. This relationship dynamic involves specific and accurate estimation and evaluation methods.

In the following, we will present an example, where the stress and strength are normally distributed S=*N(35,3)* and C= *N(40,5)*.

The following figure shows the relationship between the S and C variables, and the correlation coefficient $\rho=0.2296$.

The histograms shows, that the stress probability lies between 28 and 42, and the strength probability lies between 27 and 58, so we will estimate the failure probability looking to the interference area between the variables and taking into account the correlation coefficient.

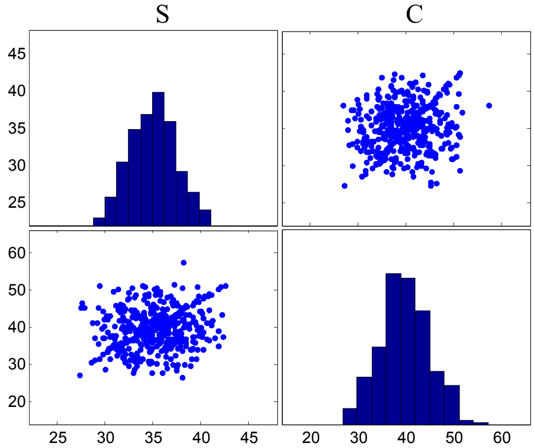

Figure 2: The relationship between S and C variables.

Methods to solve the equation (4) are essentially of three types [2]; last two methods will be used in the following:

- multidimensional integration of the original problem, using the transformation of the original variable;
- numerical integration;
- the Monte Carlo simulations.

The multidimensional integration is used when the analytical expression of reliability is necessary to know, and will be used in other applications.

Direct numerical integration of multiple integrals (4) is not usually practical if integrations number is great to 5. Rather than carry out numeric integration, in some applications it may be sufficient to bound the probability content. The failure probability is limited by a lower and an upper border, as is presented in following equation:

$$\max[\Phi(-a)\cdot\Phi(-k),\Phi(-b)\cdot\Phi(-h)]\le P_d \quad \text{and} \quad P_d \le \Phi(-a)\cdot\Phi(-k)+\Phi(-b)\cdot\Phi(-h) \tag{9}$$

where: $a=\dfrac{h-\rho k}{\left(1-\rho^2\right)^{1/2}}$ and $b=\dfrac{k-\rho h}{\left(1-\rho^2\right)^{1/2}}$.

The stress and strength variables are correlated with the coefficient $\rho=0.2296$, and the values mentioned are h=1.6667, k=1, a=1.4765, b=0.6343. Using these values, we have calculated the lower and upper borders for failure probability, as follows: LimLow = 0.0126, respectively, LimUp = 0.0237. So, the failure probability lies in [0.0126; 0.0237] interval.

In the following figure we presented the joint variable restricted in the failure domain.

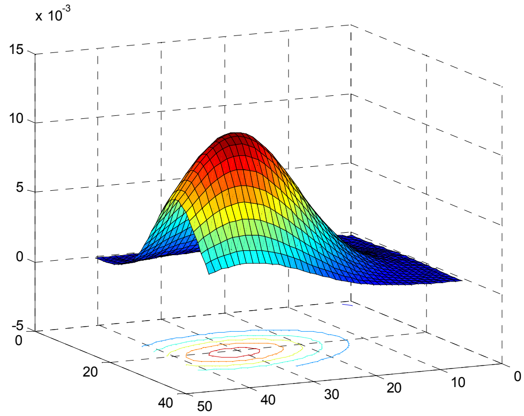

Figure 3: The joint (bivariate) density function.

The Monte Carlo techniques involve sampling at random to simulate artificially a large number of experiments and to observe the result. In case of analysis for structural reliability, this means, in the simplest approach, sampling each random variable V randomly to give a sample value v_i. If the limit stat function is violated, the element or system has failed. To apply the Monte Carlo techniques to practical problems it is necessary, because this technique take into account the relationship between variables, without a numerical calculation of correlation coefficient.

For our variables, using 1000 trials, the probability of failure aim to 0.019, its evolution depending by the trials number, is described in figure 4.
In principle, Monte Carlo methods are only worth exploiting when the number of trials or simulations is less than the number of integration points required in numerical integration.

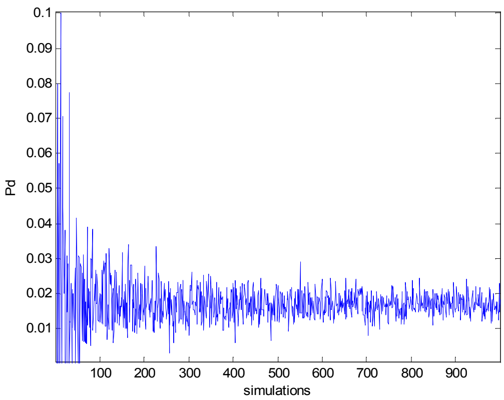

Figure 4: The Monte Carlo simulation result.

5. CONCLUSIONS

The actual tendency in the prediction of the reliability domain and of safety in the systems functioning (with a great functional responsibility) is based on the monitories of the state vectors components.
The present paper maintain the necessity of the transition from the monitories of the tendency of the absolute values of the components to the monitories of the relationship between the components and, especially, that means, the relations between the stress components and the components which represent the effects of the stress.
These probabilistic concepts can be used in the technical system design, by calculating a security coefficient taking into account the dependence and the influence of the stress to the strength components.

References

[1] Ivas D., Munteanu F, Rotariu M., Voinea E., Nemeş C. *Ingineria fiabilității sistemelor complexe*. Ed. Perfect 2001 ISBN 973-85069-3;

[2] C Nemeş „Modelarea solicitărilor componentelor sistemelor electroenergetice în calculele de fiabilitate”. Teză de doctorat. Universitatea Tehnică „Gh. Asachi” Iaşi, aprilie 2005.

[3] *Sang-Chin Yang, Joel,Nachlas Bivariate Reliability and Availability Modeling.* IEEE Transactions on reliability, vol 50, no 1 march 2001.