# ON – LINE PARAMETRIC IDENTIFICATION AND DISCRETE OPTIMAL COMMAND OF FLYGHT OBJECTS

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***Abstract*** **– This paper presents a new ON-LINE parametric identification and discrete optimal command algorithm for mono or multivariable linear systems. The method may be applied with good result to the automatic command of the flying object movement.**
**The simulation results obtained with this real time algorithm, with parametric identification of an air-air rocket's movement in vertical plain regarding to target's line are presented.**

***Keywords***: *parametric identification, algorithm, rocket*

## 1. INTRODUCTION

Because of fast flying parameters' modify for the modern aircraft and rockets, performing real time identification and optimal or adaptive command algorithms have to be made. The authors of this paper have made such an algorithm.

First of all, an off-line parametric identification is made, without command, for obtaining the initial values of these parameters for the on-line identification process.

Using the leading system (A) and model's outputs, a discrete optimal command law is projected, using a quality quadratic criterion, which assures the convergence of the difference between leading system and model's outputs. The model's parameters, obtained by the ON-LINE identification, are used for he command law calculus.

For the algorithm validation one uses as example the automatic command of the A 's movement in vertical plain; time characteristics, representing evolution of state variables of A and their estimate, are plotted. These variables' stabilization and the convergence of the errors $e_i = x_i - \hat{x}_i$ happen in maximum 2 seconds.

The proposed algorithm produces very good results in the case of longitudinal and lateral movement's stabilization for transport and fights aircrafts

## 2. CONTINOUS AND DISCRETE MODELS FOR

The leading system (the movement of A ) may be described by the input – output equations with general forms

$$\dot{x} = Ax + Bu, \tag{1}$$

$$y = cx, \tag{2}$$

where $x$ is the state vector $(n \times 1)$, $u-$ the command vector $(m \times 1)$, $A-$ the system matrix $(n \times n)$, $B-$ matrix $(m \times m)$, $y-$ output vector $(p \times 1)$, $C-$ measurement system matrix $(p \times n), p \leq n$. The estimated model is described by equations

$$\dot{\hat{x}} = \hat{A}\hat{x} + \hat{B}u, \tag{3}$$

$$\hat{y} = \hat{C}\hat{x}, \tag{4}$$

where $\hat{x}$ is the state $x$'s estimation, $\hat{y}$ - output $y$'s estimation, $\hat{A}, \hat{B}$ and $\hat{C} = C-$ estimate matrices.

The discrete variants of equations systems (1), (2), and (3), (4) are, respectively

$$x(k+1) = A_d x(k) + B_d u(k), \tag{5}$$

$$y(k) = C_d x(k); \tag{6}$$

$$\hat{x}(k+1) = \hat{A}_d \hat{x}(k) + \hat{B}_d u(k), \tag{7}$$

$$\hat{y}(k) = \hat{C}_d \hat{x}(k); \tag{8}$$

matrices $A_d, B_d, C_d$ and $\hat{A}_d, \hat{B}_d, \hat{C}_d$ are discrete variants of matrices $A, B, C$ and $\hat{A}, \hat{B}, \hat{C}$.

Another description form for the estimated system A ( A dynamics estimation) [1] is

$$\hat{y}(k+1) = \hat{x}^T(k+1)\hat{b}(k) + \hat{e}(k+1) \tag{9}$$

or

$$\hat{y}(k+1) = \hat{b}^T(k)\hat{z}(k+1) + \hat{e}(k+1), \tag{10}$$

where $e(k+1) = y(k+1) - \hat{y}(k+1)$,

$$\hat{b}^T(k) = \begin{bmatrix} \hat{\alpha}^T(k) & \hat{b}_1(k) & \hat{\beta}^T(k) \end{bmatrix}, \tag{11}$$

with

$$\begin{aligned} \hat{\alpha}^T(k) &= \begin{bmatrix} -\hat{a}_1(k) & -\hat{a}_2(k) & \ldots & -\hat{a}_n(k) \end{bmatrix}, \\ \hat{\beta}^T(k) &= \begin{bmatrix} \hat{b}_1(k) & \hat{b}_2(k) & \ldots & \hat{b}_m(k) \end{bmatrix}, \end{aligned} \tag{12}$$

$\hat{\alpha}(np \times p), \hat{b}_1(p \times m), \hat{\beta}[(m-1)p \times m]$;

$$z^T(k+1) = \begin{bmatrix} \hat{Y}^T(k) & u(k) & U^T(k) \end{bmatrix}, \tag{13}$$

with

$$\begin{aligned} \hat{Y}^T(k) &= \begin{bmatrix} \hat{y}(k) & \hat{y}(k-1) & \ldots & \hat{y}(k-n+1) \end{bmatrix}, \\ \hat{U}^T(k) &= \begin{bmatrix} u(k-1) & u(k-2) & \ldots & u(k-m+1) \end{bmatrix}; \end{aligned} \tag{14}$$

$\hat{Y}(np \times 1), U[(m-1)m \times 1]$.

If $m = p$, then equation (10) becomes

$$\hat{y}(k+1) = \hat{\alpha}^T(k)\hat{Y}(k) + \hat{b}_1(k)u(k) + \hat{\beta}^T(k)U(k); \tag{15}$$

if $m \neq p$, then $\hat{\beta}^T(k)$ matrix cannot be multiplied wit $U(k)$ vector because of their dimensions. That's why, in equation (15) the last term is expressed for each concrete case (function of $m$ and $p$ values). So that, in the case presented below (rocket's movement in vertical plain) $n = 4, m = 1$ and equation (15) becomes

$$\hat{y}(k+1) = \hat{\alpha}^T(k)\hat{Y}(k) + \hat{b}_1(k)u(k), \qquad (16)$$

where

$$\begin{aligned} \hat{Y}^T(k) &= [\hat{y}(k) \quad \hat{y}(k-1) \quad \hat{y}(k-2) \quad \hat{y}(k-3)], \\ \hat{\alpha}^T(k) &= [-\hat{a}_1(k) \quad -\hat{a}_2(k) \quad -\hat{a}_3(k) \quad -\hat{a}_4(k)], \end{aligned} \qquad (17)$$

$\hat{b}_1$ is a $(p \times 1)$ vector, $\hat{y}$ is a $(p \times 1)$ vector and $u(k) - (1 \times 1)$.

For command law $(u(k))$ obtaining, one chooses the performance indicator

$$J = [\bar{y}(k+1) - \hat{y}(k+1)]^T Q[\bar{y}(k+1) - \hat{y}(k+1)] + u^T(k)Ru(k), \quad (18)$$

where $\bar{y}(k+1)$ is the imposed output vector while $Q(p \times p)$ and $R(m \times m)$ are symmetric and positive definite matrices, $R-$ nonsingular matrix; $y(k+1)$ has the form (17). The optimal command is obtained from optimum condition $\left(\frac{\partial J}{\partial u(k)} = 0\right)$,

$$u(k) = G\left[\bar{y}(k+1) - \hat{\alpha}^T(k)\hat{Y}(k)\right], \qquad (19)$$

for $m = 1; G$ has the expression

$$G = \left[R + \hat{b}_1^T(k)Q\hat{b}_1(k)\right]^{-1} \hat{b}_1^T(k)Q. \qquad (20)$$

$Q$ and $R$ matrices may be calculated using ALGLX algorithm proposed by the authors of this paper or other algorithms [3], [4], [5].

## 3. A 'S PARAMETERS IDENTIFICATION

First of all the off – line system A 's parameters identification is made, using, for example, the least square method (LSM), resulting the parameters vector $\hat{b}_0 = \hat{b}(0)$; in this moment the system's command is uncoupled, therefore the system is an open loop one. $\hat{y}(t)$ is then computed and the vectors $\hat{Y}_0 = \hat{Y}(0)$ and $U_0 = U(0)$ are memorized. Also, the covariance matrix $P_0$ is memorized at the end of identification $(P_0 = P(0))$. Then, matrices $A_d, B_d, \hat{A}_d, \hat{B}_d$ are computed and with these state vectors $x$ and $\hat{x}$ are computed; these vectors (at the end of identification) are memorized.

For simulation of time varying of A 's parameters, the parameters of A are modified (for example with 5%) and with the new coefficients $A_d$ and $B_d$ matrices are computed.

The loop is then closed (one adds $u(k)$ command) and $Q'$ and $R$ matrices are computed with ALGLX algorithm in rapport with $\hat{A}_d$ and $\hat{B}_d$ matrices and after that the matrix $Q = \left(C^T\right)^+ Q' C^+$ is calculated.

$G$ matrix from (20) is obtained with $\hat{b}_1$ extracted from $\hat{b}(k)$. Then command $u(k)$ is computed with (19).

The vectors $x(k+1)$, $\hat{x}(k+1)$, $y(k+1)$ and $\hat{y}(k+1)$ are calculated; vectors $\hat{Y}(k+1)$ and $U(k+1)$ are memorized and the error $\hat{e}(k+1) = y(k+1) - \hat{y}(k+1)$ is computed.

The actualization of covariance matrix is made with formula [1]

$$P(k+1) = P(k)\left[I_{m+n} - \frac{\hat{z}(k+1)\hat{z}^T(k+1)}{\lambda + \hat{z}^T(k+1)P(k)\hat{z}(k+1)}P(k)\right] (21)$$

and, with this,

$$\hat{b}(k+1) = \hat{b}(k) + P(k+1)\hat{z}(k+1)\hat{e}(k+1), \qquad (22)$$

where $\hat{z}(k+1)$ has the form (13).

State variables $x_i(t)$ and $\hat{x}_i(t)$ are plotted.

## 4. IDENTIFICATION AND OPTIMAL COMMAND OF THE ROCKET'S MOVEMENT

For the identification and discrete optimal command algorithm's validation, present above, a simulation program was made in the MATLAB medium.

Considering model of A 's movement in rapport with equal signal line [6], [7], with state vector $x^T = -[y \quad \dot{y} \quad \Delta\alpha \quad \dot{\theta}], u = \delta; y$ and $\dot{y}$ are lateral deviation and lateral deviation angular velocity, respectively, $\Delta\alpha$ - incidence angle variation, $\dot{\theta}$ - pitch angular velocity .

With flying parameters' values from [8], for the 40th second of flight, following step by step the algorithm, one obtained successively the results

$$b_0^T = [4.02 \ \ \text{-}6.06 \ \ 4.05 \ \ \text{-}1.01 \ \ \text{-}10^{-3} \cdot 0.04 \ \ \text{-}10^{-3} \cdot 0.46 \ \ \text{-}10^{-3} \cdot 0.46 \ \ \text{-}10^{-3} \cdot 0.04]$$

$$\hat{b}_0^T = [3.84 \ \ \text{-}5.50 \ \ 3.46 \ \ \text{-}0.81 \ \ \text{-}0.006 \ \ 0.06 \ \ 0.13 \ \ \text{-}0.05]$$

$$A_d = \begin{bmatrix} 4.023 & 1 & 0 & 0 \\ -6.062 & 0 & 1 & 0 \\ 4.053 & 0 & 0 & 1 \\ -1.015 & 0 & 0 & 0 \end{bmatrix}; \hat{A}_d = \begin{bmatrix} 3.845 & 1 & 0 & 0 \\ -5.503 & 0 & 1 & 0 \\ 3.469 & 0 & 0 & 1 \\ -0.811 & 0 & 0 & 0 \end{bmatrix};$$

$$B_d = 10^{-3} \cdot [-0.041 \quad -0.461 \quad -0.462 \quad -0.042]^T;$$
$$C_d = [1 \quad 0 \quad 0 \quad 0]; D_d = [0]$$
$$\hat{B}_d = [-0.0064 \quad 0.0612 \quad 0.1361 \quad -0.0564]^T;$$
$$\hat{C}_d = [1 \quad 0 \quad 0 \quad 0]; \hat{D}_d = [0]$$

and $b$ vector of the system with optimal command

$$b^T = [3.82 \ \ \text{-}5.75 \ \ 3.85 \ \ \text{-}0.96 \ \ \text{-}3.97 \cdot 10^{-5} \ \ \text{-}0.43 \cdot 10^{-3} \ \ \text{-}0.43 \cdot 10^{-3} \ \ \text{-}4 \cdot 10^{-5}]$$

Also one obtained $Q = 0.1067, R = 1$.

In fig.1 state variables $x_i(t), \hat{x}_i(t)$ and $\delta(t)$ are presented ( $x_i(t)$ with blue and $\hat{x}_i(t)$ with red).

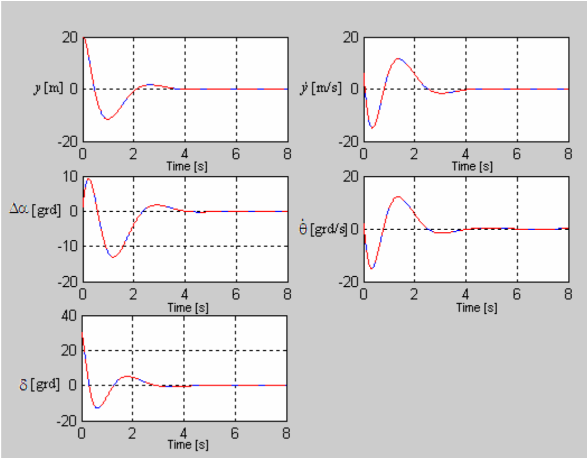


Fig.1 – Time varying of $x_i, \hat{x}_i$ and $\delta$

## 5. CONCLUSIONS

The paper presents an ON – LINE parametric identification and discrete optimal command algorithm for linear systems. For validation, it is used to automatic command of a rocket's movement in vertical plain with respect to equal signal line, which materializes target line. A simulation program based to presented algorithm was made in Matlab/Simulink. The obtained graphic plots express time evolution of the state variables $x_i(t), \hat{x}_i(t)$ and the evolution of command $\delta(t)$.

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