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OPTIMAL CONTROL OF THE ROCKET'S LATERAL DEVIATION IN RAPPORT WITH EQUAL SIGNAL LINE

Romulus LUNGU, Dipl. Eng., Ph.D., Mihai LUNGU, Dipl. Eng. Ph.D. student

Avionics Division, Faculty of Electrical Engineering, University of Craiova, Romania, romulus_lungu@yahoo.com, lma1312@yahoo.com

Abstract – The paper presents a liniarized model of rocket's vertical plain movement, in rapport with equal signal line, represented by target line direction, linear state estimator (observer) projection using a new reduced order algorithm and gain matrix of optimal command law after estimated state vector projection. The present paper's authors have obtained the observer projection algorithm and command law projection algorithm. The results of numeric simulation including state variables and estimated state variables' dynamics are presented.

Keywords: *rocket, observer, algorithm, command law.*

1. INTRODUCTION

The subject of this paper is the optimal control of lateral deviation of a rocket in rapport with equal signal line. A model for a rocket's movement is used and it expresses the rocket's movement around mass center, model that is liniarized around a reference trajectory (PD – A – T direction, PD is leading point, A – rocket, T – fixed or mobile target). State vector of the dynamic model is composed of lateral deviation in rapport with equal signal line, lateral deviation angular velocity, incidence angle and angular pitch velocity. The output is lateral deviation. Using a minimum number of transducers (velocity transducer and/or incidence angle or overload factor transducer) a linear reduced order state estimator (observer) is projected, which estimates state vector's components.

Optimal command law has to cancel the deviation and relative lateral speed of the rocket in rapport with target line, and to stabilize the incidence angle and pitch angular velocity.

A new algorithm for gain matrix of the optimal command law projection and a new algorithm for reduced order observer projection are used.

PC simulations validate theoretical results; calculus programs based on the two algorithms and state variables variation for a set of flying parameters (10^{th} flight second) are also presented.

2. ROCKET'S MOVEMENT MODEL

The rocket's movement in vertical plain is described by the following equations system [1]

$$T_V \dot{\vartheta} = \alpha - T_V \frac{g}{V} \cos\vartheta, \tag{1}$$

$$\dot{V} + \frac{c_x}{c_y^\alpha T_V} V = \left(1 + \frac{c_x}{c_y^\alpha}\right)\frac{F_T}{m} - g\sin\vartheta, \tag{2}$$

where ϑ is the trajectory slope ($\vartheta = \theta - \alpha$, θ is the pitch angle, α - the incidence angle, V - flying velocity, m - the rocket's mass, F_T - the thrust force, c_x and c_y - aerodynamic drag and lift coefficients $(c_y = c_y^\alpha \alpha)$, T_V - time constant of the flying object (rocket A)),

$$T_V = \frac{mV}{\rho \frac{V^2}{2} S c_y^\alpha + F_T}, \tag{3}$$

S - aerodynamic reference surface of A, ρ - air density.

Expressing equation (1) in α and θ variables, one obtains

$$T_V \dot{\theta} = T_V \dot{\alpha} + \alpha - T_V \frac{g}{V}\cos\vartheta \tag{4}$$

or in a liniarized form in rapport with reference $(\theta_0, \alpha_0, \vartheta_0)$

$$T_V \dot{\theta} = T_V \Delta\dot{\alpha} + \Delta\alpha - \tilde{\alpha}, \tilde{\alpha} = -\frac{g}{V} T_V \cos\vartheta_0, \tag{5}$$

$\dot{\theta} = \Delta\dot{\theta}, \tilde{\alpha}$ - perturbation.

A 's movement around mass center in vertical plain is described by equation [1]

$$J_z \ddot{\theta} + m_{\dot{\theta}} \dot{\theta} + m_\alpha \alpha = m_\delta \delta, \tag{6}$$

where J_z is the inertia moment of A in rapport with horizontal axis, $m_{\dot{\theta}}$ and m_α - dynamic damp and static stabilization moment coefficients, m_δ - command moment coefficient. With $\theta = \vartheta + \alpha$ and equation (1), equation (6) leads to linear equation

$$\Delta\ddot{\alpha} + 2\xi\omega_0 \Delta\dot{\alpha} + \omega_0^2 \Delta\alpha = k_\delta \delta + m_{\dot{\theta}} \frac{g}{J_z V}\cos\theta_0, \tag{7}$$

$\Delta\delta = \delta, \xi$ - damp coefficient and ω_0 - proper frequency of the twist movement in vertical plain;

$$2\xi\omega_0 = \frac{m_{\dot{\theta}}}{J_z} + \frac{1}{T_V}, \omega_0^2 \cong \frac{m_{\dot{\theta}}}{J_z T_V}, k_\delta = \frac{m_\delta}{J_z}. \tag{8}$$

Equation (6) is equivalent with the following one

$$\ddot{\theta} = -\frac{m_{\dot{\theta}}}{J_z}\dot{\theta} - \frac{m_\alpha}{J_z}\alpha + k_\delta\delta, \quad (9)$$

which, taking into account equation (8)

$$\frac{m_{\dot{\theta}}}{J_z} = 2\xi\omega_0 - \frac{1}{T_V} = a_1, \frac{m_\alpha}{J_z} \cong \omega_0^2 - \frac{m_{\dot{\theta}}}{J_z T_V} = \omega_0^2 - \frac{a_1}{T_V} = a_0, \quad (10)$$

leads to

$$\ddot{\theta} = -a_1\dot{\theta} - a_0\Delta\alpha + k_\delta\delta. \quad (11)$$

Considering that rocket A is leaded using a three points method (co-linearity PD – A -T), target line (PD - T) being the equal signal line, lateral deviation in rapport with this is [1]

$$\dot{y} = V\vartheta + Vf_T, \quad (12)$$

where f_T is a perturbation due target line rotation, which tends to deviate A from equal signal line and gives A a normal acceleration w_T. By equation (12) derivation and taking into account equation (1), one obtains

$$\ddot{y} = \frac{V}{T_V}\Delta\alpha + w_T, w_T = \dot{V}f_T + V\dot{f}_T - g\cos\vartheta_0; \quad (13)$$

w_T is an equivalent perturbation (normal acceleration to equal signal line direction).

For above equations coefficients calculus one may use calculus equations [2]

$$T_V = \frac{1}{d_1}, \omega_0 = \sqrt{d_1 d_4 - d_3}, \xi = \frac{d_1 + d_4}{2\sqrt{d_1 d_4 - d_3}}, k_\delta = d_2, \quad (14)$$

where d_1, d_2, d_3, d_4 are read from diagrams or graphic characteristics for different rockets variants. Thus, for an Oerlicon rocket at 10^{th} flight second $d_1 = 1.5\,\mathrm{s}^{-1}, d_2 = 40\,\mathrm{s}^{-2}$, $d_3 = -20\,\mathrm{s}^{-2}$, $d_4 = 1.2\,\mathrm{s}^{-1}$, $V = 400\,\mathrm{m/s}$; it results $T_V = 0.66\,\mathrm{s}$, $\omega_0 = 4.669\,\mathrm{s}^{-1}$, $\xi = 0.0619, k_\delta = 40\,\mathrm{s}^{-2}$, $a_1 = 0.922\,\mathrm{s}^{-1}$, $a_0 = 23.1825\,\mathrm{s}^{-2}$.

Choosing the state vector $x^T = [y \quad \dot{y} \quad \Delta\alpha \quad \dot{\theta}]$, the system formed by equations (6), (9) and (13) becomes

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= \frac{V}{T_v}x_3 + w_T, \\ \dot{x}_3 &= -\frac{1}{T_V}x_3 + x_4 + \frac{1}{T_V}\tilde{\alpha}, \\ \dot{x}_4 &= -a_0 x_3 - a_1 x_4 + k_\delta\delta. \end{aligned} \quad (15)$$

If the input is $u = \delta$ and perturbation vector is $u_p^T = [\tilde{\alpha} \quad w_T]$, system (15) becomes

$$\dot{x} = Ax + Bu + Eu_p, \quad (16)$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{V}{T_V} & 0 \\ 0 & 0 & -\frac{1}{T_V} & 1 \\ 0 & 0 & -a_0 & -a_1 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ k_\delta \end{bmatrix}, E = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ \frac{1}{T_V} & 0 \\ 0 & 0 \end{bmatrix}. \quad (17)$$

3. LINEAR OBSERVER PROJECTION

Let's consider the linear observer described by equations [3], [4]

$$M\dot{z} = Fz + Gu + Hy, \quad (18)$$

$$\hat{x} = Pz + Qy, z \quad (19)$$

with $z(r\times 1), \hat{x}(n\times 1)$, $M(r\times r), F(r\times r)$, $\det(M) = 0$ and $rank(M) \le n - q$, $G(r\times m)$, $rank(E) = q$, $H(r\times p)$, $P(n\times r)$, $Q(n\times p)$, $q \le p$; let's consider the observer's error

$$e = z - Nx, \quad (20)$$

where N is a $(r\times r)$ matrix.

By equation (20) derivation and substituting $\dot{x}$ of form (16), $y = Cx$, $\dot{z}$ of form (18) and imposing that all coefficients of x, u and u_p being null, that means

$$G = MNB, \quad (21)$$

$$HC = MNA - FN, \quad (22)$$

$$MNE = 0, \quad (23)$$

one obtains the error's equation

$$M\dot{e} = Fe. \quad (24)$$

The error $(\hat{x} - x)$ may be replaced function of e. Indeed, taking into account equations (19) and (20), one obtains

$$\hat{x} - x = Pe \quad (25)$$

if

$$PN + QC = I. \quad (26)$$

With E of form

$$E = [E_1 \quad E_2], E_1((n-p)\times q), E_2((n-q)\times q), \quad (27)$$

equation (23) is equivalent with equation system

$$\begin{aligned} M_2E_1 + M_1E_2 &= 0, \\ M_4E_1 + M_3E_2 &= 0. \end{aligned} \quad (28)$$

Choosing $M_2 = 0_{(n-p)\times(n-p)}$ and $M_4 = 0_{(n-q)\times(n-p)}$, it results

$$M = \begin{bmatrix} M_1 & 0 \\ M_3 & 0 \end{bmatrix}, M_1E_2 - 0, M_3E_2 = 0. \quad (29)$$

Condition (22) is equivalent wit equations system

$$M_1 A_3 = H_1 C_1, M_1 A_4 - F_1 = H_1 C_2,$$
$$M_3 A_3 = H_2 C_1, M_3 A_4 - F_3 = H_2 C_2; \tag{30}$$

it results $H^T = \begin{bmatrix} H_1 & H_2 \end{bmatrix}$, with

$$H_1 = M_1 A_3 C_1^+, H_2 = M_3 A_3 C_1^+, \tag{31}$$

$$F_1 = M_1 A_4 - M_1 A_3 C_1^+ C_2, F_3 = M_3 A_4 - M_3 A_3 C_1^+ C_2,$$
$$F_2 = F_4 = 0. \tag{32}$$

Matrices P and Q may be obtained from condition (26), equivalent with following equations

$$P_1 = P_3 = 0, Q^T = \begin{bmatrix} Q_1 & Q_2 \end{bmatrix}, \tag{33}$$

$$P_2 + Q_1 C_1 = 0, Q_1 C_2 = 0,$$
$$P_4 + Q_2 C_1 = 0, Q_2 C_2 = I; \tag{34}$$

one results

$$Q_2 = C_2^+, P_4 = -C_2^+ C_1, \tag{35}$$

Q_1 and P_2 are solution of system (33). G matrix is calculated with (21).

Using the presented algorithm with parameters' values for the 10th flight second and $C = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, one obtains matrices

$$N = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, M = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, G = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$
$$H = \begin{bmatrix} 0 & 0 \\ 600 & 0 \\ 0 & 0 \\ 600 & 0 \end{bmatrix}, F = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \tag{36}$$
$$P = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, Q = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

The algorithm proposed in this paper has the following steps:

1. One makes coordinates transformation $x = T\overline{x}$ for system (16), where T is a non-singular transformation; it results matrices

$$\overline{A} = T^{-1} A T, \overline{B} = T^{-1} B = \begin{bmatrix} I_m & 0 \end{bmatrix}, \tag{37}$$

where T is chosen having form $T = \begin{bmatrix} B & \widetilde{T} \end{bmatrix}$, with $\widetilde{T}$ randomly chosen so that $rank(T) = n$.

2. Gain matrix $\overline{K}$ is calculated for the optimal control of system $(\overline{A}, \overline{B})$ so that closed loop system with matrix $\overline{G} = \overline{A} - \overline{B}\overline{K}$ be stable [5].

3. $\overline{K}$ and $\overline{P}$ are partionated [6]

$$\overline{K} = \begin{bmatrix} \overline{K}_1 & \overline{K}_2 \end{bmatrix}, \overline{P} = \begin{bmatrix} \overline{P}_{11} & \overline{P}_{12} \\ \overline{P}_{21} & \overline{P}_{22} \end{bmatrix}, \overline{P}_{12} = \overline{P}_{21}^T, \overline{P}_{22} = \overline{P}_{22}^T. \tag{38}$$

The following matrices are calculated

$$\overline{P}_{11} = R\overline{K}_1, \overline{P}_{12} = \overline{P}_{21}^T = R\overline{K}_2, \overline{P}_{22} I_{n-m}. \tag{39}$$

For the studied case (one input and one output) it results

$$\overline{K} = \begin{bmatrix} k_1 & | & k_{21} & k_{22} & k_{23} \end{bmatrix}, R = [1], \tag{40}$$

$$\overline{P} = \begin{bmatrix} k_1 & | & k_{21} & k_{22} & k_{23} \\ - & | & - & - & - \\ k_{21} & | & 1 & 0 & 0 \\ k_{22} & | & 0 & 1 & 0 \\ k_{23} & | & 0 & 0 & 1 \end{bmatrix}. \tag{41}$$

4. Matrices Q and $\overline{Q}$ are calculated

$$\overline{Q} = -\left[\overline{P}\overline{A} + \overline{A}^T \overline{P} - \overline{P}\overline{B}\overline{K}\right], \tag{42}$$

$$Q = (T^{-1})^T \overline{Q} T^{-1}. \tag{43}$$

5. One solves Riccati equation in rapport with unknown P

$$PA + A^T P - PBR^{-1}B^T P + Q = 0. \tag{44}$$

6. One calculates gain matrix

$$K = R^{-1} B^T P \tag{45}$$

7. One calculates optimal command law

$$u = -K\hat{x}. \tag{46}$$

With the presented algorithm one obtains matrices

$$T = \begin{bmatrix} 0 & -0.432 & -1.146 & 0.327 \\ 0 & -1.665 & 1.19 & 0.174 \\ 0 & 0.125 & 1.189 & -0.186 \\ 40 & 0.287 & -0.037 & 0.725 \end{bmatrix}; \overline{K}^T = \begin{bmatrix} 83.917 \\ -4.592 \\ -96.105 \\ 15.416 \end{bmatrix};$$
$$\overline{A} = 10^3 \cdot \begin{bmatrix} 0.0094 & 0.0037 & 0.0302 & -0.0047 \\ -0.1314 & -0.0776 & -0.6868 & 0.1063 \\ -0.104 & -0.0196 & -0.1512 & 0.0224 \\ -0.539 & -0.1763 & -1.4335 & 0.2196 \end{bmatrix}; \overline{B}^T = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}; \tag{47}$$

$$\overline{Q} = 10^4 \begin{bmatrix} 0.08 & -0.0039 & -0.5169 & -0.0047 \\ -0.0039 & 0.021 & 0.1639 & -0.0079 \\ -0.5169 & 0.1639 & 1.5346 & -0.0984 \\ 0.1337 & -0.0079 & -0.0984 & -0.0058 \end{bmatrix};$$
$$Q = 10^4 \begin{bmatrix} 0.0005 & -0.014 & 1.541 & 0.0181 \\ -0.014 & 0.0057 & -0.287 & -0.0039 \\ 1.541 & -0.287 & 4.598 & 0.0105 \\ 0.0181 & -0.0039 & 0.0105 & 0 \end{bmatrix}; R = [1]; \tag{48}$$
$$K^T = \begin{bmatrix} -2.2361 \\ -2.3607 \\ -80.5432 \\ 2.0979 \end{bmatrix}; P = \begin{bmatrix} 10.5573 & 0.5728 & 9.5164 & -0.1118 \\ 0.5728 & 0.7892 & 9.9349 & -0.118 \\ 9.5164 & 9.9349 & 263.8207 & -4.0272 \\ -0.1118 & -0.118 & -4.0272 & 0.1049 \end{bmatrix}.$$

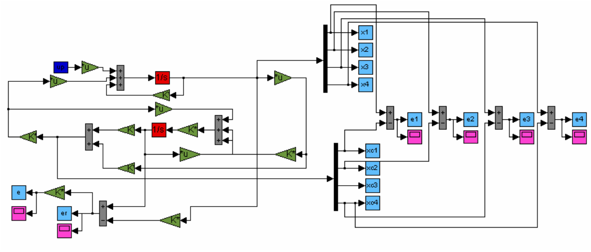

Fig.1 - Matlab /Simulink model of the rocket's movement

Using Matlab/Simulink model of the rocket's movement from fig.1 and a simulation program, one calculates $x_i(t)$ and $\hat{x}_i(t)$ (curves x_i of blue color, $\hat{x}_i$ of red color), represented in fig.2.

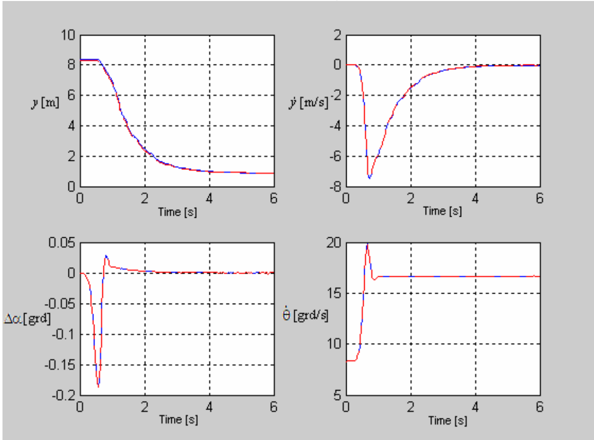

Fig.2 – Time varying of $x_i, \hat{x}_i$

The obtained results with these algorithms are superior to those obtained with algorithms from [7], [8], [9] from the point of view of precision.

4. CONCLUSIONS

The paper presents an optimal command system of lateral deviation of a rocket in rapport with equal signal line. A 's model is a liniarized model which expresses the dynamic of the main variables: lateral deviation and deviation velocity in rapport with equal signal line, incidence angle variation and pitch angular velocity. Because the lateral deviation is difficult to measure, one measures incidence angle and pitch angular velocity and estimates the full state vector using a linear state estimator (observer). A new projection algorithm of such an observer is presented. The system's command is chosen to be optimal, based on usage of a quality quadratic criterion. For gain matrix's projection of the optimal command, a new algorithm is presented. This contains weight matrices, from quality quadratic criterion, calculus after coordinates transformations have been made. With obtained matrices, a Riccati equation is solved and after that gain matrix of the optimal command is calculated. Simulation program calculates state observer, gain matrix and time functions, which expresses state variables dynamics of the system and estimated state variables dynamics.

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