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COMPARATIVE ANALYSIS OF THE EFFICIENCY OF PLANAR DIELECTRIC-CONDUCTOR OPEN WAVEGUIDES

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Abstract **– This paper presents a comparative analysis of the efficiency of active power transmission in an open planar dielectric-conductor waveguide in the TE and TM modes, respectively. The influence of frequency and waveguide parameters on the overall performance is investigated.**

Keywords: *electromagnetic fields, dielectric waveguides.*

1. INTRODUCTION

Dielectric waveguides are used for the transmission of high frequency signals (tens of GHz) mainly because for these frequencies conductor waveguides would have very small cross-sections. Two classes of waveguides fall in this category: a) dielectric-conductor waveguides, in which case the wave is transmitted through a dielectric layer placed over a solid conductor; b) pure dielectric waveguides, surrounded by a dielectric cladding. Simple planar or cylindrical geometries are usually used. The optical fiber is a well known example of cylindrical dielectric waveguide widely used in the microwave and optical frequency range because of its low attenuation (of the order of 1/100 dB/km) and small susceptibility to radio frequency interference [1], [2].
Both analytical and numerical methods are employed for the analysis of electromagnetic fields in waveguides [1], [3], and dedicated software packages have recently been developed for the study of open dielectric waveguides [4]. In a previous paper [5] the case of a planar dielectric-conductor waveguide, excited in TE mode, was investigated.
This paper presents a complete study of a planar dielectric-conductor waveguide in which both TE and TM modes may be produced.
The solution for the electric and magnetic field components is determined analytically using the separation of variables method. The complex propagation constant in the dielectric waveguide and the attenuation constant in the surrounding dielectric are determined numerically.
The transmitted power and the efficiency of power transmission is calculated for several dielectrics and the influence of frequency, waveguide thickness and permittivity is outlined.

The waveguide thickness is considered in all cases small enough in order to suppress higher modes.

2. PHYSICAL MODEL AND BASIC EQUATIONS

The physical model of a planar dielectric-conductor waveguide is presented in Fig.1. A dielectric layer with the thickness d and material constants ε_1, μ_1 covers a flat metal sheet. The conductor is considered to be perfect ($\sigma\to\infty$). The surrounding dielectric, which is usually air, has the permittivity ε_2 and the permeability μ_2.

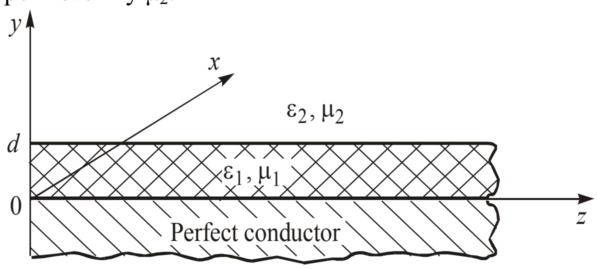

Figure 1: Planar dielectric-conductor waveguide

If the two dielectrics are lossless and free of charge, the field vectors satisfy the vector Helmholtz equation in complex form:

$$\Delta\underline{\mathbf{E}}+\omega^2\mu\varepsilon\underline{\mathbf{E}}=0 \tag{1}$$

$$\Delta\underline{\mathbf{H}}+\omega^2\mu\varepsilon\underline{\mathbf{H}}=0 \tag{2}$$

where $\omega=2\pi f$ is the angular frequency.
In the case of a lossy dielectric the permittivity is complex, of the form

$$\underline{\varepsilon}=\varepsilon'-j\varepsilon''=\varepsilon_0\varepsilon_r'(1-j\,tg\delta)\,, \tag{3}$$

where ε_r' and tgδ are the dielectric constant and the loss angle, respectively.
Depending on the source exciting the waveguide (e.g. a small loop or a small electric dipole) two modes can be established in the waveguide: TE or TM.
In the TE mode the axial component of the magnetic field satisfies the scalar Helmholtz equation

$$\Delta\underline{H}_z+\omega^2\mu\varepsilon\underline{H}_z=0 \tag{4}$$

and in the TM mode a similar equation is satisfied by the component E_z:

$$\Delta\underline{E}_z + \omega^2\mu\varepsilon\underline{E}_z = 0 . \quad (5)$$

The field propagates in direction Oz so that $\underline{H}_z = \underline{H}_z(y)\mathrm{e}^{-\underline{\gamma}z}$ and $\underline{E}_z = \underline{E}_z(y)\mathrm{e}^{-\underline{\gamma}z}$, where $\underline{\gamma}=\alpha+j\beta$ is the complex propagation constant. Denoting the constant

$$\underline{\gamma}^2 + \omega^2\mu\varepsilon = \underline{k}_c^2 , \quad (6)$$

equations (4) and (5) reduce to ordinary differential equations:

$$\frac{\mathrm{d}^2\underline{H}_z}{\mathrm{d}y^2} + \underline{k}_c^2\,\underline{H}_z = 0 , \quad (7)$$

$$\frac{\mathrm{d}^2\underline{E}_z}{\mathrm{d}y^2} + \underline{k}_c^2\,\underline{E}_z = 0 . \quad (8)$$

The constant k_c is referred to as the critical wave number, having different values in the two dielectrics. In the case of lossless dielectrics $\underline{\gamma}=j\beta$ and thus $\underline{k}_c^2$ has real positive or negative values. In order to enforce the wave attenuation in direction Oy in the surrounding dielectric, the conditions $\underline{k}_{c2}^2 = -\alpha_2^2 < 0$ for $y>d$ and, respectively, $\underline{k}_{c1}^2 > 0$ for $y \in [0,d]$, are imposed. Thus $\underline{H}_z$ and $\underline{E}_z$ have expressions of the form:

$$\begin{array}{ll} \left(C\sin k_{c_1}y + D\cos k_{c_1}y\right)e^{-\underline{\gamma}_1 z} & , y\in[0,d] \\ A e^{-\alpha_2 y}\, e^{-\underline{\gamma}_2 z} & , y>d \end{array} .$$

Imposing the condition $\underline{H}_z\big|_{y=0} = 0$, in the TE mode, and $\underline{E}_z\big|_{y=0} = 0$ in the TM mode (conservation of tangential **H** and tangential **E** on the perfect conductor-dielectric boundary), the constant D becomes 0, so that the solutions are of the form

$$\begin{array}{ll} A_1 e^{-\alpha_2 y} e^{-\underline{\gamma}_2 z} & , y>d \\ A_2 \sin k_{c_1} y\cdot e^{-\underline{\gamma}_1 z} & , y\in[0,d] \end{array} .$$

Imposing the conservation of the tangential components $\underline{E}_z$ or $\underline{H}_z$ on the surface $y=d$, two aditional conditions arise:

$$\underline{\gamma}_1 = \underline{\gamma}_2 , \quad (9)$$

$$A_2 = \frac{A_1 e^{-\alpha_2 d}}{\sin k_{c_1} d} . \quad (10)$$

Relation (9), called phase matching, leads to the following relation between the constants k_{c_1} and α_2:

$$k_{c_1}^2 - \omega^2\mu_1\varepsilon_1 = -\alpha_2^2 - \omega^2\mu_2\varepsilon_2 . \quad (11)$$

Finally, taking into account the expressions for the transversal field components in the TE and TM modes [1], the solutions for each of the field components are:

<u>TE mode</u>

$$\underline{H}_z = \begin{cases} A\dfrac{e^{-\alpha_2 d}}{\sin k_{c_1} d}\sin k_{c_1} y\cdot e^{-\underline{\gamma}z} & , y\in[0,d] \\ A e^{-\alpha_2 d}\, e^{-\underline{\gamma}z} & , y>d \end{cases} \quad (12)$$

$$\underline{E}_x = \begin{cases} -A\dfrac{j\omega\mu_1}{k_{c_1}}\dfrac{e^{-\alpha_2 d}}{\sin k_{c_1} d}\cos k_{c_1} y\cdot e^{-\underline{\gamma}z} & , y\in[0,d] \\ -A\dfrac{j\omega\mu_2}{\alpha_2} e^{-\alpha_2 d}\, e^{-\underline{\gamma}z} & , y>d \end{cases} \quad (13)$$

$$\underline{H}_y = \begin{cases} -A\dfrac{\underline{\gamma}}{k_{c_1}}\dfrac{e^{-\alpha_2 d}}{\sin k_{c_1} d}\cos k_{c_1} y\cdot e^{-\underline{\gamma}z} & , y\in[0,d] \\ -A\dfrac{\underline{\gamma}}{\alpha_2} e^{-\alpha_2 d}\, e^{-\underline{\gamma}z} & , y>d \end{cases} ,(14)$$

<u>TM mode</u>

$$\underline{E}_z = \begin{cases} C\dfrac{e^{-\alpha_2 d}}{\sin k_{c_1} d}\sin k_{c_1} y\cdot e^{-\underline{\gamma}z} & , y\in[0,d] \\ C e^{-\alpha_2 d}\, e^{-\underline{\gamma}z} & , y>d \end{cases} \quad (15)$$

$$\underline{E}_y = \begin{cases} -C\dfrac{\underline{\gamma}}{k_{c_1}}\dfrac{e^{-\alpha_2 d}}{\sin k_{c_1} d}\cos k_{c_1} y\cdot e^{-\underline{\gamma}z} & , y\in[0,d] \\ -C\dfrac{\underline{\gamma}}{\alpha_2} e^{-\alpha_2 d}\, e^{-\underline{\gamma}z} & , y>d \end{cases} \quad (16)$$

$$\underline{H}_x = \begin{cases} C\dfrac{j\omega\varepsilon_1}{k_{c_1}}\dfrac{e^{-\alpha_2 d}}{\sin k_{c_1} d}\cos k_{c_1} y\cdot e^{-\underline{\gamma}z} & , y\in[0,d] \\ C\dfrac{j\omega\varepsilon_2}{\alpha_2} e^{-\alpha_2 d}\, e^{-\underline{\gamma}z} & , y>d \end{cases} .(17)$$

The conservation of normal **B**, in the TE mode, and that of normal **D** in the TM mode, leads to a second relation between k_{c_1} and α_2:

$$k_{c_1}\tan k_{c_1} d = \alpha_2 F , \quad (18)$$

where $F=\mu_1/\mu_2$ in the TE mode and $F=\varepsilon_1/\varepsilon_2$ in the TM mode.

The constants k_{c_1} and α_2 represent the solution of the non-linear algebraic system of equations:

$$\begin{cases} k_{c_1}^2 + \alpha_2^2 = \omega^2\left(\mu_1\varepsilon_1 - \mu_2\varepsilon_2\right) \\ k_{c_1}\tan k_{c_1} d = F\alpha_2 \end{cases} . \quad (19)$$

The constants A and C are determined using the expression of the total active power transmitted by the waveguide:

$$P = P_z = \mathrm{Re}\left\{\oiint_\Sigma \left(\underline{\mathbf{E}}\times\underline{\mathbf{H}}^*\right)\cdot \mathbf{n}\,\mathrm{d}A\right\} = P_{1z} + P_{2z} \tag{20}$$

where

$$\begin{aligned} P_{1z} &= G_1 \frac{\omega\beta\, \mathrm{e}^{-2\alpha_2 d}}{k_{c_1}^2 \sin^2 k_{c_1} d}\left(\frac{\sin 2k_{c_1} d}{4k_{c_1}} + \frac{d}{2}\right), \; y\in[0,d] \\ P_{2z} &= G_2 \frac{\omega\beta\, \mathrm{e}^{-2\alpha_2 d}}{2\alpha_2^3} \qquad\qquad\qquad\qquad , \; y > d \end{aligned} \tag{21}$$

with

$$G_1 = \begin{cases} \mu_1 A^2 \text{ , TE mode} \\ \varepsilon_1 C^2 \text{ , TMmode} \end{cases}, \quad G_2 = \begin{cases} \mu_2 A^2 \text{ , TE mode} \\ \varepsilon_2 C^2 \text{ , TMmode} \end{cases}.$$

The active power transmitted in direction Oy is P_y=0, as may be easily verified.

The efficiency of power transmission may be appreciated using the relation:

$$\eta = \frac{P_{1z}}{P}. \tag{22}$$

The incidence angle θ_i on the surface $y=d$ may be expressed with the relations:

$$\tan\theta_i^{(TM)} = \left|\frac{\underline{E}_y}{\underline{E}_z}\right|_{y=d}, \; \tan\theta_i^{(TE)} = \left|\frac{\underline{H}_y}{\underline{H}_z}\right|_{y=d}.$$

In both cases θ_i satisfies the relation:

$$\tan\theta_i = \frac{\beta}{k_{c_1}\tan k_{c_1} d}, \tag{23}$$

but the values for θ_i are different in the TE and TM modes because of the different values of β and k_{c_1}. It is to be noted that the incidence angle must be always greater than the critical incidence angle in the conditions presented before:

$$\theta_i > \theta_{i_{cr}} = \arcsin\sqrt{\frac{\varepsilon_{r_2}\mu_2}{\varepsilon_{r_1}\mu_1}}. \tag{24}$$

3. RESULTS AND DISCUSSIONS

A numerical simulation was conducted in order to determine the critical wavenumber, the propagation constant and the efficiency, and their dependence on ε_1, d and f. Some of the results are summerized in Table 1. In all the cases considered in the paper the thickness of the dielectric layer was chosen so as to eliminate multiple solutions of the system (19) (the superior modes), i.e. $d<d_{max}$, where

$$d_{\max} = \frac{1}{4f\sqrt{\varepsilon_1\mu_1 - \varepsilon_2\mu_2}} \tag{25}$$

for both TE and TM modes.

No.	f (GHz)	$\frac{\varepsilon_{r1}}{\varepsilon_{r2}}$	d_{max} (mm)	d (mm)	α_2(Nep/m)		k_{c_1} (rad/m)		β (rad/m)		η (%)	
					TE	TM	TE	TM	TE	TM	TE	TM
1.	10	2.1	7.2	5	154.5	117.7	156	185.4	260.3	240.2	71.5	54.1
2.	20	2.1	3.6	2.5	301.1	235.4	312.1	370.9	520.6	480.5	71.5	54.1
3.	20	2.26	3.3	2.5	340.6	259.2	324.1	392.3	539.9	492.6	74.3	57.6
4.	10	2.56	6	6	210.3	175.5	155.6	193.9	296.8	273.3	84.3	77.4
5.	10	1.219	11	10	110.5	106.3	89.2	94.2	236.8	234.8	81.2	79.1
6.	20	8.3	1.4	1.4	912.3	571.4	669.7	97.7	1004	708.5	84.6	73
7.	60	8.3	0.46	0.46	2725	1668	2025	2957	3000	2088	84.2	70

Table 1: Propagation constant and efficiency of power transmission for different waveguide parameters

The dependence of transmitted power efficiency on the dielectric layer thickness, d, is plotted in Fig.2 for the frequency f=10GHz, ε_{r1}=2.1 (teflon) and ε_{r2}=1, in the two modes. In Fig.3 $\eta^{(TM)}$ and $\eta^{(TE)}$ are represented versus frequency when a 85% Al_2O_3 ceramic dielectric (ε_r=8.3) is used.

As may be seen, the efficiency of power transmission is always greater in TE mode than in TM mode. Fig. 2 shows that higher values of η are obtained when the dielectric thickness has the maximum value ($d=d_{max}$) for a single TE_1 mode. The efficiency increases constantly for increasing frequency values as may be seen from Fig.3.

The incidence angle calculated with relation (23) is always larger in TE mode than in TM mode, for the investigated examples, satisfying the relation $\theta_i^{(TE)}>\theta_i^{(TM)}>\theta_{i\ cr}$. This was to be expected since the wave in the surrounding medium is a surface wave (decreases in amplitude in the direction normal to the boundary between the two dielectrics).

Fig.4 presents the amplitude of the three field components in the TE mode (H_z, H_y, E_x) in direction Oy for $y\in[0,d]$, in the case $d=d_{max}$=1.4mm, f=20 GHz and ε_{r1}=8.3. The values correspond to the case P=10W. As expected, the field components decrease rather rapidly outside the waveguide, at a distance d from the plane boundary between the two dielectrics being approximatively one third of the minimum value in the waveguide.

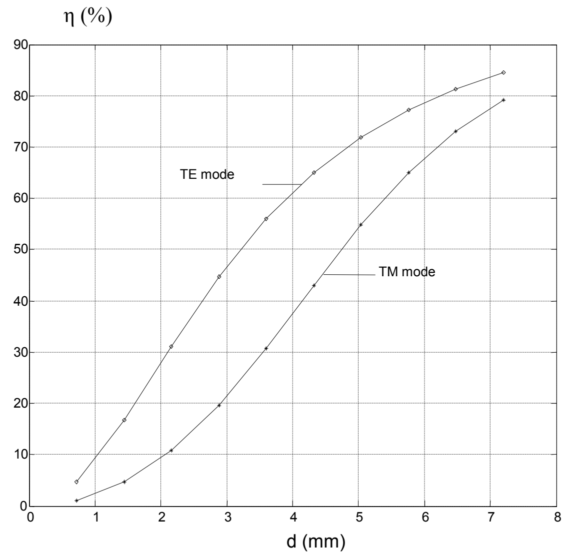

Figure 2: Power efficiency versus dielectric thickness

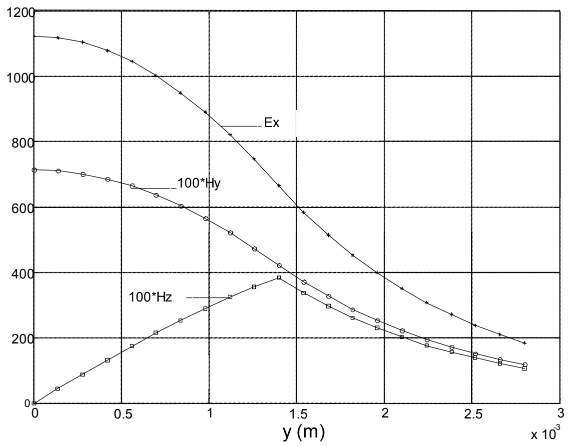

Figure 4: Amplitude of the field components

4. CONCLUSIONS

The paper presents a comparative study of the TE_1 and TM_1 modes in a planar dielectric conductor open waveguide. The *fsolve* routine in MATLAB is used in order to solve the non-linear algebraic system of equations satisfied by the critical wavenumbers. Based on the analytical expressions of the field components in the single modes TE_1 or TM_1 the expression of power transmission efficiency is established. The numerical examples considered show that the highest efficiency levels are obtained when the waveguide thickness is close (or equal) to the highest possible value that still ensures a single mode in the dielectric $d=d_{max}$.

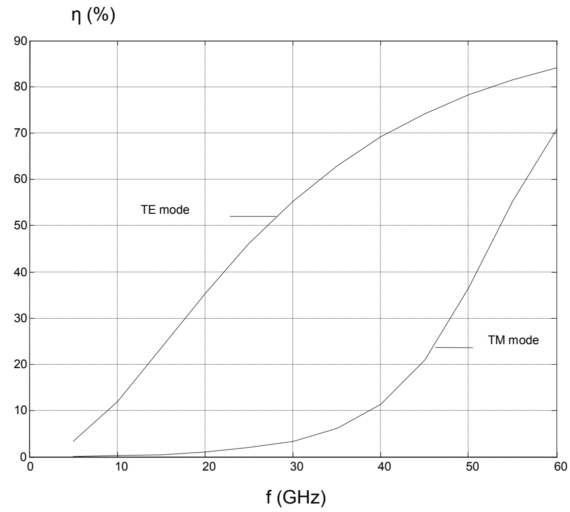

Figure 3: Efficiency of power transmission versus frequency

The relations derived in this paper may be used to evaluate the efficiency of power transmission in the case of a lossy dielectric waveguide, in which case the values of η will be significantly smaller.

Acknowledgments

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