RESONANCES AT THE ACCESS GATES OF A LINEAR AND NON-AUTONOMOUS TWO-PORT, SUPPLYING, IN HARMONIC STEADY-STATE, A NON-LINEAR, INERTIAL AND PASSIVE RECEIVER

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Abstract. **The possibility of realization of resonance at the access gates of a linear and non-autonomous two-port, supplying, in harmonic steady-state, a non-linear, inertial and passive receiver, is studied. It is proved that it is possible to realize at the most: a) two different resonances at the input gate; b) one resonance at the output gate; c) two different total resonances.**

1. INTRODUCTION

Let be a linear and non-autonomous two-port (LNT), which supplies, in harmonic steady-state, a non-linear, inertial and passive receiver (NIPR) constituted from the series connexion of a resistor, a coil and a condenser, all three non-linear, inertial (Fig. 1). When the LNT is excited by a signal whose variation in time is harmonical, the established steady-state is a harmonic one [1]. Consequently it is possible to use, in such a situation, the symbolic method based on utilization of complex numbers.

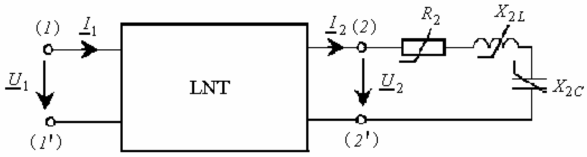

Fig. 1

The equations of a LNT in harmonic steady-state are well known [2] namely

$$\begin{bmatrix} \underline{U}_1 \\ \underline{I}_1 \end{bmatrix} = \begin{bmatrix} \underline{A}_{11} & \underline{A}_{12} \\ \underline{A}_{21} & \underline{A}_{22} \end{bmatrix} \begin{bmatrix} \underline{U}_2 \\ \underline{I}_2 \end{bmatrix}, \qquad (1)$$

where $\underline{A}_{ij}$, $(i,\ j = 1,\ 2)$, re[resent the fundamental parameters of the two-port.

As regards the parameters of NIPR represented in Fig. 1, these ones may be approximated with relations [3]

$$R_2 = R_2\left(I_{2m}\right) = a_1 + \frac{3}{4}a_3 I_{2m}^2, \qquad (a_1 > 0,\ a_3 \gtrless 0), \quad (2)$$

$$X_{2L} = X_{2L}\left(I_{2m}\right) = b_1\omega - \frac{3}{4}b_3 I_{2m}^2, \qquad (b_1 > 0,\ b_3 \gtrless 0), \quad (3)$$

$$X_{2C} = X_{2C}\left(I_{2m}\right) = \frac{c_1}{\omega} - \frac{3c_3}{\omega^3} I_{2m}^2, \qquad (c_1 > 0,\ c_3 \gtrless 0), \quad (4)$$

and consequently

$$X_2 = X_2\left(I_{2m}\right) = X_{2L}\left(I_{2m}\right) - X_{2C}\left(I_{2m}\right) = b_1\omega - \frac{c_1}{\omega} - \frac{3}{4}\left(b_3\omega + \frac{c_3}{\omega^3}\right) I_{2m}^2, \quad (5)$$

where I_{2m} is the amplitude of the secondary current.

The aim of this paper is to determine the conditions which must be fulfilled to realize: a) the resonance at the input gate of the considered LNT; b) the resonance at the output gate of the same two-port; c) the total resonance at the gates of the studied two-port.

2. RESONANCE AT THE INPUT GATE

In a previous paper [4] it was shown that the equivalent reactance, at the input gate of the two-port represented in Fig. 1 is

$$X_{e1} = \frac{\alpha I_{2m}^4 + \beta I_{2m}^2 + \gamma}{\delta I_{2m}^4 + \varepsilon I_{2m}^2 + \varphi}, \qquad (6)$$

with the values of $\alpha, \beta, \ldots, \varphi$ indicated in Appendix 1.

To realize the resonance at the input gate of the LNT represented in Fig. 1, in harmonic steady-state, it is necessary and sufficient to satisfy the equality

$$X_{e1} = 0, \qquad (7)$$

which, having in view expression (6), leads to

$$\alpha I_{2m}^4 + \beta I_{2m}^2 + \gamma = 0, \qquad (8)$$

an algebraic biquadratic equation in I_{2m}. Evidently, the resonance at the input gate of the studied LNT may be realized only for those roots of Eq. (8) which are real and positive. These roots are, in generally, of the form

$$I_{2m} = \pm\sqrt{\frac{-\beta \pm \sqrt{\beta^2 - 4\alpha\gamma}}{2\alpha}}, \qquad (9)$$

with α, β, γ given in Appendix 1. These roots are real only if the inequalities

$$\beta^2 - 4\alpha\gamma > 0,\ \ \mathrm{sign}\left(-\beta \pm \sqrt{\beta^2 - 4\alpha\gamma}\right) = \mathrm{sign}\alpha \quad (10)$$

are satisfied. In this case two real and positive roots of Eq. (8) may be discerned and consequently it exist two distinct resonance regimes at the input gate of the LNT. The number of these regimes diminishes to only one in case when the first inequality (10) being satisfied, the inequalities

$$\mathrm{sign}\left(-\beta + \sqrt{\beta^2 - 4\alpha\gamma}\right) = \mathrm{sign}\alpha,\ \text{but}\ \mathrm{sign}\left(-\beta - \sqrt{\beta^2 - 4\alpha\gamma}\right) = -\mathrm{sign}\alpha \quad (11')$$

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are also satisfied.
In the particular case when

$$\beta^2 = 4\alpha\gamma, \quad (12)$$

a single resonance regime at the input gate of the LNT may be realized, only if, in addition

$$\mathrm{sign}\beta = -\mathrm{sign}\alpha. \quad (13)$$

As regards the parameters of the NIPR in case(s) when the resonance at the input gate of the LNT is realized, these ones may be obtained replacing in relations (2) and (5) the expression (9) of the amplitude, I_{2m}, of the secondary current. Performing the calculi it results

$$\begin{cases} R_2 = a_1 + \dfrac{3\left(-\beta \pm \sqrt{\beta^2 - 4\alpha\gamma}\right)}{8\alpha}, \\ X_2 = b_1\omega - \dfrac{c_1}{\omega} - 3\left(b_3\omega + \dfrac{c_3}{\omega^3}\right)\dfrac{-\beta \pm \sqrt{\beta^2 - 4\alpha\gamma}}{8\alpha}. \end{cases} \quad (14)$$

If relations (12), (13) are satisfied, expressions (14) become

$$R_2 = a_1 - \frac{2\beta}{8\alpha},\ \ X_2 = b_1\omega - \frac{c_1}{\omega} + 3\beta\frac{b_3\omega + c_3/\omega^3}{8\alpha}. \quad (15)$$

3.RESONANCE AT THE OUTPUT GATE OF THE LNT

Strictly speaking the possibility of realization of such a regime was studied in a previous paper [5], where the realizability of ferroresonance in a non-linear inertial circuit, in harmonic steady-state, was investigated. The realization of such a regime imposes that

$$X_2\left(I_{2m}\right) = 0, \quad (16)$$

that is

$$X_{2L}\left(I_{2m}\right) = X_{2C}\left(I_{2m}\right). \quad (17)$$

Having in view relations (3) and (4) it results that in this case the amplitude of the secondary current must be

$$I_{2m} = \frac{2\omega}{\sqrt{3}}\sqrt{\frac{b_1\omega^2 - c_1}{b_3\omega^4 + c_3}}. \quad (18)$$

A resonance regime at the output gate of the LNT may be realized only if the relation

$$\mathrm{sign}\left(b_1\omega^2 - c_1\right) = \mathrm{sign}\left(b_3\omega^4 + c_3\right) \quad (19)$$

is satisfied.
As regards the receiver's parameters, it is sufficient to replace, in this case, expression (18) of the secondary current amplitude in relations (2),…,(4), resulting

$$R_2 = a_1 + a_3\omega^2\frac{b_1\omega^2 + c_1}{b_3\omega^4 + c_3},\ \ X_{2\mathrm{L}} = X_{2C} = \omega\frac{b_1c_3 + b_3c_1\omega^2}{b_3\omega^4 + c_3}. \quad (20)$$

In brief, it is necessary to underline that in case of a LNT which supplies, in harmonic steady-state, a RNIP having the structure represented in Fig. 1, it is possible to realize at the most a single resonance regime at the output gate when the relation (19) is satisfied.

4. TOTAL RESONANCE AT THE ACCESS GATE OF A LNT

It is possible to realize the regime of total resonance at the access gates of a LNT, in harmonic steady-state, only when the receiver's parameters, R_2, X_2, satisfy the equation of circle's arc [6]

$$\begin{aligned} &\Im m\left(\underline{A}_{11}\underline{A}_{21}^*\right)\left(R_2^2 + X_2^2\right) + \Im m\left(\underline{A}_{11}\underline{A}_{21}^* - \underline{A}_{12}^*\underline{A}_{21}\right)R_2 + \\ &+\left[\Re e\left(\underline{A}_{11}\underline{A}_{22}^* - \underline{A}_{12}^*\underline{A}_{21}\right) + 1\right]X_2^2 + \Im m\left(\underline{A}_{12}\underline{A}_{22}^*\right) = 0, \end{aligned} \quad (21)$$

situated in the half-plane $R_2 \geq 0$.
In view to establish the conditions which permit the realization of total resonance at the access gates of a LNT, supplying, in harmonic steady-state, an NIPR, it is necessary to replace, in relation (21), the expressions (2) and (5) of the NIPR's parameters. Performing the calculi it results

$$\alpha' I_{2m}^4 + \beta' I_{2m}^2 + \gamma' = 0, \quad (22)$$

where α', β', γ' have the expressions given in Appendix 2.
As (8), equation (22) is an algebraic biquadratic one in I_{2m}, the regime of total resonance being realized only when roots of this equation,

$$I_{2m} = \pm\sqrt{\frac{-\alpha' \pm \sqrt{\beta'^2 - 4\alpha'\gamma'}}{2\alpha'}}, \qquad (23)$$

are real and positive, α', β', γ' having the expressions indicated in Appendix 2.
For a formal point of view expressions (9) and (23) being analogous, where (9) are the roots of equation (8), the analysis of the roots (23) leads to conclusions analogous to those obtained in § **2**. Namely it is possible to obtain two, one or none total resonance regimes.

5. CONCLUSIONS

The possibilities of realization of resonance regimes at the access gates of a linear and non-autonomous two-port are investigated, when the two-port supplies, in harmonic steady-state, a non-linear, inertial and passive receiver.
Three different situations may be identified namely: a) resonance at the input gate; b) resonance at the output gate; c) total resonance. In first and third situation are possible at most two resonance regimes and in the second one, at most one resonance regime.

APPENDIX

The parameters $\alpha,\ \beta, \ldots, \varphi$ from (5) are:

$$\alpha = \frac{9}{16}\Im m\left(\underline{A}_{11}\underline{A}_{21}^{*}\right)\left[a_3^2 + \left(b_3\omega + \frac{c_3}{\omega^3}\right)^2\right],$$

$$\beta = \frac{3}{2}\Im m\left(\underline{A}_{11}\underline{A}_{21}^{*}\right)\left[a_1 a_3 - \left(b_1\omega - \frac{c_1}{\omega}\right)\left(b_3\omega + \frac{c_3}{\omega^3}\right)\right] + \frac{3}{4}a_3\Im m\left(\underline{A}_{11}\underline{A}_{22}^{*} + \underline{A}_{12}\underline{A}_{21}^{*}\right) -$$
$$-\frac{3}{4}\left(b_3\omega + \frac{c_3}{\omega^3}\right)\Re e\left(\underline{A}_{11}\underline{A}_{22}^{*} - \underline{A}_{12}\underline{A}_{21}^{*}\right),$$

$$\gamma = \Im m\left(\underline{A}_{11}\underline{A}_{21}^{*}\right)\left[a_1^2 + \left(b_1\omega - \frac{c_1}{\omega}\right)^2\right] + a_1\Im m\left(\underline{A}_{11}\underline{A}_{22}^{*} + \underline{A}_{12}\underline{A}_{21}^{*}\right) +$$
$$+\left(b_1\omega - \frac{c_1}{\omega}\right)\Re e\left(\underline{A}_{11}\underline{A}_{22}^{*} - \underline{A}_{12}\underline{A}_{21}^{*}\right) + \Im m\left(\underline{A}_{12}\underline{A}_{22}^{*}\right),$$

$$\delta = \frac{9}{16}A_{21}^2\left[a_3^2 + \left(b_3\omega + \frac{c_3}{\omega^3}\right)^2\right] > 0,$$

$$\varepsilon = \frac{3}{2}A_{21}^2\left[a_1 a_3 - \left(b_1\omega - \frac{c_1}{\omega}\right)\left(b_3\omega + \frac{c_3}{\omega^3}\right)\right] + \frac{3}{2}a_3\Re e\left(\underline{A}_{21}\underline{A}_{22}^{*}\right) - \frac{3}{2}\left(b_3\omega + \frac{c_3}{\omega^3}\right)\Im m\left(\underline{A}_{21}\underline{A}_{22}^{*}\right),$$

$$\gamma = A_{21}^2\left[a_1^2 + \left(b_1\omega - \frac{c_1}{\omega}\right)^2\right] + 2a_1\Re e\left(\underline{A}_{21}\underline{A}_{22}^{*}\right) - 2\left(b_1\omega - \frac{c_1}{\omega}\right)\Im m\left(\underline{A}_{21}\underline{A}_{22}^{*}\right).$$

$$\alpha' = \alpha = \frac{9}{16}\Im m\left(\underline{A}_{11}\underline{A}_{21}^{*}\right)\left[a_3^2 + \left(b_3\omega + \frac{c_3}{\omega^3}\right)^2\right],$$

$$\beta' = \frac{3}{2}\Im m\left(\underline{A}_{11}\underline{A}_{21}^{*}\right)\left[a_1 a_3 - \left(b_1\omega - \frac{c_1}{\omega}\right)\left(b_3\omega + \frac{c_3}{\omega^3}\right)\right] + \frac{3}{4}a_3\Im m\left(\underline{A}_{11}\underline{A}_{22}^{*} - \underline{A}_{12}^{*}\underline{A}_{21}\right) -$$
$$-\frac{3}{4}\left(b_3\omega + \frac{c_3}{\omega^3}\right)\left[\Re e\left(\underline{A}_{11}\underline{A}_{22}^{*} - \underline{A}_{12}^{*}\underline{A}_{21}\right) + 1\right],$$

$$\gamma' = \left[a_1^2 + \left(b_1\omega - \frac{c_1}{\omega}\right)^2\right]\Im m\left(\underline{A}_{11}\underline{A}_{21}^{*}\right) + a_1\Im m\left(\underline{A}_{11}\underline{A}_{22}^{*} - \underline{A}_{12}^{*}\underline{A}_{21}\right) +$$
$$+\left(b_1\omega - \frac{c_1}{\omega}\right)\left[\Re e\left(\underline{A}_{11}\underline{A}_{22}^{*} - \underline{A}_{12}\underline{A}_{21}^{*}\right) + 1\right] + \Im m\left(\underline{A}_{12}\underline{A}_{22}^{*}\right).$$

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