

INSTANTANEOUS COMPLEX APPARENT POWER THEORY AND APPARENT POWER DEFINITIONS – CASE

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Abstract - This paper suggests a new definition of apparent power based on the instantaneous power theory. The component corresponding to non-useful power and the quadratic relationship between apparent, active, reactive and non-useful powers are also highlighted. A few case-studies draw a parallel between the results according to the well-known definitions gathered from previous literature and the results based on the new expression. It is shown that, under sinusoidal and balanced conditions, all definitions of apparent power lead to the same results. However, in the case of sinusoidal unbalanced situations, the results based on the new definition are identical only with those corresponding to Buchholz's and Czarnecki's definitions. In contrast, under nonsinusoidal conditions, the suggested definition leads to higher values of the apparent power.

Keywords: Instantaneous Complex Apparent Power, Apparent Power, Power Factor.

1. INTRODUCTION

A large number of research publications and specialists have discussed and are still discussing issues related to the properties of the powers flow in three-phase loads operating under non-sinusoidal voltages and currents conditions [1]-[6]. The main phenomenon is the increasing of the apparent power of the power supply over the values corresponding to the active and reactive powers under sinusoidal conditions. The quantitative identification of this increase is very important due to the impact on the power factor in power distribution systems and electrical equipment. It is generally accepted that under non-sinusoidal conditions, along with the active power (P) and the reactive power (Q), another power – frequently named the distortion power (D) – is present. The quadratic relation between these powers and the apparent power (S) is broadly accepted too:

$$S^2 = P^2 + Q^2 + D^2.$$
 (1)

The definitions and the interpretations of the active and reactive powers are almost near unanimously accepted. Under these conditions, it is clear that defining the apparent power will determine the distortion power and vice versa. The active power has a clear physical signification and a well-argued mathematical definition, as the average instantaneous power over one cycle. In this respect, the question is whether defining the apparent power by a mathematical expression is relevant or not.

The phasor theory applied to the three-phase system proved to be a very useful tool in control applications and determined good practical results and important physical interpretations. Last but not least, applying the instantaneous complex apparent power theory to the active filters control proves its usability and its connection to the physical phenomena in three-phase systems [7]-[10].

This paper is not intended to starting a debate, but to be nothing but a point of view based on mathematical correctness.

2. THE THEORY OF APPARENT INSTANTANEOUS COMPLEX POWER

The space phasor of the supply voltages of the distortion load and distorted three-phased current, u and i, are defined in the following matrices [11]

$$\begin{bmatrix} u \\ u \\ u \\ q \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \cdot \begin{bmatrix} u_a \\ u_b \\ u_c \end{bmatrix}$$
(2)

$$\begin{bmatrix} i \end{bmatrix} = \frac{2}{3} \begin{bmatrix} A \end{bmatrix} \cdot \begin{bmatrix} i_b \\ i_c \end{bmatrix}.$$
(3)

$$\begin{bmatrix} A \end{bmatrix} = \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$$

The apparent instantaneous complex power is defined as

$$\underline{s} = p + jq = \frac{3}{2} \underline{u} \cdot \underline{i}^* =$$

$$= \frac{3}{2} \left[u_d i_d + u_q i_q + j \left(-u_d i_q + u_q i_d \right) \right]$$
(4)

The direct and alternating components can be outlined in the real and imaginary parts (sometimes named instantaneous active and reactive powers [1]), p and q:

$$p = P + p \sim$$

$$q = Q + q \sim$$
(5)

P and Q are the average values resulting from

$$P = \frac{1}{T} \int_{t-T}^{t} p dt$$

$$Q = \frac{1}{T} \int_{t-T}^{t} q dt$$
(6)

It is evidently that

$$\int_{0}^{2\pi} p_{\sim} d(\omega t) = \int_{0}^{2\pi} q_{\sim} d(\omega t) = 0.$$
 (7)

3. NEW DEFINITIONS

In the square of the instantaneous complex power modulus, the components of the instantaneous powers can be separated by

$$\frac{|\underline{s}|^{2}}{P^{2} + Q^{2} + q^{2}} = (P + p_{\sim})^{2} + (Q + q_{\sim})^{2} =$$

$$P^{2} + Q^{2} + p_{\sim}^{2} + q_{\sim}^{2} + 2(Pp_{\sim} + Qq_{\sim})$$
(8)

The root mean square values are calculated from relation (8)

$$\frac{1}{2\pi} \int_{0}^{2\pi} \left| \underline{s} \right|^{2} d(\omega t) = \frac{1}{2\pi} \int_{0}^{2\pi} P^{2} d(\omega t) + \frac{1}{2\pi} \int_{0}^{2\pi} Q^{2} d(\omega t) + \frac{1}{$$

In the relation (9), the square of active and reactive powers P and Q can be identified. In these conditions, comparing with relation (1), the relation (9) suggests a new definition for apparent power: the root mean value of instantaneous complex power modulus

$$S = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} \left|\underline{s}\right|^2 d(\omega t)} .$$
 (10)

Thus, the following relation can be obtained

$$S^{2} = P^{2} + Q^{2} + \frac{1}{2\pi} \int_{0}^{2\pi} \left(p_{\sim}^{2} + q_{\sim}^{2} \right) d(\omega t) , \qquad (11)$$

because

$$\int_{0}^{2\pi} 2(Pp_{\sim} + Qq_{\sim})d(\omega t) = 0.$$

Comparing the relations (11) and (1), a new definition for distortion power can be given

$$D = \sqrt{\frac{1}{2\pi}} \int_{0}^{2\pi} \left(p_{\sim}^{2} + q_{\sim}^{2} \right) d(\omega t) .$$
 (12)

Also, the relations (4), (6) and (12) suggest defining of following complex powers:

- the instantaneous distortion complex power,

$$\underline{d} = p_{\sim} + jq_{\sim}; \tag{13}$$

- the average apparent complex power,

$$\underline{S}_{av} = P + jQ . \tag{14}$$

So, the apparent instantaneous complex power can be expressed that the sum of the two powers,

$$\underline{s} = \underline{S}_{av} + \underline{d} . \tag{15}$$

From relation (4), the instantaneous apparent complex power modulus is obtained

$$|\underline{s}| = \frac{3}{2} |\underline{u}| \bullet |\underline{i}| = \sqrt{(u_d^2 + u_q^2)} \bullet \sqrt{(i_d^2 + i_q^2)} \quad (16)$$

The relations (11), (12) and (13) show that the distortion power and instantaneous distortion power contain all non-useful powers (distortion power and the power because of unbalanced load).

4. CASE STUDIES

Now, there are four different definitions of the apparent power in literature. Thus, in the IEEE Standard Dictionary of Electrical and Electronics Terms there are two different definitions, respectively [12]:

$$S_{A} = U_{R}I_{R} + U_{S}I_{S} + U_{T}I_{T}; \qquad (17)$$

$$S_G = \sqrt{P^2 + Q^2} . \tag{18}$$

There is a third definition, introduced by Buchholz [13],

$$S_B = \sqrt{U_R^2 + U_S^2 + U_T^2} \sqrt{I_R^2 + I_S^2 + I_T^2} .$$
(19)

The one of most consistent researchers in this area is professor Leszek S. Czarnecki from Electrical and Computer Engineering Department of Louisiana State University, Baton Rouge, USA, which developed the Currents' Physical Components (CPC) theory [13], [14]. He proposed generalizing relation (19) for nonsinusoidal conditions

$$S_{C} = \|u\|\|i\| = \sqrt{\|u_{R}^{2}\| + \|u_{S}^{2}\| + \|u_{T}^{2}\|}\sqrt{\|i_{R}^{2}\| + \|i_{S}^{2}\| + \|i_{T}^{2}\|}, (20)$$

where,

$$\left\| u_{R,S,T}^{2} \right\| = \sum_{k=I}^{\infty} U_{R,S,T(k)}^{2} .$$
 (21)

are the effective values of "k" order harmonics on each phase.

It is evidently that relation (18) can not be applied under nonsinusoidal conditions because the third term under root is missed. Also, the relation (20) is another form of relation (19).

We will compare the results obtained by relations (17) and (20) with the results obtained by the new relation (10).

Under sinusoidal conditions and balanced load all three relations became identically.

4.1. Sinusoidal voltage and nonsinusoidal and balanced currents

In this case, the currents don't have a direct component and will contain N harmonics, and the space phasor components will be

$$i_{d} = \sqrt{2} \sum_{k=1}^{N} I_{k} \sin(k\omega t + \beta_{k})$$

$$i_{q} = -\sqrt{2} \sum_{k=1}^{N} I_{k} \cos(k\omega t + \beta_{k})$$
(22)

Using relation (15) and (17), the parts of instantaneous complex apparent power are obtained

$$p = 3UI_1 \cos(\beta_1 - \alpha) + 3U \sum_{k=2}^N I_k \cos[(k-1)\omega t + \beta_k - \alpha]$$
$$q = -3UI_1 \sin(\beta_1 - \alpha) - 3U \sum_{k=2}^N I_k \sin[(k-1)\omega t + \beta_k - \alpha]$$

 α is the initial phase of voltage.

The square value of apparent power defined by relation (10) is

$$S^{2} = \frac{1}{2\pi} \int_{0}^{2\pi} |s|^{2} d(\omega t) =$$

$$9U^{2} \frac{1}{2\pi} \int_{0}^{2\pi} \left[\sum_{k=1}^{N} I_{k}^{2} + 2 \sum_{k=1}^{N-1} \sum_{i=k+1}^{N} I_{k} I_{i} \cos[(i-k)\omega t + \beta_{i} - \beta_{k}] \right] d(\omega t) =$$

$$= 9U^{2} \sum_{k=1}^{N} I_{k}^{2} = 9U^{2} I^{2}$$

In conclusion, for sinusoidal voltage and nonsinusoidal and balanced currents, the results obtained by relations (17), (20) and (10) are identically.

4.2. Sinusoidal voltage and linear and unbalanced load

We will refer to example given by professor Czarnecki in [15], respectively, a resistive load (R), connected as shown in Fig. 1, which is supplied from symmetrical source of a sinusoidal, positive voltage with $u_R = \sqrt{2}U \cos \omega t$, U = 120V. He found I=103,9A, SA = 24,9 kVA, SB = SC = 30,5 kVA and,

$$p = \sqrt{3}UI \left[1 + \cos 2(\omega_1 t + 30^0) \right];$$

$$q = -\sqrt{3}UI \sin 2(\omega_1 t + 30^0)$$
(23)



Fig. 1. Circuit with sinusoidal voltage and linear and unbalanced load

From relations (23) and (5) result $P = \sqrt{3}UI; Q = 0;$ $p_{\sim} = \sqrt{3}UI \cos 2(\omega_1 t + 30^0); q_{\sim} = -\sqrt{3}UI \sin 2(\omega_1 t + 30^0)$ We can calculate from relation (13): $|\underline{d}|^2 = 3U^2I^2 = D^2$ and

$$S = \sqrt{P^2 + Q^2 + D^2} = \sqrt{6}UI = 30,5 \, kVA$$

So, and in this case, relation (10) has given the same result as relations (17) and (20). We notice that the alternating components of the real and imaginary parts exist because the currents are unbalanced.

Consequently, in this case, the alternative components of real and imaginary parts of instantaneous complex apparent power are determined by unbalanced currents. It can show that if the alternative components are compensated by a three wire active parallel filter, because the currents on the all phases became sinusoidal.

4.3. Nonsinusoidal voltage and current

Let us consider a balanced three wire load in connection Y supplied by a nonsinusoidal and symmetrical voltage source. Each phase voltage contains $(6q \pm 1)$ order harmonics with amplitude $U_{m1}/(6q \pm)$, q=1..5 and Um1=100 V (Fig. 2). The phase load has $R = \sqrt{3/2} \Omega$; $L = 1/(\sqrt{2} \cdot 100\pi)H$.

So, the phase current is lower distorted and contains the same harmonics as voltage (Fig. 3).

We calculated all the powers and total power factor by each method. The active and reactive powers were P=9,26 kW, Q=5,55 kVAR. We can see the apparent power, the distortion power and total power factor, calculated by those three methods, in Table 1.

S	SB	S _F	D _A	D _B	D _F	λ_{A}	λ_{B}	λ_{f}
а 11,1	11,	11,3	2,5	2,5	3,3	0,8	0,8	0,8
	1	1	9	2	2	5	5	1

Table 1: The powers and total power factor under nonsinusoidal conditions



Let us precise that the values of powers from Table 1 are kVA and kVAD, respectively. In this case the apparent power calculated by relation (10) is upper and distorted power too.

Using relation (10) it can obtain,

$$S_{f}^{2} = 9U^{2}I^{2} +$$

$$18\sum_{j=1}^{N-1}\sum_{k=1}^{N-j}\sum_{i=1}^{N-j}U_{k}U_{k+j}I_{i}I_{i+j}\cos(\alpha_{k} - \alpha_{k+j} - \beta_{i} + \beta_{i+j})$$
(24)

The relation (24) shows that under nonsinusoidal voltage and current, the new definition of apparent power contains a term depended by voltage and current different harmonics and their phases (α_k and β_k).

4.4. Nonsinusoidal voltage and unbalanced load

Let us consider the same non-sinusoidal and symmetrical voltage source which supplied unbalanced load, as show in Fig. 4.

For R=3 Ω the currents on the R and S phases are strong distorted and opposite (Fig. 5).

In this case, the active and reactive powers have been P=5,43 kW and Q=0. The values obtained for apparent and distortion powers (in kVA and kVAD) are showed in Table 2.



Fig. 4. Three wire, nonsinusoidal voltage source and unbalanced load



Fig. 5. The wave shape of R and S phase currents

We can see that, if nonsinusoidal voltage and current and unbalanced load there are simultaneously, the apparent power values calculated by relations (10), (17) and (20) are different.

Because the active power are the same P=5,43 kW, it means that the total power factor depends on the selection of the apparent power definitions. Thus, it seems to be unclear what the true value of total power factor is. It is unclear as well, what the power rating of a compensator is needed for the power factor improvement to unity value.

S	S_B	S _F	D _A	D_B	$D_{\rm F}$	$\lambda_{\rm A}$	λ_{B}	λ_{f}
А	7,6	8,1	3,1	5,4	6,1	0,8	0,7	0,6
6,27	8	8	3	3	2	6	1	6

Table 2: The powers and total power factor under nonsinusoidal conditions and unbalanced load

So, three values for compensator power rating are obtained:

$$S_{CA} = 3,13VA; \quad S_{CB} = 5,43VA; \quad S_{Cf} = 6,12VA$$
.

The differences are great. So the power rating of a compensator calculated by apparent power given by relation (10) is upper with 95%, respectively with 12,7% than the powers rating of a compensator calculated by apparent power given by relations (17) and (20).

CONCLUSION

The active power P is average value of real part of instantaneous complex apparent power, this is unanimously accepted. The reactive power Q is average value of imaginary part of instantaneous complex apparent power, this is large accepted. In these conditions, look at relation (9), correctly from mathematical point of view, certainly. Because the fourth term is zero, the relation contains, in right part, two known terms (P^2 and Q^2) and one unknown term. So, is naturally to introduce relations (10) and (12). The mathematical correctness is not enough, but it is necessary, we think so. These relations represent new definitions for apparent power and distortion power.

We notice that apparent power is defined as the root mean square value of instantaneous complex apparent power modulus. The distortion power is defined similarly as the root mean square value of instantaneous complex distortion power modulus. From the case studies considered, we established:

- for sinusoidal voltage and balanced linear or nonlinear load, the apparent power introduced give the same value as relations (17) and (20);
- for sinusoidal voltage and unbalanced linear or nonlinear load, the apparent power introduced give the same value as relation (20) but upper than (17);
- for nonsinusoidal voltage and balanced or unbalanced linear or nonlinear load, the apparent power introduced give upper value than relations (17) and (20);
- the alternate components of real and imaginary parts of instantaneous complex apparent power, characterize all the supplementary powers (unbalanced load and distortion load).

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