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CONSIDERATIONS ON THE STABILITY OF NONLINEAR SYSTEMS. PART II: JUMP RESONANCE PHENOMENA

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***Abstract* – Jump resonance is one of the phenomena which characterize a category of nonlinear systems and whose analysis reveals specific behavioral particularities. The paper presents two graphical-analytical methods which enable us to determine the occurrence conditions of the jump resonance phenomenon in nonlinear feedback systems. The jump resonance is produced around the system's resonant frequency and consists of multiple values of the amplitude and phase of the nonlinear element's input signal (and also of the system's output signal) when the amplitude or frequency of the harmonic input signal varies continuously.**

Keywords: *jump resonance, resonant frequency, critical curve.*

1. THEORETICAL CONSIDERATIONS

We have shown that using the concept of incremental input describing function it is possible to estimate the stability of the forced oscillation of a perturbed nonlinear feedback system, depending on the relative position of the linear element's transfer locus and of the nonlinear element's inverse negative describing locus, [3].
Let us consider the nonlinear system in Fig.1.

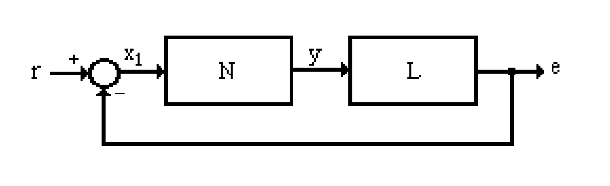

Figure 1: Nonlinear feedback system.

It was demonstrated, [4], that if the system is perturbed such that the nonlinear element's input signal contains a supplementary component of the same frequency as the harmonic input signal but of very small amplitude, and with a different phase, the characteristic equation of the autonomous system is:

$$1 + H(j\omega)N_i(A,\varphi) = 0 \qquad (1)$$

where, $N_i(A,\varphi)$ is the incremental input describing function and $H(j\omega)$ is the transfer locus of the nonlinear system, considered of the following form:

$$H(j\omega) = H_R(\omega) + jH_I(\omega) \qquad (2)$$

Let us note:

$$N_i(A,\omega) = N_{iR}(A,\varphi) + N_{iI}(A,\varphi) \qquad (3)$$

and therefore equation (1) becomes:

$$\begin{aligned} &1 + H_R(\omega)N_{iR}(A,\varphi) + H_I(\omega)N_{iI}(A,\varphi) + \\ &+ j\left[H_R(\omega)N_{iI}(A,\varphi)\right] + \\ &+ j\left[H_I(\omega)N_{iR}(A,\varphi)\right] = 0 \end{aligned} \qquad (4)$$

which is transformed in:

$$\begin{cases} 1 + H_R(\omega)N_{iR}(A,\varphi) + \\ + H_I(\omega)N_{iI}(A,\varphi) = 0 \\ \\ H_R(\omega)N_{iI}(A,\varphi) + H_I(\omega)N_{iR}(A,\varphi) = 0 \end{cases} \qquad (5)$$

We have shown, [4], that the real and imaginary parts of the incremental input describing function have the following expressions:

$$\begin{cases} N_{iR}(A,\varphi) = N(A) + \dfrac{AN'(A)}{2}(1+\cos 2\varphi) \\ N_{iI}(A,\varphi) = -\dfrac{AN'(A)}{2}\sin 2\varphi \end{cases} \qquad (6)$$

and thus equations (5) become:

$$\begin{cases} 1 + H_R(\omega)N_{iR}(A)\left[1+\cos 2\varphi\right] + \\ + H_I(\omega)N_{iI}(A)\sin 2\varphi = 0 \\ \\ H_I(\omega)N_{iR}(A)\left[1+\cos 2\varphi\right] - \\ - H_R(\omega)N_{iI}(A)\sin 2\varphi = 0 \end{cases} \qquad (7)$$

where we have designated:

$$N_{iR}(A) = N(A) + \frac{AN'(A)}{2}$$

$$N_{iI}(A) = \frac{AN'(A)}{2}$$

We eliminate the argument 2φ between the two equations (7) and after further processing we obtain:

$$\left[H_R(\omega)+h_R(A)\right]^2+\left[H_I(\omega)\right]^2=\left[r(A)\right]^2 \qquad (8)$$

which, for a constant value of amplitude $A=A_1$, represents the equation of a circle with the center in the point of coordinates $\left(-h_R(A_1,0\right)$ and radius $r(A_1)$, where we noted:

$$h_R(A)=\frac{2N(A)+AN'(A)}{N(A)+AN'(A)} \qquad (9)$$

$$r(A)=\frac{AN'(A)}{2N(A)\left[N(A)+AN'(A)\right]} \qquad (10)$$

The curve described by equation (8) is a critical curve of jump resonance, parameterized with the values of the amplitude A.

The relative position of the transfer locus of the system's linear part to the circle described by equation (8) enables us to estimate the stability of the nonlinear system, and thus of the occurrence of the jump resonance phenomenon as well.

If the transfer locus of the linear element does not intersect the jump resonance critical curve, this phenomenon does not take place, and the output oscillation is stable.

If equation (8) has solutions, therefore the transfer locus $H(j\omega)$ intersects o portion of the locus $-\frac{1}{N_i(A,\varphi)}$ corresponding to a constant value of the amplitude A, the jump resonance phenomenon does occur. As a result, the output harmonic oscillation is unstable and undergoes an abrupt variation (resonant jump) of amplitude and phase, having multiple values for the same frequency of the harmonic input signal.

Results equivalent to the ones provided by equation (8) can also be obtained in a different way. Let us consider the system with the structure in Fig.1 for which *N(A)* is the nonlinearity's describing function, considered a real function, and $H(s)$ the transfer function of the linear element, having a low pass filter characteristic. If the input signal is harmonical with amplitude R and frequency ω, the frequency response of the error is:

$$\frac{X}{R}(j\omega,A)=\frac{1}{1+H(j\omega)N(A)} \qquad (11)$$

The jump resonance phenomenon means that, for a constant value of the frequency ω (close to the resonant frequency), the amplitude and phase of the nonlinearity's input signal has multiple values for a continuous variation of the amplitude of the system's input signal. Thus the general condition for jump resonance is:

$$\frac{\delta R}{\delta A}<0, \quad \omega=const. \qquad (12)$$

in the resonant points the following conditions being fulfilled:

$$\frac{\delta R}{\delta A}=0, \quad \omega=const. \qquad (13)$$

and characterizes an S-shaped *A(R)* curve. In absolute values, equation (11) may be written:

$$\frac{A}{R}=\frac{1}{N(A)\sqrt{\left[H_R(\omega)+\frac{1}{N(A)}\right]^2+\left[H_I(\omega)\right]^2}} \qquad (14)$$

where $H_R(\omega), H_I(\omega)$ are the real and imaginary parts of the linear element's frequency response. Assuming the describing function *N(A)* is differentiable with respect to A, we impose condition (13) to equation (14), which, becomes, following differentiation:

$$\left[H_R(\omega)+\frac{1}{N(A)}\right]\left[H_R(\omega)+\frac{1}{N(A)+AN'(A)}\right]+ \\ +V^2(\omega)=0$$

or, in a compact form:

$$f(H_R,H_I,A)=0 \qquad (15)$$

It is clear that for a constant value of the amplitude A corresponding to the point on the *A(R)* characteristic in which the resonant jump takes place, in the complex plane (H_R, jH_I) equation (15) represents a family of curves whose envelope satisfies:

$$\begin{cases} f(H_R,H_I,A)=0 \\ \dfrac{\delta f(H_R,H_I,A)}{\delta A}=0 \end{cases} \qquad (16)$$

Imposing conditions (16), equation (15) becomes:

$$\begin{cases} H_R=-\dfrac{n_1(A)}{n(A)} \\ H_I=\pm\dfrac{n_2(A)n_3(A)}{n(A)} \end{cases} \qquad (17)$$

where we have noted:

$$n_1(A) = N'(A)\big(N(A) + AN'(A)\big) +$$

$$+ N(A)\big(2N'(A) + AN''(A)\big)$$

$$n_2(A) = \sqrt{N'(A)\big(N(A) + AN'(A)\big) - N(A)N'(A)}$$

$$n_3(A) = \sqrt{\begin{aligned}&\big((N(A) + AN'(A)\big)\big(2N(A) + N''(A)\big) - \\ &- N(A)\big(N(A) + AN'(A)\big)\end{aligned}}$$

$$n(A) = 2N'(A) + N''(A)$$

where $N'(A), N''(A)$ are the first and second derivatives of the function $N(A)$ with respect to A.

Equations (17), with the above notations, describe the jump resonance critical curve, which separates a surface in the complex plane in which jump resonance occurs only if the transfer locus of the linear element intersects it.

2. SIMULATION RESULTS

Let us consider that the transfer function of the linear element L is:

$$H(s) = \frac{k}{s(T_1 s + 1)(T_2 s + 1)}$$

and the system contains a cubic nonlinearity with the describing function:

$$N(A) = \frac{3}{4} b A^2$$

Representing in the complex plane the jump resonance critical curve provided by equation (8) for a constant value of the amplitude $A = A_1$:

$$\left[H_R(\omega) + \frac{8}{9{A_1}^2}\right]^2 + \left[H_I(\omega)\right]^2 = \left[\frac{4}{9{A_1}^2}\right]^2$$

and the transfer locus of the linear element corresponding to the transfer function *H(s)*:

$$H_R(\omega) = -\frac{k(T_1 + T_2)\omega^2}{(T_1 + T_2)^2 \omega^4 + (1 - T_1 T_2 \omega^2)^2}$$

$$H_I(\omega) = -\frac{k\omega(T_1 + T_2 - T_1 T_2 \omega^2)}{T_1 + T_2)^2 \omega^4 + (1 - T_1 T_2 \omega^2)^2}$$

We observe, Fig.2, that the two curves intersect, and the forced oscillation of frequency ω is unstable, the intersection points indicating the frequencies at which the resonant jumps occur.

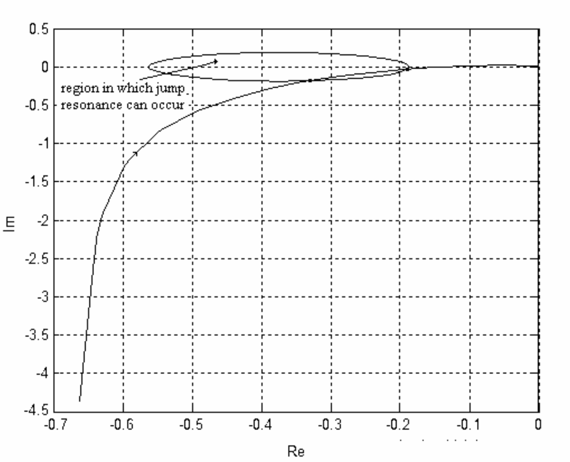

Figure 2: Illustration of the graphical-analytical method; A=0.5, b=1; k=1; T_1 = 0.4 sec., T_2=1.1 sec.

In the opposite situation, Fig.3, equation (1) is not fulfilled, and consequently the forced oscillation of the system's output is stable.

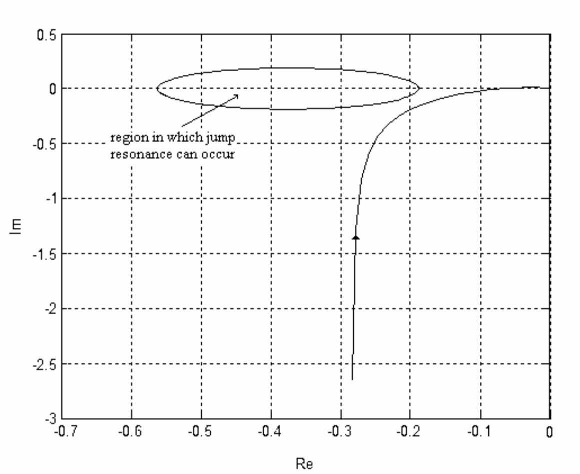

Figure 3: Illustration of the graphical-analytical method; A=0.5, b=1; k=1; T_1 = 0.4 sec., T_2=0.5 sec.

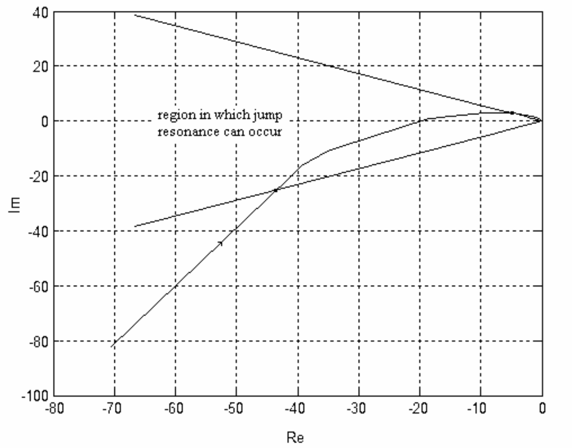

Figure 4: Critical jump resonance curve for cubic nonlinearity; A=0.5, b=1; k=1; T_1 = 0.4 sec., T_2=1.1 sec.

Similar conclusions are obtained if we use the method which implies fulfilling the conditions (16).

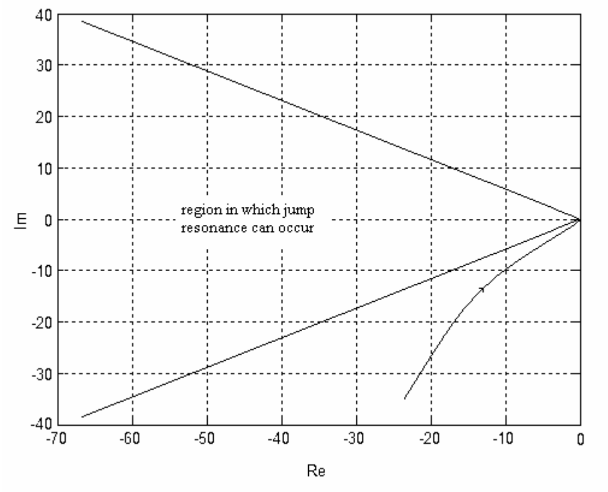

Figure 5: Critical jump resonance curve for cubic nonlinearity; A=0.5, b=1; k=1; T_1 =0.4 sec., T_2=0.5sec.

Thus, for the same system structure, we provided examples that if the transfer locus of the linear element passes through the zone delimited by the jump resonance critical curve resonant jumps do occur in the system, Fig.4, and if not, the forced oscillation of the system's output is stable, Fig.5.

3. CONCLUSIONS

We analyzed the behavior of a nonlinear automatic system form the point of view of the stability of the output forced oscillation, with examples for the occurrence conditions of the jump resonance phenomenon. For this, we presented two graphical-analytical methods to estimate if resonant jumps occur in the system. The theoretical considerations are confirmed by the results obtained through numerical simulation.

References

[1] S. Sastry, *Nonlinear Systems*, Springer-Verlag, New-York, 1999.

[2] J.Ch. Gille, P. Decaulne, M. Pélegrin, *Méthodes d'étude des systèmes asservis non linéaires*, Éditeur Dunod, Paris, 1964.

[3] A. Gelb, W.E. Vander Velde, *Multiple-Input Describing Functions and Nonlinear Systems Design*, McGraw-Hill Book Company, New-York, 1968.

[4] M. Cleju, *Considerations on the Stability of Nonlinear Systems. Part. I: Incremental Input Describing Function*, 6th International Conference on Electromechanical and Power Systems, Chişinău, R.Moldova, 2007.