TRANSITORY REGIMES ANALYSIS METHODS FOR ELECTRICAL TRANSFORMERS

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Abstract **– Electrical transformers transitory regimes analysis importance is enforced by the reducing cost strategy demanded by the manufacturers. Thus, knowing the areas that are most affected by these regimes, supplementary protection measures may be taken only for these areas, leading to an important decrease of material consumption. This paper makes an overview of a developed simulation platform, created in the MATLAB-SIMULINK environment, for modeling of an electrical transformer. Based on this simulation platform, a transitory regime study for the energizing and short-circuit processes may be performed during the early stages of the transformer's development.**

Keywords: *electrical transformer, transitory regime, transformer model.*

1. ELECTRICAL TRANSFORMER THEORY

The electrical transformer theory is treated from two points of view: the physical theory and the technical theory.

The first one neglects the magnetic core saturation. Thus the inductances have constant values, regardless of windings electrical currents, and core losses are neglected [1]. In this case the dependency between the fascicular magnetic flux and the magneto-motive-force (m.m.f.) is linear and the transformers equations use self and mutual inductances.

The second one (technical theory) takes into account the magnetic core saturation. The magnetizing inductances depend on the windings electrical currents and core losses are no longer neglected [1]. Nevertheless if we consider that the magnetic core is made of superior quality sheets (characterized by reduced magnetic losses) then the core losses may be neglected. The dependency between the fascicular magnetic flux and the m.m.f. is non-linear and the transformers equations use magnetizing and leakage inductances.

2. THE INSTANTANEOUS ELECTRICAL TRANSFORMER EQUATIONS

The voltage equations for the primary and secondary circuits of the electrical transformer, based on the technical theory, are [1], [2]:

$$\begin{cases} u_1 = R_1 \cdot i_1 + L_{\sigma 1}\dfrac{di_1}{dt} + w_1\dfrac{d\varphi}{dt} \\ -u_2 = R_2 \cdot i_2 + L_{\sigma 2}\dfrac{di_2}{dt} + w_2\dfrac{d\varphi}{dt} \end{cases} \tag{1}$$

$$\theta = w_1 \cdot i_1 - w_2 \cdot i_2 = w_1 \cdot i_{10} \tag{2}$$

$$\varphi = f(\theta) \text{ or } \varphi = f(i_{10}) \tag{3}$$

$$u_2 = R \cdot i_2 + L\frac{di_2}{dt} + \frac{1}{C_d}\int i_2 \cdot dt \tag{4}$$

where:
R_1, R_2 – primary/secondary winding resistance,
$\Phi_1 = L_{\sigma 1}\dfrac{di_1}{dt} + w_1\dfrac{d\varphi}{dt}$, $\Phi_2 = L_{\sigma 2}\dfrac{di_2}{dt} + w_2\dfrac{d\varphi}{dt}$ – primary/secondary circuit total flux,
φ – magnetizing fascicular flux,
$L_{\sigma 1}$, $L_{\sigma 2}$ – primary/secondary leakage inductance,
i_{10} – no-load current,
R, L and C_d – load resistance, inductance and capacitance (series RLC – see figure 1).

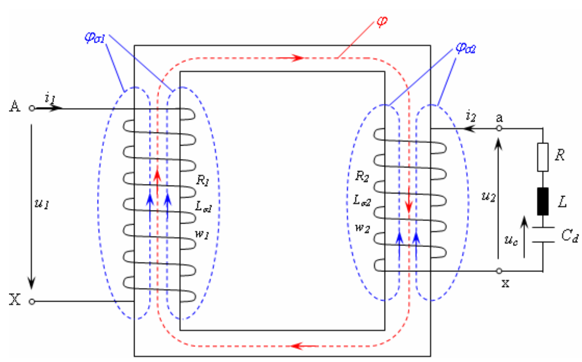


Figure 1. Electrical transformer sketch.

If we neglect the core losses then (3) is the initial magnetization curve of the magnetic core's sheets.

The system's ((1) – (4)) unknowns are: i_1, i_2, u_2, θ (i_{10}) and φ. Because of the core's non-linear characteristic (3) this system is also nonlinear.

3. THE ELECTRICAL TRANSFORMER'S MATLAB-SIMULINK MODEL

In order to be able to implement the system ((1) – (4)) in the MATLAB-SIMULINK environment the non-linear characteristic (3) has to be approximated with an analytical expression.

Due to the simplicity of the calculations but also of the proper approximation the following expression [3] was chosen for the non-linear characteristic (3):

$$\varphi = i_{10} / \left(a_1 + a_2 \cdot |i_{10}| \right) \tag{5}$$

where a_1 and a_2 are analytically obtained by imposing specific coordinates (φ, i_{10}) to the characteristic.

Starting from the above mentioned system ((1) – (4), also considering (5)) the mathematical model based on the primary i_1 and the secondary i_2 currents was obtained:

$$\begin{cases} \dfrac{di_1}{dt} = \dfrac{D + E \cdot B}{1 - E \cdot C} \\ \dfrac{di_2}{dt} = \dfrac{B + C \cdot D}{1 - E \cdot C} \\ \dfrac{du_c}{dt} = \dfrac{1}{C_d} \cdot i_2 \end{cases} \tag{6}$$

where:

$$A = \left(\frac{a_1 \cdot w_1^2}{\left(a_1 \cdot w_1 + a_2 \cdot |w_1 \cdot i_1 - w_2 \cdot i_2| \right)^2} \right) \tag{7}$$

$$\begin{cases} B = \dfrac{(-u_1 + R_1 \cdot i_1)}{w_2 \cdot A};\ C = \left(\dfrac{L_{\sigma 1}}{w_2 \cdot A} + \dfrac{w_1}{w_2} \right); \\ D = \dfrac{(-u_c - (R + R_2) \cdot i_2)}{w_2 \cdot A};\ E = \left(\dfrac{w_2}{w_1} - \dfrac{L + L_{\sigma 2}}{w_2 \cdot A} \right); \end{cases} \tag{8}$$

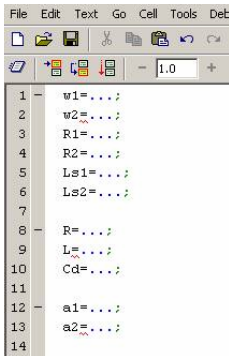
$u_c = \dfrac{1}{C_d} \int i_2 \cdot dt$ – the voltage over the load's capacitor.

This model was implemented into the MATLAB-SIMULINK environment. It has as input signal the supplying voltage. The transformer's parameters w_1, w_2, R_1, R_2, $L_{\sigma 1}$, $L_{\sigma 2}$, the load's parameters R, L, C_d and a_1, a_2 constants are known. The output signals that coincide with the model's state-variables are: the primary i_1 and the secondary i_2 currents and the voltage over the load's capacitor u_c.

Figure 2 shows the *trafo_param_fractie.m* file where the user defines the MATLAB-SIMULINK model's parameter values.

Figure 3 shows the S-function [4] file (*trafo_sfunc_fractie.m*) which is the MATLAB transcription of (6), (7) and (8).

Figure 4 shows the electrical transformer's SIMULINK diagram. The *Transformer* block comprises the S-function file depicted in figure 3. This model was used for the study of the transitory energizing and short-circuit regimes.

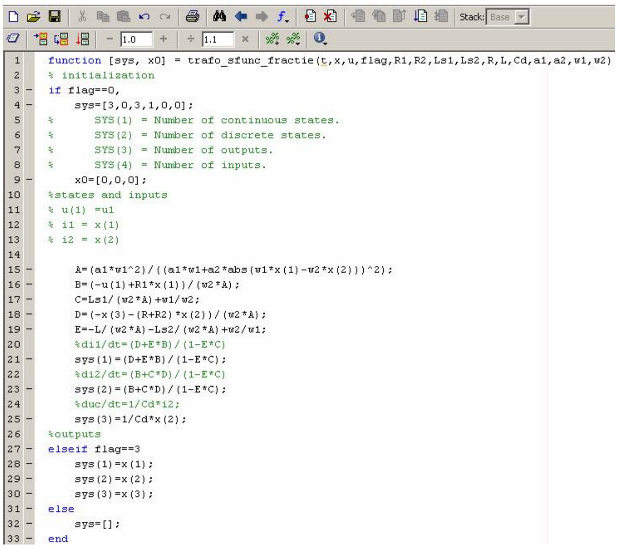


```
w1=...;
w2=...;
R1=...;
R2=...;
Ls1=...;
Ls2=...;

R=...;
L=...;
Cd=...;

a1=...;
a2=...;
```

Figure 2. The *trafo_param_fractie.m* file.



```
function [sys, x0] = trafo_sfunc_fractie(t,x,u,flag,R1,R2,Ls1,Ls2,R,L,Cd,a1,a2,w1,w2)
% initialization
if flag==0,
    sys=[3,0,3,1,0,0];
%       SYS(1) = Number of continuous states.
%       SYS(2) = Number of discrete states.
%       SYS(3) = Number of outputs.
%       SYS(4) = Number of inputs.
    x0=[0,0,0];
%states and inputs
% u(1) =u1
% i1 = x(1)
% i2 = x(2)

    A=(a1*w1^2)/((a1*w1+a2*abs(w1*x(1)-w2*x(2)))^2);
    B=(-u(1)+R1*x(1))/(w2*A);
    C=Ls1/(w2*A)+w1/w2;
    D=(-x(3)-(R+R2)*x(2))/(w2*A);
    E=-L/(w2*A)-Ls2/(w2*A)+w2/w1;
    %di1/dt=(D+E*B)/(1-E*C)
    sys(1)=(D+E*B)/(1-E*C);
    %di2/dt=(B+C*D)/(1-E*C)
    sys(2)=(B+C*D)/(1-E*C);
    %duc/dt=1/Cd*i2;
    sys(3)=1/Cd*x(2);
%outputs
elseif flag==3
    sys(1)=x(1);
    sys(2)=x(2);
    sys(3)=x(3);
else
    sys=[];
end
```

Figure 3. The S-function file (*trafo_sfunc_fractie.m*).

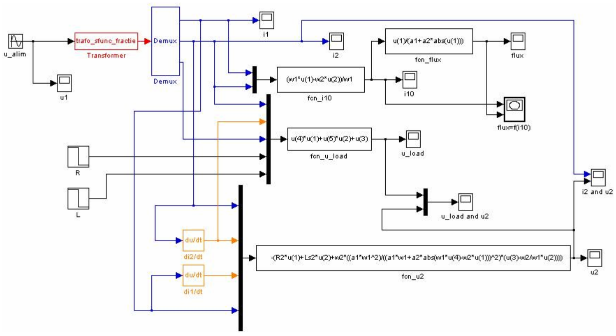


Figure 4. The electrical transformer's SIMULINK diagram.

4. THE ELECTRICAL TRANSFORMER'S ENERGISATION REGIME

Figure 5 depicts the core's dimensions [5] (expressed in meters) of the transformer that was used for the transitory energizing process study. This transformer is characterized by the following rated data [5]:

S_n = 5 kVA;
U_{1n}/U_{2n} = 115/230 V/V;
f_n = 60 Hz;
w_1 = 18 turns;
w_2 = 36 turns;
R_1 = 0.38 mΩ
R_2 = 1.5 mΩ
$L_{\sigma 1}$ = 0.06 mH;
$L_{\sigma 2}$ = 0.24 mH.

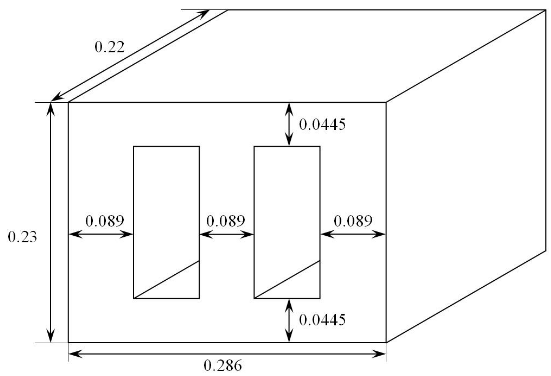


Figure 5. The transformer's core dimensions (expressed in meters).

The initial magnetization curve $\varphi = f(i_{10})$ of the magnetic core's sheets is depicted in figure 6 [5].

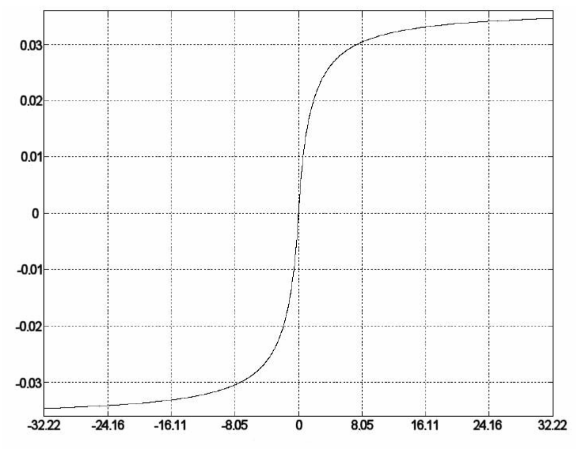


Figure 6. The initial magnetization curve $\varphi = f(i_{10})$ of the magnetic core's sheets.

When using (5) to approximate the above $\varphi = f(i_{10})$ magnetization curve (figure 6) the following values are obtained:
a_1 = 85.4721;
a_2 = 27.55097;

Figure 7 shows the (5') characteristic.
The no-load coupling transitory regime was obtained by applying the rated voltage to the primary winding while the secondary winding circuit was opened. The considered load's parameters were: $R = 10^8$ Ω, $L = 10^7$ H and $C_d = 10^{-10}$ F. The full-load coupling transitory regime (at rated primary voltage) was also analyzed. In this case the considered load's parameters were: R = 10 Ω, L = 0.0185 H and $C_d = 6.63 \cdot 10^{-4}$ F.

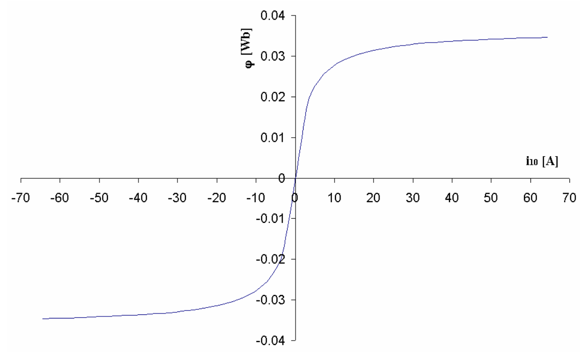


Figure 7. The estimated magnetization curve of the magnetic core's sheets.

The transformer's feeding voltage was supplied (in all the analyzed cases) from a voltage source characterized by the following parameters: R_s = 115 mΩ, L_s = 0.7 mH. The source's impedance was considered in series with the transformer's primary impedance. This leaded to an equivalent impedance ($Z_e = R_e + j \cdot L_e$) that was considered to be the primary winding impedance in the MATLAB-SIMULINK model.

Case a – The supplying voltage phase influence over the primary inrush current for the no-load coupling transitory regime
The table below shows the relation between the primary no-load inrush current amplitude ($\hat{i}_{10}$) and the supplying voltage's coupling phase (γ). The feeding voltage amplitude was considered constant ($\hat{u}_1 = 115 \cdot \sqrt{2}$ V). The magnetization curve that was used is the one shown in figure 7.
In accordance with table 1 the inrush current amplitude reaches its maximum value when the supplying voltage's coupling phase is null ($u_1 = U \cdot \sqrt{2} \cdot \sin(\omega \cdot t + 0)$).

$\hat{i}_{10}$	A	221.75	216.06	198.45	169.87	129.55	84.35	41.78	18.84	9.79	5.76
γ	°	0	10	20	30	40	50	60	70	80	90

Table 1. The $\hat{i}_{10} = f(\gamma)$ dependence.

Figure 8 shows the primary no-load current time variation (first ten periods) considering that the coupling phase of the rated supplying voltage is null ($u_1 = 115 \cdot \sqrt{2} \cdot \sin(\omega \cdot t + 0)$). The no-load current amplitude reaches its maximum value after the first half-period of time.

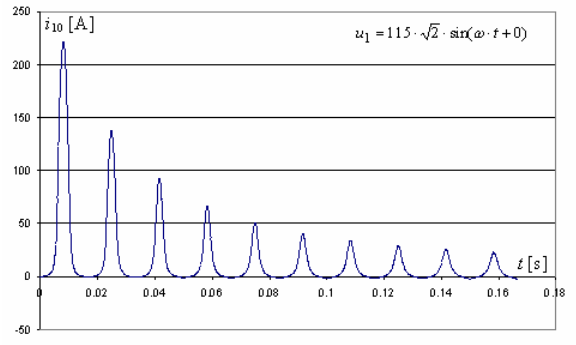


Figure 8. The no-load current time variation.

Case b – The magnetization curve flatness influence over the primary inrush current for the no-load coupling transitory regime

In this case a comparison has been performed between the maximum values of the no-load primary current obtained when using the magnetization curves presented in figures 7, 9 (the flat curve - $\varphi = i_{10} / (60 + 32 \cdot |i_{10}|)$) and 10 (the less flatness curve - $\varphi = i_{10} / (120 + 23 \cdot |i_{10}|)$).

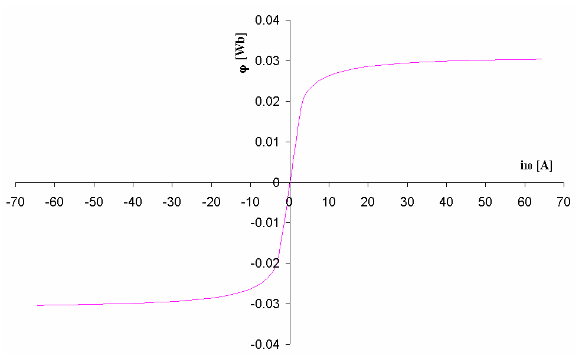


Figure 9. The flat magnetization curve.

Figure 11 shows the no-load primary current time variation (first ten periods) considering the coupling phase of the rated supplying voltage to be null ($u_1 = 115 \cdot \sqrt{2} \cdot \sin(\omega \cdot t + 0)$) and the magnetization curve presented in figure 9.

Figure 12 is similar to figure 11 but in this case the considered magnetization curve is the one presented in figure 10.

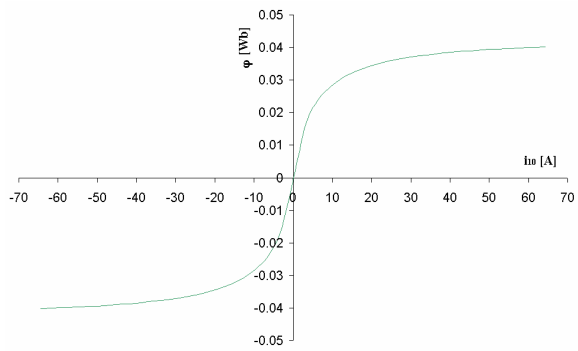


Figure 10. The less flatness magnetization curve.

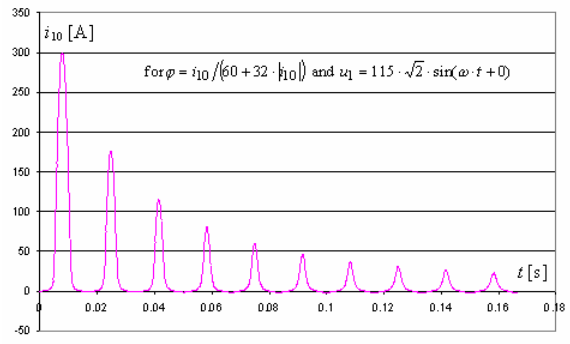


Figure 11. The no-load current time variation based on the magnetization curve in figure 9.

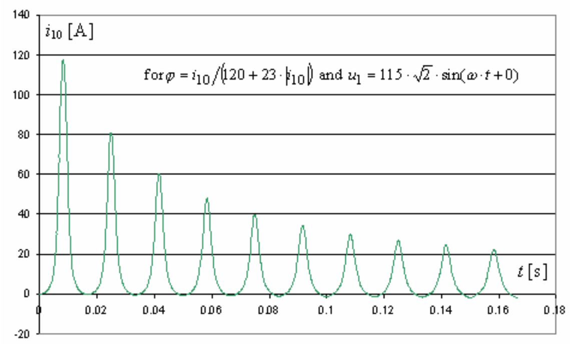


Figure 12. The no-load current time variation based on the magnetization curve in figure 10.

Analyzing figures 8, 11 and 12 one may observe that the no-load current amplitude reaches its maximum value after the first half-period of time when the used magnetization curve is the one depicted in figure 9.

Case c – The load's value influence over the primary inrush current for the load coupling transitory regime

This time the transformer is considered to be fully loaded.

Figure 13 shows the primary current time variation (first ten periods) considering that the coupling phase of the rated supplying voltage is null ($u_1 = 115 \cdot \sqrt{2} \cdot \sin(\omega \cdot t + 0)$) and the load's parameters are R = 10 Ω, L = 0.0185 H and C_d = 6.63·10^{-4} F.

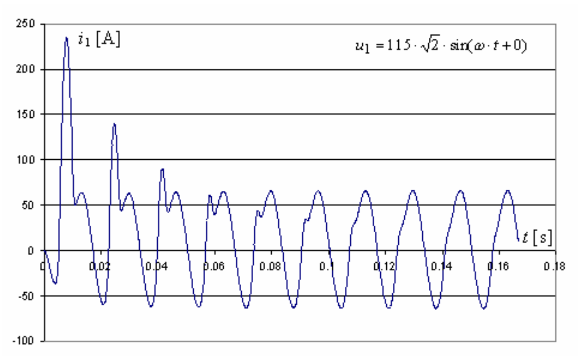


Figure 13. The primary current time variation for the fully loaded transformer.

One may see that the primary current amplitude also reaches its maximum value after the first half-period of time. The difference between this value and the one obtained for the no-load coupling transitory regime (case a) is less than 6 %.

5. THE ELECTRICAL TRANSFORMER'S SHORT-CIRCUIT REGIME

For the study of this regime the MATLAB model described in section 3 was modified as follows:

- the transformer's load is series RL, therefore the D constant (from (8)) does not contain anymore the u_c voltage and only the first two equations from (6) are used;
- the transformer's feeding voltage was supplied from an infinite power grid (R_s = 0 Ω, L_s = 0 H), therefore the considered transformer's primary impedance is given only by the winding's parameters.

The used magnetization curve is the one shown in figure 7 (defined by (5')).

The supplying voltage phase (γ) influence over the primary inrush current for a sudden secondary winding short-circuit was studied.

Before the short-circuit took place the transformer was operating under load (the load's parameters are R = 10 Ω and L = 0.0185 H) being supplied by rated voltage $u_1 = 115 \cdot \sqrt{2} \cdot \sin(\omega \cdot t + \gamma)$. Therefore the analyzed case is that of a rated voltage sudden short-circuit.

The short-circuit is modeled by a sudden change of the load's parameters (the new values of the load's parameters are R = 6.7·10^{-2} Ω and L = 10^{-3} H).

Table 2 reflects the relation between the primary inrush current amplitude ($\hat{i}_{1k}$) and the rated supplying voltage phase (γ) for a sudden secondary winding short-circuit.

$\hat{i}_{1k}$	A	2648.34	2636.32	2594.15	2518.08	2414.08	2270.36	2109.3	1918.29	1715.42	1685.84
γ	°	0	10	20	30	40	50	60	70	80	90

Table 2. The $\hat{i}_{1k} = f(\gamma)$ dependence.

In accordance with table 2 the primary inrush current amplitude for a sudden secondary winding short-circuit reaches its maximum value ($\hat{i}_{1k\_m}$) when the supplying voltage phase is null ($u_1 = U \cdot \sqrt{2} \cdot \sin(\omega \cdot t + 0)$).

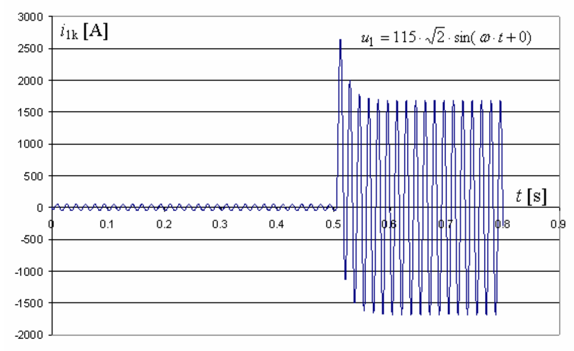


Figure 14. The primary current time variation for a sudden secondary winding short-circuit.

Figure 14 shows the primary current (i_{1k}) time variation for a sudden secondary winding short-circuit considering that the phase of the rated supplying voltage is null ($u_1 = 115 \cdot \sqrt{2} \cdot \sin(\omega \cdot t + 0)$).

The primary current amplitude reaches its maximum value at about half-period of time from the short-circuit moment.

This value is very close to the maximum theoretical current value given by:

$$(i_{1k})_{\max} = \frac{100 \cdot I_{1n} \cdot \sqrt{2} \cdot \left(1 + e^{\frac{-\pi \cdot R_k}{2 \cdot \pi \cdot f \cdot L_k}}\right)}{\sqrt{R_k^2 + (2 \cdot \pi \cdot f \cdot L_k)^2}} = 2648.59 \text{ A} \quad (15)$$

where:

I_{1n} = 43,48 A – the transformer's effective rated current value,

$$R_k = R_1 + R_2 \cdot \left(\frac{w_1}{w_2}\right)^2 = 0.755\ \mathrm{m}\Omega,$$

$$L_k = L_{\sigma 1} + L_{\sigma 2} \cdot \left(\frac{w_1}{w_2}\right)^2 = 0.12\ \mathrm{mH}$$ – the transformer's short-circuit parameters.

6. CONCLUSIONS

An overview of an electrical transformer developed simulation model, created in the MATLAB-SIMULINK environment, is made. Based on this simulation model, a transitory regime study for the no-load and short-circuit processes may be performed during the early stages of the transformer's development.

The main contribution of this work has been the development of the S-function file (as a transcription of (6), (7) and (8) – see figure 3), which is the MATLAB mathematical model based on the primary i_1 and the secondary i_2 currents, and of the electrical transformer's SIMULINK diagram (see figure 4).

In accordance with table 1 and figure 8 the primary no-load current amplitude reaches its maximum value after the first half-period of time when the supplying voltage's coupling phase is null.
In the case of three-phase transformers there is always a phase for which the supplying voltage's coupling phase is approximately null. Therefore on that phase the primary no-load current amplitude reaches its maximum value while on the other two phases the currents have much smaller values.
Figures 8, 11 and 12 show that the no-load current amplitude reaches its maximum value after the first half-period of time when the used magnetization curve is the one depicted in figure 9. This fact proves that a transformer with a flat magnetization curve saturates faster, and so, the no-load transformer's current reaches a higher value.

For the case of the fully loaded transformer one may see that the primary current amplitude also reaches its maximum value after the first half-period of time (see figure 13). As already mentioned the difference between this value and the one obtained for the no-load coupling transitory regime (case a) is less than 6 %. This means that the primary current amplitude reaches its maximum value after the first half-period of time, when the coupling phase of the supplying voltage is null, and has approximately the same value regardless of the transformer's load.

As it has already been shown (in section 5) the electrical transformer's short-circuit regime leads to high current values through its windings. These currents generate electro-dynamical forces that act over the transformer's windings. These forces lean upon the maximum current amplitude and so they have very high values. By reason of this the transformer's windings have to be able to stand this kind of forces, and so, a transitory regime study for the short-circuit processes must be performed during the early stages of the transformer's development.

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