FUEL FLOW RATE CONTROLLER WITH RESPECT TO THE COMPRESSOR'S PRESSURE RATIO

Alexandru Nicolae TUDOSIE

University of Craiova, Faculty of Electrical Engineering, Avionics Department,
E-mail: atudosie@elth.ucv.ro

***Abstract* – The paper deals with a controller for the transient aircraft turbo-engines' regimes (acceleration/deceleration, atmospheric disturbances, unstable burning), which is meant to assure the engine's over-speed and over-heating avoiding during its operating. One has elaborate the non-linear mathematical model, based on the motion equations for each system's part; using a linearization method, one has also obtained the linear and the non-dimensional equation system form, as well as a simplified mathematical model form, the transfer function and some functional block diagrams (block diagrams with transfer functions), which are useful for the controller's studying and pre-design.**

Keywords: *controller, turbo-engine, over-heating, over-speed, transient, fuel, air flow rate.*

1. INTRODUCTION

The engine's dynamic regimes controllers are based on the fuel flow rate's control during the transient operating regimes (acceleration/deceleration), in order to ensure for the engine's combustor the co-relation between the fuel injection and the compressor's air delivery. The fuel injection velocity is higher then the air's one, because of the engine's turbo-compressor's spool inertia, so the dynamic regime's control must be based on the fuel flow rate's control with respect to one of the spool's characteristic parameters (as the rotation speed, the air flow rate or the compressor's pressure ratio are), in order to assure the engine's over-speed and over-heating avoiding during its operating [3,6,8].

Such a controller, with respect to the compressor's pressure ratio, is presented in figure 1[5,6]. The controller's action above the engine's operating is an indirect one, because of its using as fuel flow rate limiter, acting on the fuel pump's controller, not on the injectors or on the injection dosing element(s). Its particularity is the presence of a barometric correction system, which assures an appropriate fuel flow rate control for any flight regime, in a very large range of flight altitudes and speeds. The controller's parts are: 1-main slide-valve; 2-elastic membrane; 3-main spring; 4-profiled needle; 5-drossel; 6-adjustment block's elastic membrane; 7-adjustment block's spring; 8-barometric correction block's spring; 9-barometric correction block's lever; 10- barometric correction block's rod; 11-sylphon block; 12-drossel; 13- barometric correction block's adjustment screw; 14-controller's case; 15-pressure intakes; 16-spring's elastic adjustment screw; 17-actuactor's connection.

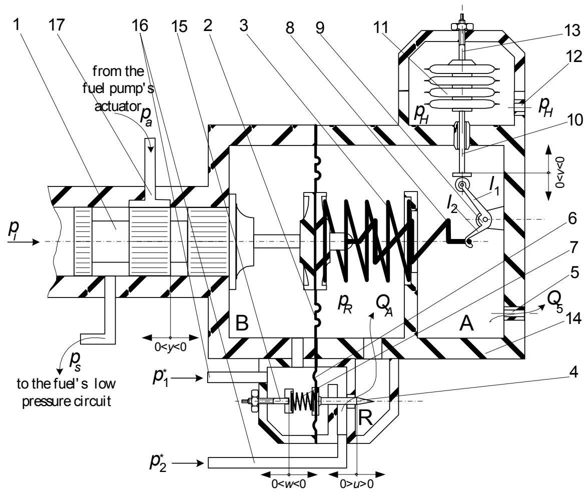

Figure 1. Fuel flow rate controller with indirect action, with respect to the compressor's pressure ratio

The above described controller can be integrated into a rotation speed control system, as the ones in [4] and [7], acting by discharging the actuator's active chambers (through the 17 connector, by the main slide-valve 3), under the combined pressure forces ($p_A - p_B$, injection pressure p_i), respectively 7 and 8 springs' elastic forces action. The pressures in A and B chambers are depending on the compressor's intake and exhaust pressure, that means on its pressure ratio, so the slide-valve's positioning is with respect to this parameter, as well as the actuator's active chamber's discharging, so the fuel injection velocity "follows" the air flow modifying velocity. The "corrected" (reference) pressure p_A is realized by the R-chamber system (membrane 6, spring 7, profiled needle 8 and adjustment screw 16).

The barometric corrector is meant to adjust the fuel/air flow rate co-ordination with respect to the flight regime (flight altitude) that means the engine's time response at different flight regimes (altitudes and speeds).

2. SYSTEM'S MATHEMATICAL MODEL

2.1. Non-linear equation system

The non-linear mathematical model is built of the motion equation of the controller's main parts, as follows:

1) *profiled needle's positioning equation*:

$$S_6\left(p_2^* - p_1^*\right) = m_4 \frac{\mathrm{d}^2 u}{\mathrm{d}t^2} + \xi \frac{\mathrm{d}u}{\mathrm{d}t} + k_7(u+w), \quad (1)$$

where S_6 is the membrane's surface area, p_1^*, p_2^* – pressure before/after the compressor, m_4 - needle 4 mass, ξ – viscous friction co-efficient, k_7 – spring's (7) elastic constant, u – needle's displacement, w – adjustment (screw's displacement);

2) A *pressure chamber's equation*:

$$Q_A = \mu_A \frac{\pi(d_A - d)^2}{4}\sqrt{\frac{2}{\rho}}\sqrt{p_2^* - p_A}\,, \quad (2)$$

$$d = d(u)\,, \quad (3)$$

$$Q_5 = \mu_5 \frac{\pi d_5^2}{4}\sqrt{\frac{2}{\rho}}\sqrt{p_A}\,, \quad (4)$$

$$Q_A - Q_5 = \frac{\mathrm{d}V_\mathrm{A}}{\mathrm{d}t} + \beta V_A \frac{\mathrm{d}p_A}{\mathrm{d}t} \quad (5)$$

$$V_A = V_{A0} - \frac{1}{3} S_1 (y_r - y_{r0})\,, \quad (6)$$

where d is the variable (with respect to the u needle's displacement) diameter of the connection between the two pressure chambers R and A; μ_A, μ_5 – flow rate co-efficient; d_A, d_5 – drossel's diameters; ρ – fuel's density; β – fuel's isothermic compressibility co-efficient; p_A -A chamber's pressure; V_A, V_{A0} – A chamber's current / initial volume; S_1 – membrane's (1) surface; y_r – membrane's center's displacement;

3) *slide-valve's* (3) *displacement:*

$$S_3 p_i - S_1 (p_A - p_1^*) = m_3 \frac{\mathrm{d}^2 y_r}{\mathrm{d}t^2} + \xi \frac{\mathrm{d}y_r}{\mathrm{d}t} + k_2 y_r\,, \quad (7)$$

where S_3 – slide-valve's frontal area's surface; p_i - fuel's injection pressure; m_3 – slide-valve's mass; k_7 – spring's (7) elastic constant;

4) *barometric corrector's equations:*

$$v = v(p_m - p_H), \quad (8)$$

$$v_1 = \frac{l_1}{l_2} v\,, \quad (9)$$

where v -corrector's rod displacement, p_m – sylphon's internal pressure; p_H – atmospheric pressure at the H flight altitude; l_1, l_2 – lever's arms' length; v_1 - spring extremity's displacement;

5) *corrected slide-valve's motion equation:*

$$S_3 p_i - S_1 (p_A - p_1^*) = m_3 \frac{\mathrm{d}^2 y_r}{\mathrm{d}t^2} + \xi \frac{\mathrm{d}y_r}{\mathrm{d}t} + k_2 y_r + $$
$$+ k_8 (y_r + v_1)\,, \quad (11)$$

All of these equations represent the controller's non-linear mathematical model.

2.2. Linearised mathematical model

The non-linear equation system can be linearised, using the small disturbances hypothesis, close to a completely determined steady state operating regime. One assumes that a generic variable X has the form

$$X = X_0 + \frac{\Delta X}{1!} + \frac{(\Delta X)^2}{2!} + \ldots + \frac{(\Delta X)^2}{n!}, \quad (12)$$

where X_0 is the steady state value and ΔX is the deviation; neglecting the terms which contains $(\Delta X)^n, n \geq 2$, it remains

$$X = X_0 + \Delta X\,. \quad (13)$$

So, expressing each variable of the non-linear system as in Eq. (13), one obtains

$$S_6\left(\Delta p_2^* - \Delta p_1^*\right) = m_4 \frac{\mathrm{d}^2(\Delta u)}{\mathrm{d}t^2} + \xi \frac{\mathrm{d}(\Delta u)}{\mathrm{d}t} + $$
$$+ k_7 (\Delta u + \Delta w), \quad (14)$$

$$k_{p2} \Delta p_2^* - k_{Au} \Delta u + \frac{1}{3} S_1 + \frac{\mathrm{d}(\Delta y_r)}{\mathrm{d}t} = \beta V_{A0} \frac{\mathrm{d}(\Delta p_A)}{\mathrm{d}t} + $$
$$+ (k_{pA} + k_{5A}) \Delta p_A\,, \quad (15)$$

$$\Delta v = -K_m \Delta p_H\,, \quad (16)$$

$$\Delta v_1 = \frac{l_1}{l_2} \Delta v\,, \quad (17)$$

$$S_3 \Delta p_i - S_1 (\Delta p_A - \Delta p_1^*) = m_3 \frac{\mathrm{d}^2(\Delta y_r)}{\mathrm{d}t^2} + \xi \frac{\mathrm{d}(\Delta y_r)}{\mathrm{d}t} + $$
$$+ (k_2 + k_8)(\Delta y_r) + k_8 \Delta v_1\,. \quad (18)$$

For the above equation system one has used the following annotations:

$$k_{p2} = \left(\frac{\partial Q_A}{\partial p_2^*}\right)_0 = \mu_{A0} \frac{\pi(d_A - d_0)^2}{8}\sqrt{\frac{2}{\rho}} \frac{1}{\sqrt{p_{20}^* - p_{A0}}}\,;$$

$$k_{pA} = -\left(\frac{\partial Q_A}{\partial p_A}\right)_0 = -\mu_{A0}\frac{\pi(d_A - d_0)^2}{8}\sqrt{\frac{2}{\rho}}\frac{1}{\sqrt{p_{20}^* - p_{A0}}} =$$

$$= k_{p2} = k_p;\; K_m = -\left(\frac{\partial v}{\partial p_H}\right)_0;$$

$$k_{Au} = -\left(\frac{\partial Q_A}{\partial u}\right)_0 = \mu_{A0}\sqrt{\frac{2}{\rho}}\pi \mathrm{tg}\alpha\sqrt{p_{20}^* - p_{A0}};$$

$$k_{5A} = \left(\frac{\partial Q_5}{\partial p_A}\right)_0 = \mu_5\frac{\pi d_5^2}{8}\sqrt{\frac{2}{\rho}}\frac{1}{\sqrt{p_{A0}}}. \quad (19)$$

The equations (14)…(18), with the annotations (19), are the controller's linear mathematical model.

2.3. System's non-dimensional mathematical model and the block diagram with transfer functions

Formally, one can apply the Laplace transformer to the above determined equation system. One can also obtain its non-linear form, using the formal transforming

$$\overline{X} = \frac{\Delta X}{X_0}, \quad (20)$$

where ΔX and X_0 have the same significance as in (13) and $\overline{X}$ is the non-dimensional deviation.

So, the non-dimensional mathematical model has the following form:

$$k_{2p}\overline{p_2^*}(\mathrm{s}) - k_{1p}\overline{p_1^*}(\mathrm{s}) = \left(T_4^2\mathrm{s}^2 + T_{\xi 4}\mathrm{s} + 1\right)\overline{u}(\mathrm{s}) +$$
$$+ k_w\overline{w}(\mathrm{s}), \quad (21)$$

$$k_{2A}\overline{p_2^*}(\mathrm{s}) - k_{1u}\overline{u}(\mathrm{s}) + \tau_y\mathrm{s}\overline{y}_r(\mathrm{s}) = \left(T_A\mathrm{s} + 1\right)\overline{p_A}(\mathrm{s}), \quad (22)$$

$$\overline{v}(\mathrm{s}) = -k_m\overline{p_H}(\mathrm{s}), \quad (23)$$

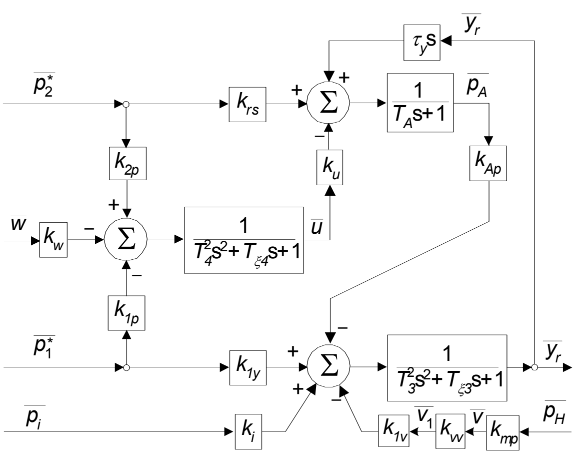

Figure 2. Controller's block diagram with transfer functions

$$\overline{v_1}(\mathrm{s}) = k_{vv}\overline{v}(\mathrm{s}), \quad (24)$$

$$k_i\overline{p}_i(\mathrm{s}) - k_{Ap}\overline{p_A}(\mathrm{s}) + k_{1y}\overline{p_1^*}(\mathrm{s}) - k_{1v}\overline{v}_1(\mathrm{s}) =$$
$$= \left(T_3^2\mathrm{s}^2 + T_{\xi 3}\mathrm{s} + 1\right)\overline{y}_r(\mathrm{s}), \quad (25)$$

where $k_{2p} = \dfrac{S_6 p_{20}^*}{k_7 u_0}$; $k_{1p} = \dfrac{S_6 p_{10}^*}{k_7 u_0}$; $T_4 = \sqrt{\dfrac{m_4}{k_7}}$;

$$T_{\xi 4} = \frac{\xi}{k_7};\; k_w = \frac{u_0}{w_0};\; k_{2A} = \frac{k_{p2}p_{20}^*}{\left(k_{pA} + k_{5A}\right)p_{A0}};$$

$$k_{vv} = \frac{l_2 v_0}{l_1 v_{10}};\; k_u = \frac{k_{Au}u_0}{\left(k_{pA} + k_{5A}\right)p_{A0}};\; T_A = \frac{\beta V_{A0}}{k_{pA} + k_{5A}};$$

$$\tau_y = \frac{S_1 y_0}{\left(k_{pA} + k_{5A}\right)p_{A0}};\; k_i = \frac{S_3 p_{i0}}{\left(k_2 + k_8\right)y_{r0}};$$

$$k_{Ap} = \frac{S_1 p_{A0}}{\left(k_2 + k_8\right)y_{r0}};\; k_m = -\frac{K_m p_{H0}}{v_0};\; T_3^2 = \frac{m_3}{k_2 + k_8};$$

$$T_{3\xi} = \frac{\xi}{k_2 + k_8};\; k_{1y} = \frac{S_1 p_{10}^*}{\left(k_2 + k_8\right)y_{r0}};\; k_i = \frac{S_3 p_{i0}}{\left(k_2 + k_8\right)y_{r0}};$$

$$k_{1v} = \frac{k_8 v_{10}}{\left(k_2 + k_8\right)y_{r0}}. \quad (26)$$

Based on the above determined equation system one has built the block diagram with transfer functions, as the figure 2 shows.

3. SYSTEM'S SIMPLIFIED MATHEMATICAL MODEL AND THE TRANSFER FUNCTIONS

In order to operate more efficient with the mathematical model, one can make some observation which can simplify its form. So, one can neglect the fuel's compressibility, $(\beta = 0)$, the viscous friction, $\xi = 0$ and the inertial effects; consequently one obtains $T_A = T_3 = T_{\xi 3} = T_4 = T_{\xi 4} = 0$.

The system's new simplified form becomes:

$$k_{2p}\overline{p_2^*}(\mathrm{s}) - k_{1p}\overline{p_1^*}(\mathrm{s}) - k_w\overline{w}(\mathrm{s}) = \overline{u}(\mathrm{s}), \quad (27)$$

$$k_{2A}\overline{p_2^*}(\mathrm{s}) - k_{1u}\overline{u}(\mathrm{s}) + \tau_y\mathrm{s}\overline{y}_r(\mathrm{s}) = \overline{p_A}(\mathrm{s}), \quad (28)$$

$$\overline{v}_1(\mathrm{s}) = -k_{vv}k_m\overline{p_H}(\mathrm{s}), \quad (29)$$

$$k_i\overline{p}_i(\mathrm{s}) - k_{Ap}\overline{p_A}(\mathrm{s}) + k_{1y}\overline{p_1^*}(\mathrm{s}) - k_{1v}\overline{v}_1(\mathrm{s}) = \overline{y}_r(\mathrm{s}). \quad (30)$$

Eliminating the intermediate variables (u, p_A) between their equations, one obtains

$$k_i\overline{p}_i(\mathrm{s}) - k_{Ap}\left(k_{2A} - k_u k_{2p}\right)\overline{p_2^*}(\mathrm{s}) + \left(k_{1y} - k_{Ap}k_u k_{1p}\right)\overline{p_1^*}(\mathrm{s}) -$$
$$- k_{Ap}k_u k_w\overline{w}(\mathrm{s}) + k_{1v}k_{vv}k_m\overline{p_H}(\mathrm{s}) = \left(\tau_y k_{Ap}\mathrm{s} + 1\right)\overline{y}_r(\mathrm{s}), \quad (31)$$

where the co-efficient in the Eq. (31) right member

$$T_{yr} = \tau_y k_{Ap} = \frac{S_1^2}{(k_2 + k_8)(k_{pA} + k_{5A})} \qquad (32)$$

is the controller's time constant. One can observe that, if $k_{Ap} > 1$, it results that $T_{yr} > \tau_y$.

System's transfer functions are:

- with respect to the compressor's pressures

$$H_{p1}(\mathrm{s}) = \frac{k_{1y} - k_{Ap} k_u k_{1p}}{T_{yr}\mathrm{s} + 1}, \qquad (33)$$

$$H_{p2}(\mathrm{s}) = -\frac{k_{Ap}(k_{2A} - k_u k_{2p})}{T_{yr}\mathrm{s} + 1}; \qquad (34)$$

- with respect to the fuel's injection pressure

$$H_i(\mathrm{s}) = \frac{k_i}{T_{yr}\mathrm{s} + 1}; \qquad (35)$$

- with respect to the flight regime (flight altitude)

$$H_H(\mathrm{s}) = \frac{k_{1v} k_{vv} k_m}{T_{yr}\mathrm{s} + 1} \qquad (36)$$

- with respect to the controller's adjustment

$$H_w(\mathrm{s}) = \frac{k_{Ap} k_v k_u}{T_{yr}\mathrm{s} + 1}. \qquad (37)$$

One can observe that all the above described transfer functions are defining the controller as a non-periodical first order system.

Considering that the adjustment is executed before the controller's service entry and the injection pressure is practically constant, assured by the engine's fuel control system, the most important transfer functions become the ones with respect to the compressor's pressures (air flow rate) and with respect to the flight regime.

The new block diagram with transfer functions is presented in figure 3.

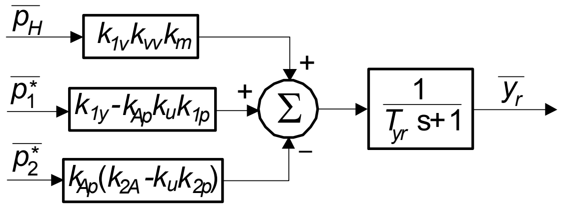

Figure 3. Controller's simplified block diagram with transfer functions

4. SYSTEM'S PRE-DESIGN BASED ON THE TRANSFER FUNCTIONS STUDIES

Analysing the Eq. (32) form, one can observe that the amplifying constant (the right member's numerator) must be a positive one, in order to assure the controller's correct operating (actuator's discharging when the compressor's exhaust pressure diminishes). So, assuming that k_{Ap} is always positive, it results that the condition

$$k_{Ap}(k_{2A} - k_u k_{2p}) > 0 \qquad (38)$$

leads to

$$k_{2A} > k_u k_{2p}, \qquad (38')$$

equivalent to

$$k_7 > \frac{8 S_6 \mathrm{tg}\alpha (p_{20}^* - p_{A0}^*) d_A}{(d_A - d_0)^2}, \qquad (38'')$$

which represents a condition for the command pressure forming block's spring's elastic constant choice, with respect to the elastic membrane's surface S_6 or to the drossel's diameter d_A. Figure 4 shows the relation between the spring's elastic constant k_7 and the other two variables.

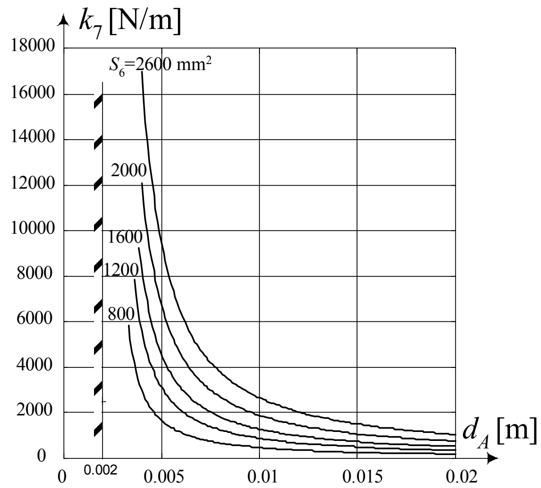

Figure 4. Spring's elastic constant versus drossel's diameter (parameter - elastic membrane's surface area)

One can observe that the permitted domain for d_A is the one at the right side of the vertical $d_0 = \mathrm{const.}$ and the permitted domain for the elastic constant is the one above the limit curve (curves) $S_6 = \mathrm{const.}$

Another observation can be made, that the spring must become more rigid when the drossel's diameter diminishes.

Concerning the system's behavior with respect to the flight regime, the transfer function (36) numerator shows the amplifying constant, also known as the system's sensitivity with respect to the flight regime and gives information concerning the system's adaptability

$$k_H = k_{1v}k_{vv}k_m = K_m\left(\frac{v_0}{y_{r0}}\right)\left(\frac{l_2}{l_1}\right)\frac{1}{1+\frac{k_2}{k_8}}, \quad (39)$$

where $\left(\frac{l_2}{l_1}\right)$ is the (9) lever's arms length ratio, $\left(\frac{v_0}{y_{r0}}\right)$-transmission ratio corrector/slide valve and $\frac{k_2}{k_8}$ - spring elastic constant ratio (for the main spring and the corrector's spring). Observing that the sylphon's elastic constant K_m maintains its value, one can affirm that the amplifying depends on the springs' constants ratio as $\frac{1}{1+\frac{k_2}{k_8}}$. In figure 5 lays the above described variation graphical representation.

The controller's time constant has the expression in Eq. (32), depending on the invariable values S_6, k_2 and k_8, as well as on the co-efficient k_{pA}, k_5 which are influenced both by the flight regime and the engine's operating regime.

If one chooses the drossels diameters as equal $\left(d_A = d_5 = d\right)$, one obtains for the controller's time constant

$$T_{yr} = \frac{S_1^2}{k_8\left(1+\frac{k_2}{k_8}\right)}\frac{1}{\mu\frac{\pi d^2}{8}\sqrt{\frac{2}{\rho}}\left[1-\frac{\left(1-\frac{d_0}{d}\right)^2}{\sqrt{\frac{p_{20}^*}{p_{A0}}-1}}\right]}. \quad (40)$$

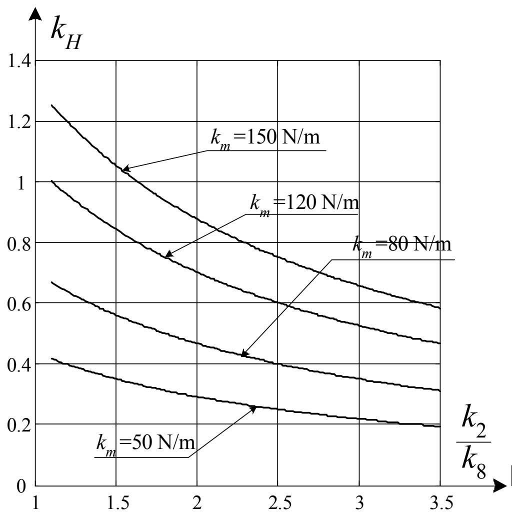

Figure 5. Amplifying constant (for the flight regime) versus springs' elastic constant ratio (parameter - sylphon's elastic constant)

It is obvious that, because of the dependence of the compressor's exhaust pressure on the flight regime (altitude H and speed V), $p_2^* = p_2^*(H,V)$, the controller's time constant depends on the same parameters. The above expression has sense because always $p_{20}^* > p_{A0}$ and when the flight regime becomes more intense, the pressure p_2^* diminishes, so the time constant grows and the engine's time response grows too.

If the springs become more rigid, or the drossels' diameters grow, the time constant diminishes.

Figure 6 presents the controller's time constant variation versus the main spring elastic constant, having the drossel's (5) diameter as parameter.

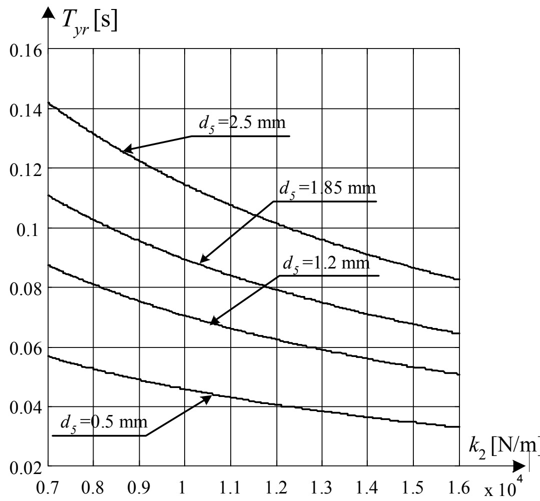

Figure 6. System's time constant versus elastic membrane's surface area (parameter - drossel's diameter)

5. CONCLUSION

One has realized a mathematical description of a fuel flow rate controller with indirect action, elaborating more forms of its mathematical model and transfer functions. Studying the transfer functions, one could obtain some relations between the main geometrical and functional parameters of the controller, relation useful for the controller's pre-design and/or performances (behavior) estimation.

Concerning the flight behavior, one can observe that at low flight altitudes, the controller's barometric corrector is practically inactive, its domain of influence being over 3500 - 4000 m. At high flight altitudes, where the mechanical compression diminishes (pressure ratio $\pi_c^* = \frac{p_2^*}{p_1^*}$ tends to 1) and the dynamical compression grows, the corrector realizes a supplementary charge of the main spring (2) through the charging of the spring (8); therefore,

the slide-valves moves back much slower, the pump's actuator's discharge is longer and, obviously, the engine's rotation speed's growing is also much slower. So, the higher is the flight altitude, the longer are the engine's time responses.
Obviously, the time response and the controller's time constant is important and necessary to be as small as possible, especially for the military aircraft engines.
The time response adjustment could be realized through the screw (16) adjustment, which modifies the spring (7) charging, or through the drossel's (5) replacing.
The modern controllers are with indirect action and can be assisted, when necessary, by some automatic limitation units (for temperature and/or rotation speed), which also control the fuel flow rate.

References

[1] Abraham, R. H. *Complex dynamical systems.* Aerial Press, Santa Cruz, California, 1986.

[2] Dodescu, Gh., a. o. *System's simulation.* Military Publishing, Bucharest, 1986.

[3] Goewski, S. A. *Aircraft Gas Turbine Propulsion Systems' Automation* U.S.S.R, Military Publishing, Moscow, 1980.

[4] Lungu, R. Tudosie, A. *Single-spool Turbo-jet Engines' Control System Based On the Fuel Flow Rate's Control.* (XXVII-th International Scientific Contribution Session, Technical Military Acade-my in Bucharest, 13-14th november 1997, section 4, pp. 74-81).

[5] Manole, I. *Design solutions for Aircraft Turbo-jet Engines*, vol. I, II; Technical Military Academy in Bucharest Publisher, 1978.

[6] Mattingly, J. D. *Elements of Gas Turbine Pro-pulsion.* McGraw-Hill, New York, 1996.

[7] Stoenciu, D. *Aircraft Engines' Automation. Turbo-jet Engines' Rotation Speed Automatic Control.* Technical Military Academy in Bucharest Publisher, 1976.

[8] Stoicescu, M., Rotaru, C. *Turbo-Jet Engines. Characteristics and control Methods* Technical Military Academy in Bucharest Publisher, 1999.