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ELECTROMECHANICAL MOVING OF CONDUCTING LIQUIDS WITH THERMAL INHOMOGENEITIES IN AN ELECTRIC FIELD

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Abstract **– This paper presents a theoretical analysis of the effect of Joule heating on the stability of mechanical equilibrium and development of electroconvective phenomena in liquids with conductivity on the order of 10^{-3}-10^{-6} S/m.**

Keywords: *conducting liquid, Joule heating, electric field, stability of electromechanical equilibrium, electrothermal convection.*

1. INTRODUCTION

Ostroumov was apparently the first to indicate Joule heating of a liquid as a source of electrohydrodynamic (EHD) phenomena in sharply nonuniform fields [1]. His "*thermal hypothesis*" seemed to be confirmed experimentally in Petrichenko's studies [2].
However, in our view [3] and in the opinion of other researchers [4], there is no basis for such confirmation in the case of dielectric liquids, which is indicated, first of all, by simple quantitative evaluations [3]. Furthermore, this hypothesis does not explain isothermal electroconvection in uniform fields [3,4], when the currents in the whole interelectrode gap are so small (< 1 μA) that the thermal hypothesis of Joule heating obviously has to be thrown out. Finally, in explaining electric wind in dielectric liquids and deviations from Ohm's law, the author of [1] himself does not make any use of the fact of Joule heating and does not fit his conclusions to the hypothesis that was expressed.
In more conductive media, researchers do not risk making an allowance for Joule heating, which "obscures" EHD phenomena and superimposes associated electrochemical processes on them, the strongest argument being that, in the final analysis, Coulomb forces are determined completely by the density of space charges $\rho = \nabla(\varepsilon \cdot \mathbf{E}) \sim \varepsilon E/l$ and field strength E, so that the density of these forces $\rho E \sim \varepsilon E^2/l$. This last formula is the main thing limiting the extension of electrohydrodynamics to more conductive media, for the greater the liquid's conductivity is, the lower the permissible strength of the electric field in them is and, it would seem, the weaker EHD effects should be.
In this article, we try to reconsider the existing positions in relation to the possibility of manifestation of EHD effects in conducting liquids and establish what ought to be expected in practice when the appropriate experiments are conducted.

2. COULOMB FORCE IN CONDITIONS OF JOULE HEATING

The appropriate information about the forces acting on a liquid in conditions of Joule heating can be drawn from the following system of equations:

$$-\nabla P + \rho \cdot \mathbf{E} + \gamma \cdot \mathbf{g} = 0 ; \quad (1)$$

$$\lambda \nabla^2 T + \sigma E^2 = 0 ; \quad (2)$$

$$\mathbf{E} = \mathbf{j}/\sigma ; \quad (3)$$

$$\nabla \mathbf{j} = 0 ; \quad (4)$$

$$\rho = \nabla(\varepsilon \cdot \mathbf{E}) ; \quad (5)$$

$$\mathbf{E} = -\nabla \varphi . \quad (6)$$

From (3)-(5) follows the well-known formula

$$\rho = \mathbf{j} \cdot \nabla \tau , \quad (7)$$

where $\tau = \varepsilon/\sigma$. In the case of Joule heating, as a consequence of $\tau = \tau(T)$, we have $\nabla \tau = (d\tau/dT)\nabla T$ and from (7) it follows that

$$\rho = \mathbf{j} \cdot (d\tau/dT) \cdot \nabla T . \quad (8)$$

Then the density of Coulomb forces due to the liquid's thermal nonuniformity can be determined by the expression

$$\mathbf{f}_C = \frac{1}{\sigma} \cdot \frac{d\tau}{dT} \cdot (\mathbf{j} \cdot \nabla T) \cdot \mathbf{j} . \quad (9)$$

We can see that, with fixed values of j, in principle, by independently raising ∇T we can increase $\mathbf{f}_C$ as much as we want, even above the values determined by the order of magnitude of ($\sim \varepsilon E^2/l$).
For liquids such as distilled water, τ ~ 10-100 *ms*,

which is much less than the period of commercial-frequency voltage oscillations. This means that in a commercial-frequency field the liquid's charge transfer can follow the field, which does not change the direction of the force ($\sim j^2$ according to (9)). This circumstance advantageously distinguishes Joule EHD effects from ordinary ones: first of all, there is no need for rectifiers, and secondly, an alternating field will make it possible to avoid electrochemical parasitic phenomena, to a significant extent.

To evaluate the forces (9), we must first determine the order of the liquid's temperature excess over the ambient temperature caused by Joule heating. According to (2),

$$\theta \sim \frac{\sigma \cdot (\nabla\varphi)^2}{\lambda}, \tag{10}$$

and the order of Coulomb force is

$$f_c \sim \frac{d\tau}{dT} \cdot \frac{\sigma^2 \cdot (\Delta\varphi)^4}{\lambda \cdot l^3}, \tag{11}$$

i.e., in contrast to ordinary EHD effects, which are determined the quadratic dependence of the force on the potential difference $\Delta\varphi$, the Joule EHD effect is characterized by a stronger dependence. For example, the Joule effect in distilled water is 12 orders higher than in purified transformer oil, and the force (11) with θ = 10 K in on the order of the buoyancy force ($f_A = \gamma\beta g\theta$). This indicates that Joule heating has a significant force action on the liquid with a rise in conductivity, which is always ignored in the case of high-resistance media.

3. STABILIZATION OF A LIQUID'S HYDROSTATIC EQUILIBRIUM BY AN ELECTRIC FIELD ON ACCOUNT OF JOULE HEATING

We will assume that the forces of gravity and a vertical electric field act on a layer of liquid. Applying the operation rot to equation (1), we get the condition of equilibrium in the form [3]

$$\nabla\rho \cdot \mathbf{E} + \nabla\gamma \cdot \mathbf{g} = 0, \tag{12}$$

which means that the vectors entering into it are parallel. In [3], it was shown that if the equality (12) is observed, the conditions $\nabla\rho \cdot \mathbf{E} > 0$ and $\nabla\gamma \cdot \mathbf{g} > 0$ correspond to a stable equilibrium. Obviously, a stronger inequality takes place, which considers the joint action of the two fields ($\mathbf{E}$ and $\mathbf{g}$):

$$\nabla\rho \cdot \mathbf{E} + \nabla\gamma \cdot \mathbf{g} > 0, \tag{13}$$

where, taking into account (4) ($\mathbf{j} = \mathbf{const}$) and (7),

$$\nabla\rho \cdot \mathbf{E} = \sigma E^2 \nabla^2 \tau. \tag{14}$$

Thus, the condition of stability as applied to an electric field is

$$\nabla^2 \tau > 0, \tag{15}$$

whatever the nature of the nonuniformities may be.

In our case,

$$\nabla^2 \tau = \frac{d^2\tau}{dT^2} \cdot (\nabla T)^2 + \frac{d\tau}{dT} \cdot \nabla^2 T > 0, \tag{16}$$

or, according to (2),

$$\frac{\partial^2\tau}{\partial T^2} \cdot (\nabla T)^2 - \frac{d\tau}{dT} \cdot \frac{\sigma E^2}{\lambda} > 0. \tag{17}$$

For liquids in which $d\tau/dT < 0$ and $d^2\tau/dT^2 > 0$, the electric field will play a stabilizing role, regardless of the type of uniform boundary conditions.

Taking into account the gravitational field, and also (14) and (16), the condition of stability (13) has the form

$$\alpha_2 \sigma E^2 A^2 + \gamma\beta g A + \alpha_1 \sigma^2 E^4 / \lambda > 0, \tag{18}$$

where

$$\alpha_1 \equiv -\frac{d\tau}{dT}; \; \alpha_2 \equiv \frac{d^2\tau}{dT^2}; \; \beta \equiv -\frac{1}{\gamma} \cdot \frac{d\gamma}{dT};$$

$$\mathbf{A} \equiv A \cdot \mathbf{k} \equiv \nabla T. \tag{19}$$

In the absence of an electric field, A = const [6]. In the general case, A is a function of the coordinates, which also depends on the boundary conditions. However, no matter what this function is, if the discriminant on the left side of (18) in relation to A is negative, then the equilibrium will be absolutely stable. From this, it follows that there is a critical value of the field strength E_{cr} above which $(E > E_{cr})$ complete mechanical equilibrium sets in:

$$E_{cr} \equiv \left(\lambda\gamma^2\beta^2 g^2 / 4\alpha_1\alpha_2\sigma^3\right)^{1/6}, \text{ if } \alpha_2 > 0; \tag{20}$$

$$E_{cr} \equiv \left(-\gamma g\beta A\lambda / \alpha_1\sigma^2\right)^{1/4}, \text{ if } \alpha_2 = 0, \tag{21}$$

where $A < 0$ means heating from below (if the liquid is heated from above, then according to (18) when $\alpha_1 > 0$ the equilibrium is stable by itself). In conditions of weightlessness, as follows from (18), the liquid should be stabilized to any kind of disturbance with any E. This method of stabilizing the equilibrium seems expedient for the purposes of space flight.

3.1 Equilibrium of a flat horizontal layer with first-order boundary conditions. Conditions of occurrence of Joule electric convection

We will consider a plane-parallel horizontal layer of liquid in a vertical field of a plane capacitor in conditions of passage of an electric current (j = const). In this case, the equation (2) has the form

$$d^2T(z)/dz^2 = -j^2/\lambda\sigma(z), \qquad (22)$$

the approximate solution of which with first-order boundary conditions, $\theta|_{z=0}$ and $\theta|_{z=l} = \theta_s$, was derived with accuracy to within the quadratic terms with respect to z in the form of expansion into a Maclaurian series:

$$\theta(z) = \theta_s \cdot \frac{z}{l} + j^2 lz \cdot \frac{l-z}{\lambda\sigma_0 l}. \qquad (23)$$

From (23), we can determine $grad\ T$:

$$\mathbf{A} = \left(\frac{\theta_s}{l} + j^2 l \cdot \frac{l-2z}{2\lambda\sigma_0 l}\right) \cdot \mathbf{k} \equiv A \cdot \mathbf{k} \qquad (24)$$

and then the Coulomb and buoyancy forces:

$$\mathbf{f}_\mathbf{C} = \frac{d\tau}{dT} \cdot \frac{j^2}{\sigma_0} \cdot \left(\frac{\theta_s}{l} + j^2 l \cdot \frac{l-2z}{2\lambda\sigma_0 l}\right) \cdot \mathbf{k}\ ; \qquad (25)$$

$$\mathbf{f}_\mathbf{A} = \gamma_0 \beta g \cdot \left(\theta_s \cdot \frac{z}{l} + j^2 lz \cdot \frac{l-2z}{2\lambda\sigma_0 l}\right) \cdot \mathbf{k}\ . \qquad (26)$$

In (25), the first term characterizes the electrothermal force on account of external heat exchange; and the second one, on account of internal Joule heating. Consequently, their ratio characterizes the contribution of Joule heating in comparison to external heat exchange:

$$L \cong \sigma \cdot \frac{(\Delta\varphi_s)^2}{2\lambda\theta_s}, \qquad (27)$$

which makes sense when $\theta_s \neq 0$. It is not hard to ascertain that $L \sim 1$ when σ = 0.1 *mS/m*, i.e., for liquids such as distilled water, which is also given by the evaluation (10). If we set $\theta_s = 0$ in (25) and (26) (the same temperatures of the capacitor plates), then we can get expressions for the force factors due to "pure" Joule heating:

$$K(z_1) = \frac{f_C|_{\theta_s=0}}{f_A|_{\theta_s=0}} = \frac{d\tau}{dT} \cdot \frac{2jz_1}{\beta g \gamma_0 \sigma_0 \left[z_1 - \left(l^2/4\right)\right]}, \qquad (28)$$

where $z_1 = z - l/2$.

The ratio $K(z_1)$ is of a local nature: near the electrodes, Coulomb forces predominate; and in middle layers of the liquid, buoyancy forces. The boundaries between layers correspond to equality of Coulomb and buoyancy forces. The width of the middle layers is equal to

$$\delta \equiv l \cdot \left[-B + \sqrt{\left(1+B^2\right)}\right] \equiv z_1^{(2)} - z_1^{(1)}, \qquad (29)$$

where $B \equiv (d\tau/dT) \cdot \left(2j^2/\gamma_0 \beta \cdot g\sigma_0 l\right)$; and $z_1^{(1)}$ and $z_1^{(2)}$ are the roots of the equations $K(z_1) = \pm 1$. When $B \to 0$ $(j \to \infty)$, $\delta \to 0$ and forces of Coulomb origin are totally predominant in the liquid. Thus, we come to the conclusion that EHD phenomena caused by Joule heating appear in the form of layered structures such that nonstationary oscillatory movements should be observed near the surface of the electrodes, and "still" Bénard cells, in the central region.

3.2 Equilibrium and electroconvection of a layer enclosed between cylindrical capacitor plates

Consideration of a problem analogous to the preceding one, but for a cylindrical layer, in which condition (12) is not fulfilled and gravitational forces are ignored, shows that if the temperature of the liquid rises from the center to the outside capacitor plate, then the equilibrium is absolutely stable. If the radial temperature gradient goes in the opposite direction, then, in contrast to the case of a plane layer, Joule electrothermal convection is possible, which is also spatially nonuniform in intensity. It is possible only with negative temperature gradients.

On the other hand, with isothermal boundaries surrounding the liquid, the temperature gradient in the interelectrode space changes sign, i.e., there is an inversion layer for the sign of **grad** T. Therefore, the condition of occurrence of electroconvection cannot be fulfilled in the whole interelectrode space. In this case, there is a critical temperature difference above which such conditions are provided.

4. HEAT EXCHANGE IN CONDITIONS OF JOULE ELECTROTHERMAL CONVECTION

Previously, the concept of an effective Reynolds number was introduced for the case of electrothermal convection [3], which is determined in approximation of a laminar boundary layer by the expression

$$\mathrm{Re_{ef}} = \left(\frac{\mathrm{Gr_E}}{1 + kM\ \mathrm{Re_{ef}}} + \mathrm{Re}^2 + k \cdot \frac{\mathrm{Gr}}{\sqrt{\mathrm{Re_{ef}}}}\right), \qquad (30)$$

where $\mathrm{Gr_E} = \varepsilon\beta_\tau\theta_s(\Delta\varphi_s)^2/\gamma\nu^2$ is an "electric" analog of the Grasshoff number, which is derived by isolating the moving part of the force (9); $k \sim \mathrm{Pr}^{-1/3}$ is a coefficient depending on the Pr number; $M \equiv \varepsilon\nu/\sigma \cdot l^2$ is the ratio of the electric and hydrodynamic relaxation times; $\beta_\tau = (l/\tau)\cdot(d\tau/dT) = \beta_\varepsilon + \beta_\sigma$; $\beta_\varepsilon \equiv (l/\varepsilon)\cdot(d\varepsilon/dT)$; and $\beta_\sigma \equiv (l/\sigma)\cdot(d\sigma/dT)$.

Taking $\theta_s \equiv \sigma(\Delta\varphi_s)^2/\lambda$ according to (10) and substituting this relationship in the expressions for the Gr and $\mathrm{Gr_E}$ numbers, we get

$$\mathrm{Gr} = \frac{\beta g\sigma\cdot l^3(\Delta\varphi_S)^2}{\lambda\nu^2}; \quad \mathrm{Gr_E} = \frac{\varepsilon\sigma\beta_\tau(\Delta\varphi_S)^4}{\lambda\gamma\nu^2}. \quad (31)$$

In the same approximation of the boundary layer, the Nusselt number is determined from the familiar dimensionless dependence

$$\mathrm{Nu} = f(\mathrm{Pr})\cdot\mathrm{Re}_{\mathrm{ef}}^{0.5}. \quad (32)$$

Checking of equation (30) with the help of dependence (32) for particular cases demonstrated its validity. For example, with pure forced movement (Gr = 0, $\mathrm{Gr_E}$ = 0), the familiar dependence Nu(Re) follows from (30) and (32). With $\mathrm{Gr_E}$ = 0 and Re = 0, the following relationship can be derived from (32) and (30):

$$\mathrm{Nu_E} = f(\mathrm{Pr})\cdot\mathrm{Gr}^{0.2} \sim (\Delta\varphi_S)^{0.4}, \quad (33)$$

which is valid, as it should be, for large Gr numbers ($\mathrm{Gr} \geq 10^6$) in the case of heat transfer in horizontal layers of liquid [3]. If Gr = 0 and Re = 0, then from (30) and (32) it follows that

$$\mathrm{Re}_{\mathrm{ef}}^2(l + k\cdot M\cdot\mathrm{Re}_{\mathrm{ef}}) = \mathrm{Gr_E}. \quad (34)$$

From expressions (34) and (32), taking into account (31), two asymptotic cases of electrothermal convection can be derived - *moderate* and *intensive*:

$$\begin{aligned} \mathrm{Nu_E} &= f(\mathrm{Pr})\cdot\mathrm{Gr_E}^{1/4} \sim \Delta\varphi_S, \text{ if } kM\mathrm{Re}_{\mathrm{ef}} << 1; \\ \mathrm{Nu_E} &= f(\mathrm{Pr})M^{-1/6}\,\mathrm{Gr_E}^{1/6} \sim (\Delta\varphi_S)^{2/3}, \text{ if } kM\mathrm{Re}_{\mathrm{ef}} >> 1. \end{aligned} \quad (35)$$

Dependences of the same type as (35) were derived theoretically in [6]. In all likelihood, for the more general case we can expect a dependence of the type

$$\mathrm{Nu_E} = f(\mathrm{Pr})\cdot\left(\frac{\sigma\cdot l^2}{\varepsilon\nu}\right)^n\cdot\mathrm{Gr_E}^m, \quad (36)$$

in which 0 < *n* < 1/6, with the exponent *n* rising as convection develops, and *m* falling.

5. CONCLUSIONS

Distinguishing characteristics of joule heating of liquids such as distilled water consist in the appearance of an electric force proportional to the potential difference to the fourth power, the existence of conditions of the liquid's dominant tendency toward electrohydrodynamic equilibrium (even in alternating commercial-frequency fields), and development of electrothermal convection of the Joule type, which is characterized by formation of layered structures such that nonstationary wave movements are observed near the electrodes, and stationary currents like Benard cells in the core of the flow. Depending on the heat-exchange conditions at the boundaries with the electrodes, convection may occur in a limited region of the liquid with an inversion surface for the sign of the temperature gradient, or it may encompass the whole volume if certain conditions are observed.

In our view, experimental checking of dependences (33) and (35) will make it possible to confirm the physical hypotheses in relation to electric convection of the Joule type and its practical use.

List of notations

ε - absolute permittivity of the liquid, [*F/m*]; l - interelectrode distance, [*m*]; *P*- pressure, [*Pa*]; γ - density of the liquid, [*kg/m*^{3}]; λ - thermal conductivity, [*W*/(*m*·*K*)]; σ - electric conductivity of the liquid, [*S/m*]; j - current density, [*A*/*m*^{2}]; φ -potential of the electric field, [*V*]; τ - electric relaxation time, [*s*]; f - density of forces, [*N*/*m*^{3}]; θ - temperature difference, [*K*]; T - temperature, [*K*]. *Subscripts*: s - on the surface of the electrodes; E - electric parameter; 0 - initial (boundary) value; c - critical value.

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