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MULTIPLE LOOP NONLINEAR CONTROL OF SINGLE LINK MANIPULATOR WITH FLEXIBLE JOINT

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Abstract –**In the paper you will implement a set point control system for the single-link manipulator with flexible joint using a multiloop controller based on feedback linearization (inner loop) and linear state-feedback control (outer loop). Then, you will modify the multiloop controller to solve a trajectory tracking problem.**
The simulations demonstrate that the multi-loop control law it is a viable method for control motion of the flexible joint manipulator.

Keywords: *single-link manipulator, flexible joint, multiloop controller, equations of motion, feedback linearization.*

1. INTRODUCTION

In the past fifteen years a large amount of research has been focused on the control of flexible joint robots . The relevant nonlinear control methods are grouped into three categories, singular perturbation, cascaded systems, and feedback linearization. The singular perturbation technique divides the dynamics by time-scale into the fast, or elastic subsystem dynamics, and the slow, or rigid-body dynamics. A rigid-body control law can be designed to stabilize the slow system and a second controller is designed to stabilize the fast system. The cascade system approach separation between the slow and fast dynamics.
The cascade system approach first solves the problem as if the links themselves can be directly controlled, then determines the actual control that would be required to account for the joint flexibility. In this approach, the dynamics of the joint flexible system are separated into two cascaded equations, first the manipulator equations and then the actuator equation. The "input" to the manipulator set of equations is the force applied by the flexible joint between the motor and the link. After the desired "input" to the links has been determined, the next loop controls the force to the actuator so that the flexible joint provides the desired "input" to the link.
Spong and Forrest-Barlach showed that, through a state transformation, the equations of motion for a flexible-joint robot are globally feedback linearizable. In this approach, the equations are transformed to a single equation that is a function of the position, velocity, acceleration, and jerk of the link to a single equation that is a function of the position, velocity, acceleration, and jerk of the link. The transformed equations are then in a form that can be linearized by feedback control, as the dynamics are expressed in a single equation with the joint torques as the input.
In the paper, you will implement a set point control system for the single-link manipulator with flexible joint using a multiloop controller based on feedback linearization (inner loop) and linear state-feedback control (outer loop). Then, you will modify the multiloop controller to solve a trajectory tracking problem.

2. FLEXIBLE JOINT MANIPULATOR MODEL

A flexible joint is modeled as a motor and link separated by a torsional spring as shown in Figure 1 The torque is applied to the motor rotor, modeled as a disc. The motor can be viewed as an intermediate link, with the next link passively connected by a torsional spring. The model used in this paper is that proposed by Spong [1]. The derivation of the model includes only the kinetic energy of the motor that comes from its own rotation. The assumption is that the off-axis rotations of the motor are any geared system. For planar robots, such as used in this research, the model is exact.

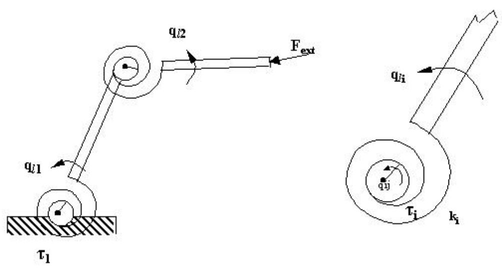


Fig.1- Multi joint flexible manipulator

The resulting equations of motion for a flexible-joint manipulator can be written as :

$$M(q_l)\ddot{q}_l + C(q_l,\dot{q}_l) = K(q_j - q_l) + J^T(q_l)F_{ext}$$

$$D\ddot{q}_j + K(q_j - q_l) = \tau \quad (1)$$

where $q_l \in R^n$ represents the link angles and $q_j \in R^n$ represents the joint angles of a robot with n degrees of freedom. $M(q_l) \in R^{nxn}$ is the inertia matrix, $C_l(q_l, \dot{q}_l) \in R^n$ represents the Coriolis, centrifugal forces and gravitational forces, and $\tau \in R^n$ is the input torque vector. The diagonal matrix $D \in R^{nxn}$ contains the inertia of the joint motors and the diagonal matrix $K \in R^{nxn}$ represents the spring constants. As before, $J(q_l) \in R^{mxn}$ is the Jacobian relating link velocities to end –point velocities, and $F_{ext} \in R^m$ is the external force at the end-point of the robot.

The first equation, Equation 1, describes the link motion, and the second equation, Equation 2, describes the joint behaviour. Note the similarities between Equation 1 and the equation describing the rigid robot. The equations are the same, except that in the case of the flexible-joint robot, the input to the links is the torque in a spring rather than a direct torque. The rigid-robot equation has the input on the right side, whereas the flexible robot has the term $K(q_j - q_l)$. Then the joint equations, Equations 1 can be used to control q_j using the actual joint torques

The equations of motion can be written in the new form. We rewrite the equations of motion so that the Coriolis and centrifugal term is written as

$$C_2(q_j, \dot{q}_l)$$

such that $C_2(q_l, \dot{q}_l)\dot{q}_l = C(q_l, \dot{q}_l)$ and the external force is set to zero:

$$M(q_l)\ddot{q}_l + C_2(q_l, \dot{q}_l)\dot{q}_l + Kq_l = Kq_j$$

$$D\ddot{q}_j + K(q_j - q_l) = \tau \quad (2)$$

where the flexible-joint position q_j is the virtual input to Equation 2.

3. DINAMIC STATE DECUPLING AND LINIARIZATION THEORY OF NONLINEAR SYSTEMS

a) The regulated output of your system is $y_i = q_{li}$. and all the state variable $(q_{li}, \dot{q}_{li}, \ddot{q}_{li})$ are available for feedback. First, you need to implement the inner loop (feedback linearization). The state-space model for (2) is a nonlinear multivariable system, described by a vector of state x, of dimension n*1 and output y of dimension m*1

$$\dot{x} = A(x) + B(x)u \qquad y = C(x) = Cx \quad (3)$$

where u is the command vector, of dimension m*1 and C represents a constant matrix.

In order to achieve the liniarization, it is analyzed a nonlinear control law u(t), which accomplishes this operation:

$$u = F(x) + G(x)w \quad (4)$$

Where:

$$F(x) = -D^{*(-1)}(x)C^*(x)$$
$$G(x) = D^{*(-1)}(x)L$$
$$L = diag(l_1, l_2, .., l_n) \quad (5)$$

where w(t) represents a new command of dimension m*1 and $C^*(x)$, $D^*(x)$ matrices of dimension n*n and vector of dimension n*1 :

$$C^*(x) = [C_1^*(x), C_2^*(x)...C_n^*(x)]^T$$
$$D^*(x) = [D_1^*(x), D_2^*(x),..., D_n^*(x)]^T \quad (6)$$

First, you need to implement the inner loop (feedback linearization). Build a block that produces the change of coordinates In the matricial form:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ . \\ \dot{x}_{(n-1)} \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} x_2 \\ x_3 \\ . \\ x_n \\ C^*(x) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ D^*(x) \end{bmatrix} u = C^*(x) + D^*(x)u \quad (7)$$

Then, implement the outer loop control

$$w = Kx \quad (8)$$

where K has been obtained using favorite linear design method.

4. MULTILOOP TRACKING CONTROLLER

In the second part of the experiment, the outer loop controller will be modified to let the output y(t)=Cx(t)=Ix(t) of the system track a time-varying reference trajectory yd(t). In order to have x(t) track yd(t), we need to drive the state x(t) to an appropriate time-varying equilibrium (a reference state trajectory function of yd(t) and its derivatives). Assume that yd(t) and its derivatives unto order 2 are known. Remember that, after feedback linearization, the system is given by:

$$\dot{x}(x) = Ax + Bw$$
$$y = Cx \quad (9)$$

And x_d is impose to satisfies

$$\dot{x}_d = Ax + B\ddot{y}_d$$
$$y_d = Cx_d \quad (10)$$

We define the error:

$$\dot{e} = \dot{x} - \dot{x}_d \quad (11)$$

and we obtain the error dynamics:

$$\dot{e} = Ae + B[w - \ddot{y}_d] \quad (12)$$

In order to regulate e(t) to zero, we need to choose an outer-loop control input v that performs two distinct actions:

1. Rendering the point e = 0 an equilibrium for the error dynamics
2. Stabilizing the newly created equilibrium at the origin.

The first action is performed by the feedforward control

$$w_{ff} = \ddot{y}_d \quad (13)$$

while the second is performed by the usual state-feedback

$$w_{fb} = Ke. \quad (14)$$

Letting

$$w = w_{ff} + w_{fb}, \quad (15)$$

we obtain the closed loop system $\dot{e} = (A + BK)e$ which has an asymptotically stable equilibrium at the origin, as required. Note that the stabilizing controller is exactly the same as before, only the way the error is defined changes, and a feedforward control action is needed.

The last thing to do is to generate yd and its derivatives. The best way to do so, is to implement a reference model, that is, to filter a reference signal through a stable linear systems. Let yd be given in the Laplace domain as the output of the transfer function

$$y_d(s) = M(s)y_r(s) \quad (16)$$

where yr(s) is an external reference signal, and M(s) is the reference model. Assume that

$$M(s) = \frac{a_0}{a_2 s^2 + a_1 s + a_0} \quad (17)$$

Where the roots of $s^2 + a_1 s + a_0$ are all Re[s]<0

5. SIMULATIONS RESULTS

As an example system, let k=100, d=1, I=2 and l=1.For the control design on choose $K_1 = 20$ and $K_2 = 20$, $a_0 = a_2 = 1; a_1 = 4$. First we consider $y_r(t)$ an Sine Wave signal with amplitude 5 and frequency 0.3 [rad. /sec], and the last, $y_r(t) = \sigma(t)$.

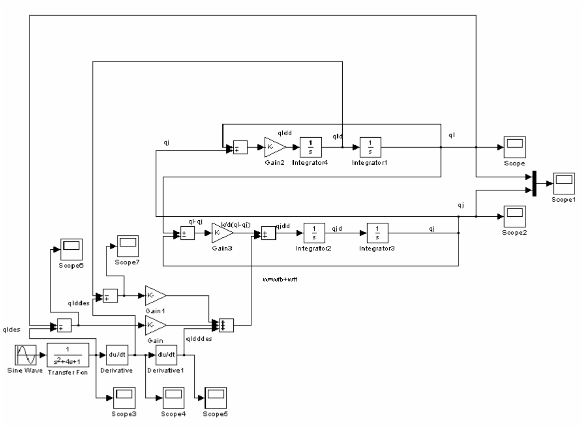
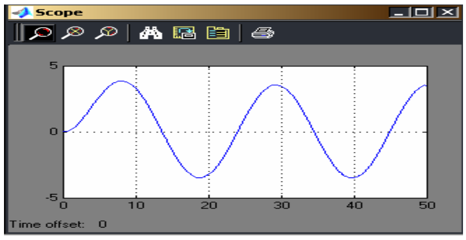


Fig.3. Simulations of multiple-loop controller



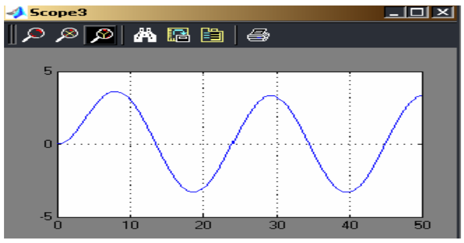


Fig.4. q_{ld}, q_l for $q_{lr}(t) = A\sin(\varpi t)$

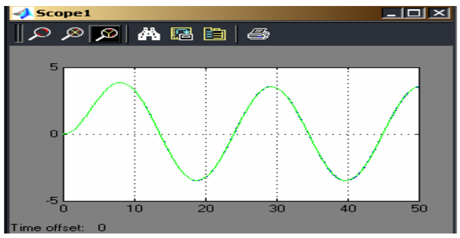
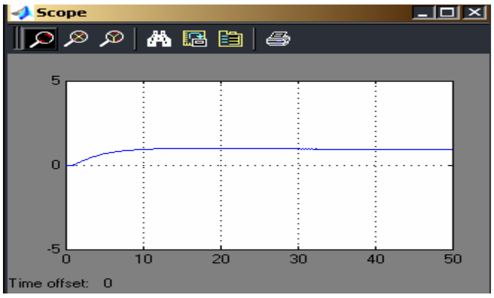


Fig.5. q_l, q_j for $q_{lr}(t) = A\sin(\varpi t)$



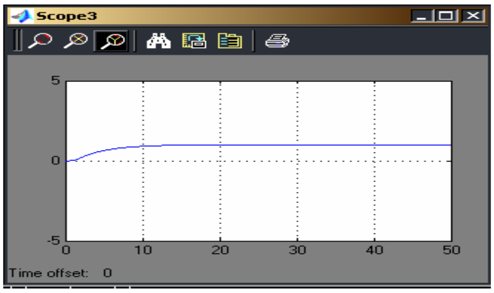


Fig. 6. q_{ld}, q_l for $q_{lr}(t) = \sigma(t)$

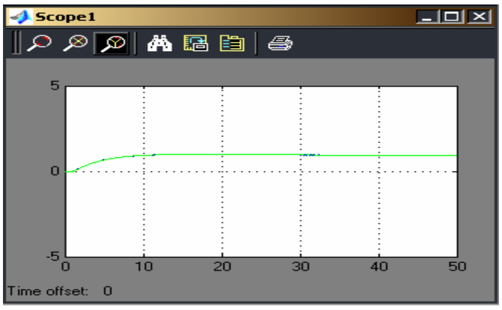


Fig.7. q_l, q_j for $q_{lr}(t) = \sigma(t)$

6. CONCLUSIONS

The paper analyses and shows the design principle of the multiple loop nonlinear controller for manipulator with flexible joint. The simulations demonstrate that the multi-loop control law it is a viable method for control motion of the flexible joint manipulator

Acknowledgments

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My main research areas are: automatic control, intelligent motion control, nonlinear control systems, algorithms for control robots, optimal design of electric machines, application of VSS theory, neural networks.

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