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# SDRE TECHNIQUE, METODOLOGY FOR DIRECT SYNTHESIS OF NONLINEAR FEEDBACK CONTROLLERS FOR D.C. SERIES MOTOR

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*Abstract* – **State Dependent Riccati Equation(SDRE) method is a recently emerged nonlinear control system design methodology for direct synthesis of nonlinear feedback controllers. Using a special form of the system dynamics, this approach permits the designer to employ linear optimal control methods such as the LQR methodology design technique for the synthesis of nonlinear control systems.**
**The analysis and simulations demonstrate that the SDRE technique it is a good methodology for direct synthesis of nonlinear feedback controller for D.C. Series motor.**

***Keywords***: *nonlinear control systems, State Dependent Riccati Equation method, linear optimal control, D.C. Series motor, nonlinear feedback controller.*

## 1. INTRODUCTION

Linear quadratic regulation (LQG) is an effective theory for the synthesis of control laws for linear systems. However, most mathematical models are inherently nonlinear. One of the highly promising and rapidly emerging methodologies for designing nonlinear controllers is the state-dependent Riccati equation (SDRE) approach in the context of the nonlinear regulator problem. In essence, the SDRE method is a systematic way of designing nonlinear feedback controllers that approximate the solution of the infinite time horizon optimal control problem and can be implemented in real-time for a broad class of applications. In most cases, the theory developed also involves using nonlinear weighting coefficients for the state and control in the cost functional to produce near optimal solutions. This methodology is quite useful and also quite difficult to implement for complex systems. Therefore, it is of general interest to explore the use of constant weighting matrices to produce a suboptimal control law that has the advantage of ease of configuration and implementation. In [5], an efficient computational methodology was proposed that requires splitting the state-=dependent coefficient matrix $A(x)$ into a constant matrix part and a state-dependent part as: $A(x) = A_0 + \Delta A(x)$. This method is effective locally for systems with constant control coefficients and if the function $\Delta A(x)$ is not too complicated (e.g., when it has the same function of in all entries) then the SDRE can be solved through a series of constant-valued matrix Lyapunov equations. The assumption on the form of $\Delta A(x)$, however, does limit the problems for which this SDRE approximation method is applicable.

In this paper, we examine the SDRE technique with constant weighting coefficients. In Section 2, we review the SDRE design technique. In Section 3 we present numerical approach for obtaining the SDC model. In Section 4 is presented the suboptimal control using SDRE solution. In Section 5 and 6 we realize the suboptimal control using SDRE solution for D.C. Series Motor. The simulations demonstrate that the SDRE technique it is a good methodology for direct synthesis of nonlinear feedback controller for D.C. Series Motor.

## 2. THE SDRE DESIGN TECHNIQUE

The SDRE design technique requires the dynamic model of the system to be placed in the state dependent coefficient (SDC) form. The SDC form has the structure:

$$\dot{x} = A(x)x + B(x)u \tag{1}$$

Note that the SDC form has the same structure as a linear dynamic system, but with the matrix A and the control influence matrix B being functions of the state variables.

The second ingredient of the SDRE design technique is the definition of a quadratic performance index in state dependent form:

$$J = \frac{1}{2}\int_{t_0}^{\infty}\left[x^T Q(x)x + u^T R(x)u\right]dt \tag{2}$$

The state dependent weighting matrices Q(x) and R(x) can be chosen to realize the desired performance objectives.

Next, a state dependent algebraic Riccati equation (SDRE), (3):

$$A^T(x)P(x)+P(x)A(x)-P(x)B(x)R^{-1}(x)B^T(x)P(x)+Q(x)=0$$

is formulated and is solved for a positive definite state dependent matrix P(x). The nonlinear state variable feedback control law is then constructed as:

$$u = -R^{-1}(x)B^T(x)P(x)x \tag{4}$$

The two main steps in the SDRE nonlinear control system design method are the computation of the SDC matrices A(x) and B(x), and the solution of the algebraic matrix Riccati equation for P(x). The remaining steps involve matrix inversion and multiplication.

## 3. METHODS FOR OBTAINING THE SDC MODEL

It will be assumed that the dynamic system is given in the standard form:

$$\dot{x} = f(x) + B(x)u \tag{5}$$

Here x is the state and u is the control vector. Note that the control vector appears linearly in the system dynamics. If the control variables appear nonlinearly in the system dynamics, an input dynamic compensator can be introduced to transform the model into standard form

Any input dynamic compensators can be employed, provided that the redefined control variable appears linearly in the dynamics. At any given value of x, finding the SDC form:

$$\dot{x} = A(x)x + B(x)u \tag{6}$$

From the given nonlinear dynamic system requires the solution of the system of n equations:

$$A(x)x = f(x) \tag{7}$$

Instantaneous SDC parameterization can be obtained by evaluating the vector nonlinear function f(x) using a set of linearly independent probe vectors $\zeta_2,...,\zeta_n$.

The probe vectors can be constructed by adding magnitude perturbation vectors $\sigma_2, \sigma_3,..,\sigma_n$ to the nominal state vector to yield a set of linearly independent vectors:

$$\zeta_2 = x + \sigma_2,...,\zeta_n = x + \sigma_n \tag{8}$$

The nonlinear function f(x) is next evaluated using these linearly independent vector. Assemble the matrix equation (9)

$$[f(x) \quad f(\zeta_2) \quad ... \quad f(\zeta_n)] = A(x)[x \quad \zeta_2 \quad ... \quad \zeta_n]$$

At any given value of x, this linear matrix equation can be solved for the elements of A(x). Since the probe vectors and the state vector are linearly independent, this equations is well-conditioned , and can be solved using well-known linear algebraic methods.

Note that the foregoing computations will have be carried out at every sample. The SDC matrix A(x) from these computations can next be used to formulate and solve the SDRE control problem.

As an aside, it is interesting to examine the relationship between the numerical construction of the SDC model and the conventional Taylor series approximation. If the perturbation vectors $\sigma_2, \sigma_3,..., \sigma_n$ are small, it can be found that:

$$A \cong \frac{\partial f}{\partial x} \quad \text{at } x=0 \tag{10}$$

Note that this corresponds to the Taylor series linearization of the system dynamic about the origin. Thus, the present methodology for constructing the SDC model automatically reverts to Taylor series linearization of the system dynamics near the origin of the state space

## 4. SUBOPTIMAL CONTROL USING SDRE SOLUTION

The focus of this section will to be formulate the suboptimal control using SDRE solution. In (6) an efficient computational methodology was proposed that requires splitting the state-dependent coefficient matrix A(x) into a constant matrix part and a state-dependent part as $A(x) = A_0 + \varepsilon\Delta A(x)$. This method is effective locally for systems with constant control coefficients and if the function $\Delta A(x)$ is not too complicated.. Then the SDRE can be solved trough a series of constant valued matrix Lyapunov equations. Likewise, A(x) and B(x) can be rewritten using the constant matrices $A_0$ and $B_0$ and no constant matrices $\Delta A(x)$ and $\Delta B(x)$ defined as:

$$\begin{aligned} A(x) &= A_0 + \Delta A(x) \\ B(x) &= B_0 + \Delta B(x) \end{aligned} \tag{11}$$

with $\Delta A(0) = 0; \Delta B(0) = 0$ This leads to the control u(x) being represented as a the sum of a constant matrix and an incremental matrix,

$$u(x) = -(K_0 + \Delta K(x))x \tag{12}$$

Where

$$K_0 = R^{-1}B_0{}^T P_0 \tag{13}$$

and

$$\Delta K(x) = R^{-1}(B^T(x)\Delta P(x) + \Delta B^T P_0) \qquad (14)$$

By construction $\Delta P(x)$ and $\Delta B(x)$ are zero at the origin so that $\Delta K(0) = 0$.Under continuity assumptions on A(x) and B(x), along with the assumption that the SDC parameterization is a detectable and stabilizable parameterization, it follows that P(x) is continuous.
Assume that A(x) and B(x) defined by (6) are continuous and (7) is a detectable and stabilizable parameterization Then the system with the control given by (12) is locally asymptotically stable.
The controlled nonlinear dynamics can be rewritten in the form (15):

$$\dot{x} = A(x)x - B(x)K(x)x =$$
$$= (A_0 + \Delta A(x))x - (B_0 + \Delta B(x))(K_0 + \Delta K(x))x =$$
$$= (A_0 - B_0K_0)x + (\Delta A(x) - B(x)\Delta K(x) - \Delta B(x)K_0)x$$

If we let:

$$g(x) = \Delta A(x) - B(x)\Delta K(x) - \Delta B(x)K_0 \qquad (16)$$

And h(x)=g(x)x, then the system is given by:

$$\dot{x} = (A_0 - B_0K_0)x + h(x) \qquad (17)$$

Examination of h(x) reveals that we are dealing with an almost linear system satisfying the property

$$\lim_{\|x\|\to 0} \frac{\|h(x)\|}{\|x\|} = 0$$

We can then obtain the solution to (17) using the variation of constants formula :

$$x(t) = e^{(A_0 - B_0K_0)t}x_0 + \int_0^t e^{(A_0 - B_0K_0)((t-s)}h(x(s))ds \qquad (18)$$

With the relation (3), for $A \equiv \Delta A(x)$

$$\Delta A^T P + P\Delta A - P\Delta BR^{-1}\Delta B^T P + Q = 0 \qquad (19)$$

It is resulte P(x) and for x=0, $P_0$ and

$$\Delta P(x) = P(x) - P_0 \qquad (20)$$

## 5. EXEMPLE. D.C. SERIES MOTOR. SIMULATIONS AND RESULTS

To investigate the performance of this control approach, the D.C. Series motor is used. The detailed computations show that the engine and its command is represented by a system of nonlinear equations:

$$\frac{dx_1}{dt} = b_{11}x_1 + b_{12}x_2{}^2 = f_1(x) + B_1(x)u \qquad (21)$$
$$\frac{dx_2}{dt} = b_{21}x_1x_2 + b_{22}x_2 + b_{23}x_3 = f_2(x) + B_2(x)u$$

where,

$$x_1 = \omega\,;\; x_2 = i\,;\; x_3 = u \qquad (22)$$
$$b_{11} = -\frac{B}{J}; b_{12} = \frac{K_m L_f}{J}; b_{21} = -K_m; b_{22} = -\frac{(R_a + R_f)}{L_f}; b_{23} = \frac{1}{L_f}$$

We can write:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} b_{11}x_1 + b_{12}x_2{}^2 \\ b_{21}x_1x_2 + b_{22}x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ b_{23} \end{bmatrix} u \qquad (23)$$

An SDC parameterization is given as:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12}x_2 \\ b_{12}x_2 & b_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ b_{33} \end{bmatrix} u \qquad (24)$$

The resulting constant and incremental matrices (11 ) have the form:

$$A_0 = \begin{bmatrix} b_{11} & 0 \\ 0 & b_{22} \end{bmatrix}; \Delta A(x) = \begin{bmatrix} 0 & b_{12}x_2 \\ b_{21}x_2 & 0 \end{bmatrix}$$
$$B_0 = B = \begin{bmatrix} 0 \\ b_{23} \end{bmatrix}; \Delta B(x) = 0 \qquad (25)$$

The cost functional for the example under is

$$J(x_0u) = \int_0^\infty \left( x^T \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} x + \frac{1}{2}u^2 \right) dt \qquad (26)$$

## 6. SIMULATIONS RESULTS

As explained in the previous section, to illustrate the proposed controller algorithms
The first, the d.c. drive system is operating at the steady state x1=90 deg. and x2=10 amp. At the time t=0 sec. a decrease step of the speed on impose, x1ref=0 deg.
The second, the operating point of the d.c. drive system is selected at x1=0 deg. and x2=10 amp. At the time t=0 sec a increase step of the speed on impose, x1ref=90 deg.

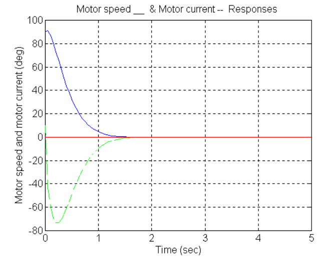


Fig.1

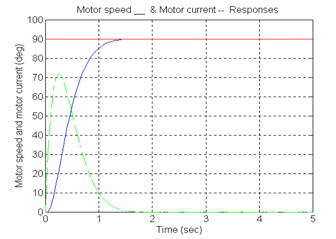


Fig.2

Figs. (1) and (2) show the state responses, the rotor speed and the current motor in the first and second case.

## 7. CONCLUSIONS

In this paper we have considered SDRE techniques for the general design and synthesis of feedback controllers, of nonlinear systems. In particular, the SDRE methods were formulated for cost functional with constant weighting coefficients. In addition, the Kleinman algorithm were presented for the numerical approximation of the solution to the SDRE for a large class of nonlinear problems. This approach is very easy to implement and was shown to perform very well on a wide class of nonlinear systems.

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My main research areas are: automatic control, intelligent motion control, nonlinear control systems, algorithms for control robots, optimal design of electric machines, application of VSS theory, neural networks. Research projects: the exploring and the estimation of new algorithms for controlling industrial robots.

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