

## STUDY OF THE UNIFORM MAGNETIC FIELD DOMAINS (3D) IN THE CASE OF THE HELMHOLTZ COILS

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**Abstract** – Helmholtz coils are used to establish a uniform magnetic field with the known value necessary for different applications from the technical field, such as: the calibrating of the magnetic field sensor, electromagnetic immunity (EMI) testing research regarding the magnetic field effects against human body. Generally the practical realization of the Helmholtz coils is based on the classic study of the magnetic field, namely the magnetic field computation only on the coils axis. In this paper are shown on the basis of the magnetic field analytical computation in any point of the space (3D), that there are other domains in which the magnetic field is uniform.

**Keywords:** uniform magnetic field, Helmholtz coils.

### 1. INTRODUCTION

Both in the field of the technical applications and in physics and in the field of the medical apparatus realization it is necessary to be created uniform magnetic field.

One of the variants frequent utilized is that which uses the Helmholtz coils.

The system (fig. 1) is made from two thin circular coils, with the equal radii  $a$ , placed co-axial to the distance  $2L$  opposite one another, crossed by the same total current  $I$ , in the same direction.

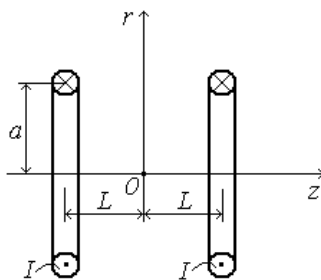


Figure 1: Helmholtz coils

In the majority of the specialty papers [1], [3], [4] the study of the uniform magnetic field domains is done only on the  $Oz$  axis. The question is if there are other domains in which the magnetic field is uniform. Such study can be done only with the analysis methods (analytical or numerical) which permit a 3D visualization of the calculation results.

The present paper has just such object, utilizing, for example the cylindrical coordinates  $(r, \varphi, z)$ .

### 2. CLASSIC STUDY (1D) OF THE HELMHOLTZ COILS

In a point placed on the common axis of the coils, to the distance  $z$  from the mediatory plane, the magnetic field strength (axial) is:

$$H(r) = \frac{I}{2R} \left[ \frac{R^3}{\sqrt{(R^2 + (L+r)^2)^3}} + \frac{R^3}{\sqrt{(R^2 + (L-r)^2)^3}} \right]. \quad (1)$$

It is observed without difficulty that in the middle of the distance  $r=0$  the field has an extreme point for any distance  $2L$  between coils:

$$\frac{dH}{dr} = 0 \text{ for } r=0. \quad (2)$$

An optimum distance  $2L$  can be chosen, which assures as possible a larger domain with uniform magnetic field. With that end in view it is set the condition as well as the second order differential  $d^2H/dr^2$  to be annulled in the middle of the distance:

$$\frac{dH}{dr} = \frac{3IR^2}{2} \left[ \frac{L+r}{\sqrt{(R^2 + (L+r)^2)^5}} - \frac{L-r}{\sqrt{(R^2 + (L-r)^2)^5}} \right],$$

$$\frac{d^2H}{dr^2} = \frac{3IR^2}{2} \left[ \frac{1}{\sqrt{(R^2 + (L+r)^2)^5}} + \frac{1}{\sqrt{(R^2 + (L-r)^2)^5}} \right] +$$

$$+ \frac{15IR^2}{2} \left[ \frac{(L+r)^2}{\sqrt{(R^2 + (L+r)^2)^7}} + \frac{(L-r)^2}{\sqrt{(R^2 + (L-r)^2)^7}} \right].$$

For  $r=0$  the annulment condition is obtained:

$$\frac{5L^2}{R^2+L^2}-1=0, \quad (3)$$

that is  $R = 2L$ .

Noting  $H_0 = I/2R$ , the field obtained in the center of a single coil, the following values results:

$$\begin{aligned} H(0) &= 1,431H_0, \\ H(L/2) &= 1,4251H_0 = 0,9958H(0), \\ H(L) &= 1,356H_0 = 0,9475H(0). \end{aligned}$$

Therefore, in an important region the field is constant in practice, the region size depending on the uniformity degree which is required.

### 3. STUDY OF THE (3D) DOMAINS WITH UNIFORM MAGNETIC FIELD

For the analytical determination of the magnetic field generated by the Helmholtz coils in the whole space, it is calculated firstly the vector magnetic potential produced by a flat circular coil utilizing the direct method by means of Biot-Savart-Laplace formula and then the superposition theorem is applied.

The expressions of the vector magnetic potential components in a cylindrical system of coordinates  $(r, \varphi, z)$ , with the origin in the center of the Helmholtz coil and the  $Oz$  axis along the Helmholtz coil axis, are:

$$\begin{aligned} A_r &= 0, \\ A_\varphi &= \frac{\mu NI}{2\pi r} \frac{1}{\sqrt{(r+a)^2+(L+z)^2}} \cdot \left[ \left( r^2+a^2+(L+z)^2 \right) \cdot K \left( 2\sqrt{\frac{ar}{(r+a)^2+(L+z)^2}} \right) - \left( (r+a)^2+(L+z)^2 \right) \cdot E \left( 2\sqrt{\frac{ar}{(r+a)^2+(L+z)^2}} \right) \right] + \\ &+ \frac{\mu NI}{2\pi r} \frac{1}{\sqrt{(r+a)^2+(L-z)^2}} \cdot \left[ \left( r^2+a^2+(L-z)^2 \right) \cdot K \left( 2\sqrt{\frac{ar}{(r+a)^2+(L-z)^2}} \right) - \left( (r+a)^2+(L-z)^2 \right) \cdot E \left( 2\sqrt{\frac{ar}{(r+a)^2+(L-z)^2}} \right) \right], \\ A_z &= 0. \end{aligned} \quad (4)$$

The magnetic flux density is calculated with the known relation  $\vec{B} = \text{rot } \vec{A}$ . It follows:

$$\begin{aligned} B_r &= \frac{\mu NI}{2\pi r} \frac{(L+z)}{\left( (r-a)^2+(L+z)^2 \right) \sqrt{(r+a)^2+(L+z)^2}} \cdot \left[ \left( r^2+a^2+(L+z)^2 \right) \cdot E \left( 2\sqrt{\frac{ar}{(r+a)^2+(L+z)^2}} \right) - \left( (r-a)^2+(L+z)^2 \right) \cdot K \left( 2\sqrt{\frac{ar}{(r+a)^2+(L+z)^2}} \right) \right] + \\ &+ \frac{\mu NI}{2\pi r} \frac{(L-z)}{\left( (r-a)^2+(L-z)^2 \right) \sqrt{(r+a)^2+(L-z)^2}} \cdot \left[ \left( r^2+a^2+(L-z)^2 \right) \cdot E \left( 2\sqrt{\frac{ar}{(r+a)^2+(L-z)^2}} \right) - \left( (r-a)^2+(L-z)^2 \right) \cdot K \left( 2\sqrt{\frac{ar}{(r+a)^2+(L-z)^2}} \right) \right], \\ B_\varphi &= 0. \end{aligned} \quad (5)$$

$$\begin{aligned} B_z &= \frac{\mu NI}{2\pi} \frac{1}{\left( (r-a)^2+(L+z)^2 \right) \sqrt{(r+a)^2+(L+z)^2}} \cdot \left[ \left( (r-a)^2+(L+z)^2 \right) \cdot K \left( 2\sqrt{\frac{ar}{(r+a)^2+(L+z)^2}} \right) - \left( r^2-a^2+(L+z)^2 \right) \cdot E \left( 2\sqrt{\frac{ar}{(r+a)^2+(L+z)^2}} \right) \right] + \\ &+ \frac{\mu NI}{2\pi} \frac{1}{\left( (r-a)^2+(L-z)^2 \right) \sqrt{(r+a)^2+(L-z)^2}} \cdot \left[ \left( (r-a)^2+(L-z)^2 \right) \cdot K \left( 2\sqrt{\frac{ar}{(r+a)^2+(L-z)^2}} \right) - \left( r^2-a^2+(L-z)^2 \right) \cdot E \left( 2\sqrt{\frac{ar}{(r+a)^2+(L-z)^2}} \right) \right]. \end{aligned}$$

In the relations (4) and (5) the terms  $K(\cdot)$ , respectively  $E(\cdot)$ , are the complete elliptic integral of the first kind, respectively of the second kind.

For the determination of the uniformity domains of the magnetic field it is calculated firstly the magnetic flux density modulus with the relation:

$$B = \sqrt{B_r^2 + B_\varphi^2 + B_z^2}. \quad (6)$$

Developing in Taylor series (until the terms of the fourth order inclusive) the magnetic flux density modulus in the center of the Helmholtz coil  $(r,z)=(0,0)$  it follows:

$$|\bar{B}| = \frac{\mu NI a^2}{(a^2 + L^2)^{\frac{3}{2}}} \left[ 1 - \frac{3(5L^2 - 2a^2)}{8(a^2 + L^2)^2} r^2 + \frac{3(4L^2 - a^2)}{2(a^2 + L^2)^2} z^2 + \frac{90a^4 - 648L^2 a^2 + 351L^4}{128(a^2 + L^2)^4} r^4 - \frac{90a^4 - 837L^2 a^2 + 468L^4}{16(a^2 + L^2)^4} r^2 z^2 + \frac{15a^4 - 180L^2 a^2 + 120L^4}{8(a^2 + L^2)^4} z^4 \right]. \quad (7)$$

The magnetic field is uniform if the terms of the second and the fourth order from Taylor series development are annulled. For example the annulment condition for the term in  $z^2$  is  $a = 2L$ . In the following table are presented the annulment conditions for the terms from the relation (6).

Term	Condition
$r^2$	$a = \sqrt{\frac{5}{2}}L$
$z^2$	$a = 2L$
$r^4$	$a = \frac{\sqrt{360 \pm 10\sqrt{906}}}{10}L$
$r^2 z^2$	$a = \frac{\sqrt{465 \pm 5\sqrt{6569}}}{10}L$
$z^4$	$a = \sqrt{6 \pm 2\sqrt{7}}L$

Table 1: Annulment conditions.

For the effectuation of the analytical calculi which have led to the relations (4), (5) and (7) the MAPLE program for the symbolic manipulation of the expression have been utilized.

The main 3D uniformity regions of the magnetic field function of the different annulment conditions of the terms from Taylor series development of the magnetic flux density modulus are given back in the figures 2.10. The values of the parameters are: ampere-turns of the coils  $NI = 100$  A, the coils radius  $a = 20$  cm.

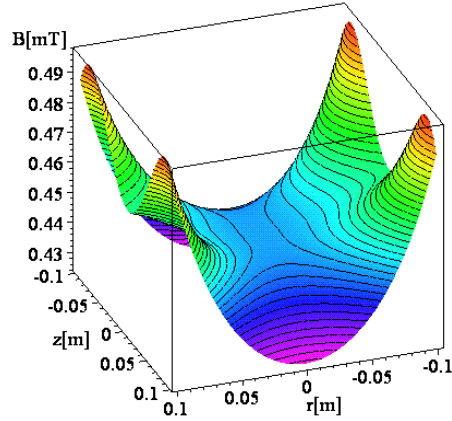


Figure 2: 3D variation of the magnetic flux density in an axial section of Helmholtz coil for  $a = 2L$ .

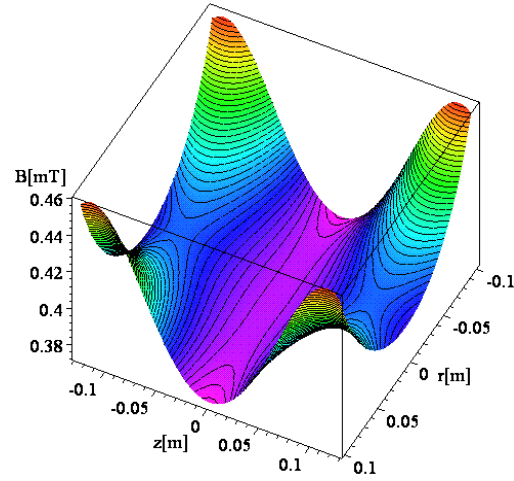


Figure 3: 3D variation of the magnetic flux density in an axial section of Helmholtz coil for  $a = \sqrt{5/2}L$ .

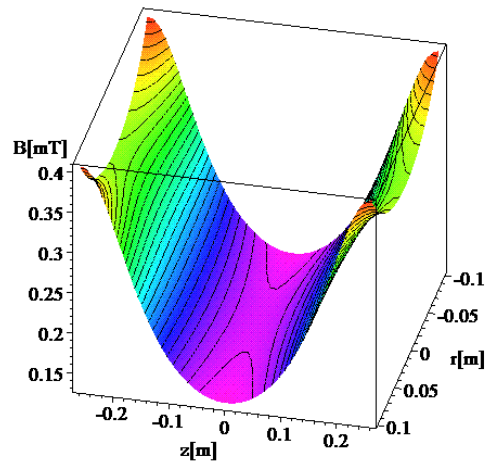


Figure 4: 3D variation of the magnetic flux density in an axial section of Helmholtz coil for  $a = \sqrt{360 \pm 10\sqrt{906}}/10L$

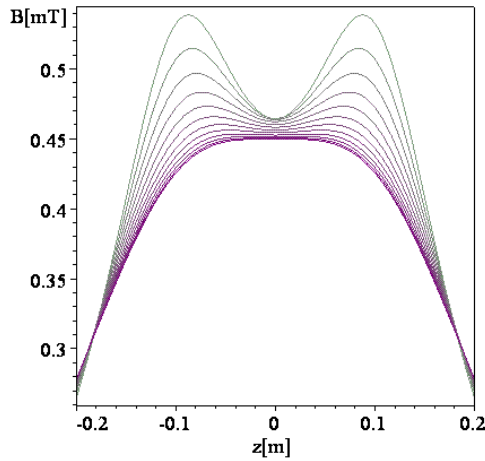


Figure 5: Magnetic flux density variation along  $Oz$  axis of Helmholtz coil in case  $a = 2L$  (for the different values of the coordinate  $z$ )

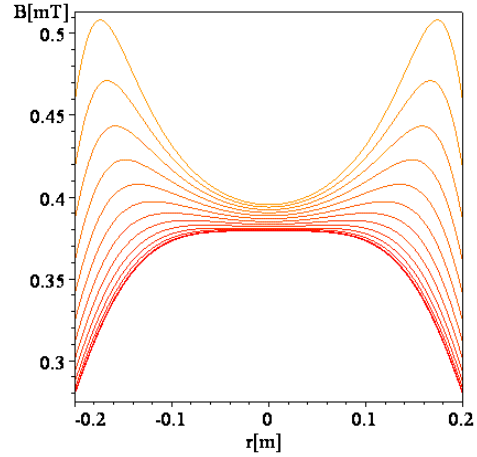


Figure 8: Magnetic flux density variation along  $O$  axis of Helmholtz coil in case  $a = \sqrt{5}/2L$  (for the different values of the coordinate  $z$ )

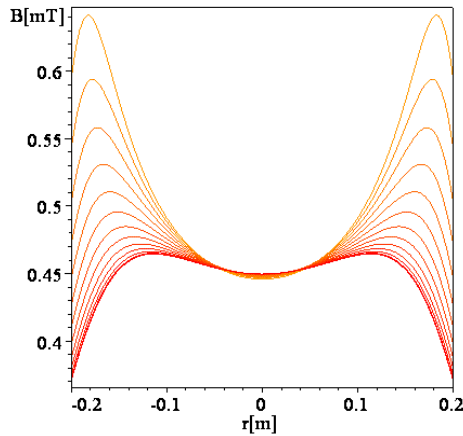


Figure 6: Magnetic flux density variation along  $O$  axis of Helmholtz coil in case  $a = 2L$  (for the different values of the coordinate  $z$ )

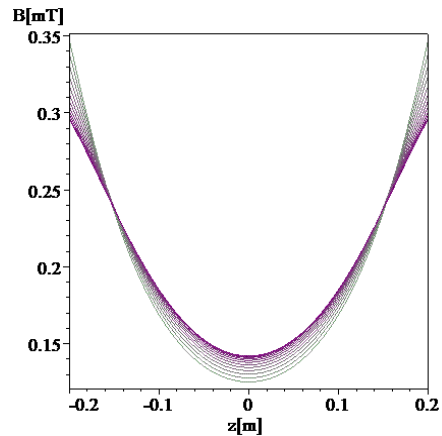


Figure 9: Magnetic flux density variation along  $Oz$  axis of Helmholtz coil for  $a = \sqrt{360 \pm 10\sqrt{906}}/10L$  (for the different values of the coordinate  $z$ )

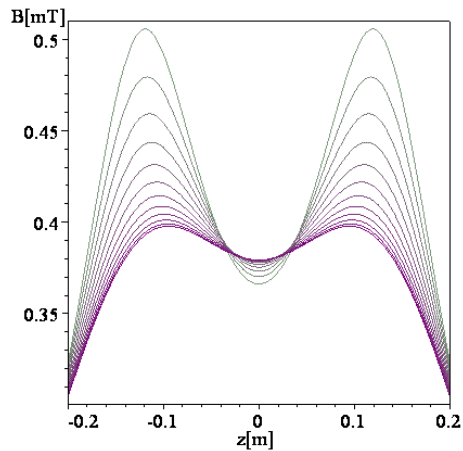


Figure 7: Magnetic flux density variation along  $Oz$  axis of Helmholtz coil in case  $a = \sqrt{5}/2L$  (for the different values of the coordinate  $z$ )

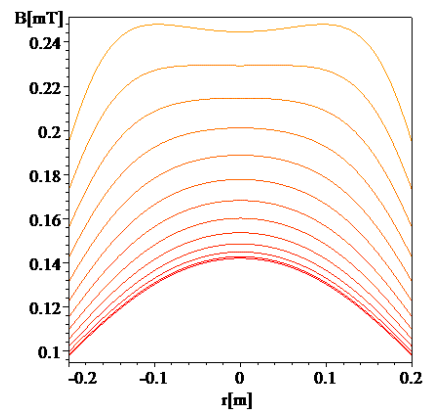


Figure 10: Magnetic flux density variation along  $O$  axis of Helmholtz coil for  $a = \sqrt{360 \pm 10\sqrt{906}}/10L$  (for the different values of the coordinate  $z$ )

For the comparison and for the results validation the magnetic field is calculated also numeric, utilizing the analysis program of the electromagnetic field with finite elements FEMM4.0 (fig.11..16).

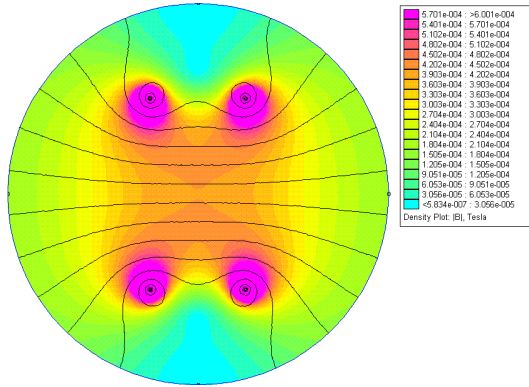


Figure 11: Magnetic field spectrum in an axial section of Helmholtz coil in case  $a = 2L$ .

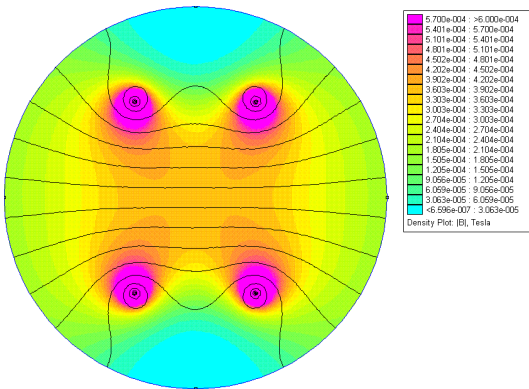


Figure 12: Magnetic field spectrum in an axial section of Helmholtz coil in case  $a = \sqrt{5}/2L$

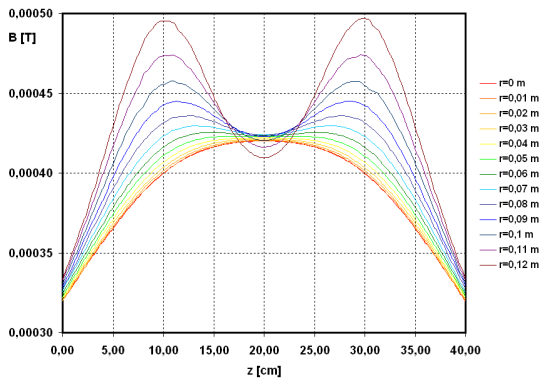


Figure 13: Magnetic flux density variation along  $Oz$  axis of Helmholtz coil in case  $a = 2L$  (for the different values of the coordinate  $r$ )

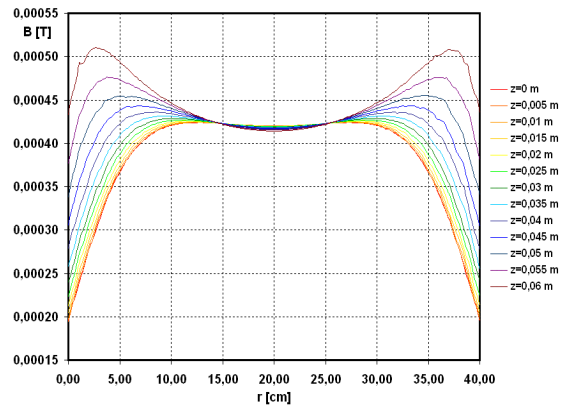


Figure 14: Magnetic flux density variation along  $O$  axis of Helmholtz coil in case  $a = 2L$  (for the different values of the coordinate  $z$ )

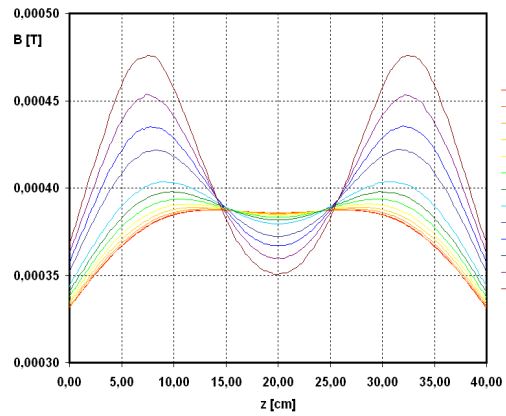


Figure 15: Magnetic flux density variation along  $Oz$  axis of Helmholtz coil in case  $a = \sqrt{5}/2L$  (for the different values of the coordinate  $r$ )

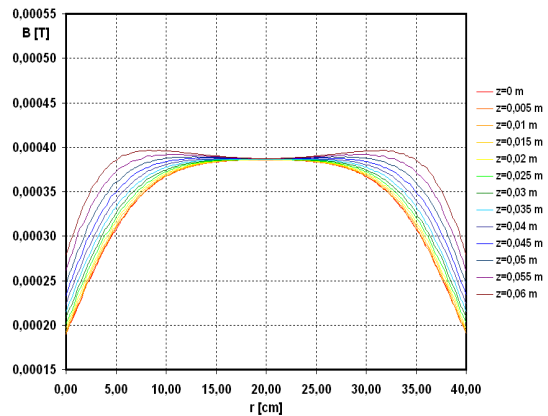


Figure 16: Figure 8: Magnetic flux density variation along  $O$  axis of Helmholtz coil in case  $a = \sqrt{5}/2L$  (for the different values of the coordinate  $z$ )

#### 4. CONCLUSIONS

The paper presents a new approach regarding the calculus of the uniformity regions of the magnetic field generated by Helmholtz coils.

From the effectuated analysis it is observed that, function de imposed conditions for the annulment of one or another from the terms of Taylor series development of the magnetic flux density modulus, new regions of uniform magnetic field appear, in comparison with the region made evident by the classic study. The localization of these new regions with the uniform magnetic field can be useful in many applications from the physics and technique, for example the electromagnetic lens designing.

It has to be specified that this study can be done only analytical, the numerical methods being used only for the validation of the obtained results.

The complex analytical calculi from this study have been effectuated using the facilities offered by the symbolic manipulator MAPLE. In addition this program offers the possibility for the effectuation of the accurate numerical calculi and contains a series of function with which can be realized suggestive graphical representations 1D, 2D and 3D (variation curves, field spectra, equipotential surfaces, etc.)

The study will be continued by the utilization of a classic nonlinear optimization program or with genetic algorithms what permits the determination of the geometrical conditions necessary for the obtaining of the largest regions with the uniform magnetic field.

#### References

- [1] C. I. Mocanu, *Teoria câmpului electromagnetic*, Ed. Didactică și Pedagogică, București, 1991
- [2] Gh. Gavrilă, *Electrostatica*, Ed. Didactică și Pedagogică, București, 1998
- [3] A. Moraru, *Complemente de teoria câmpului electromagnetic*, Ed. MATRIX ROM, București, 2003
- [4] Gh. Gavrilă, *Bazele electrotehnicii vol. IV. Teoria câmpului electromagnetic (Electrocinetică și Electrodinamică)*, Ed. Academiei Tehnice Militare, 1997
- [5] I. Munteanu, D. Ioan, F. Enache, *Symbolic Computation in Electromagnetics: A Classroom Experiment*, Proceedings of CEFC'00, Milwaukee, Wisconsin, U.S.A., June 4-7, 2000
- [6] Gh. Gavrilă, F. Enache, *Computation of the Electrostatic Fields in the Case of the True Electric Charges Distributions*, 5th International Conference on Electromechanical and Power Systems (SIELMEN 2005), October 6-8, 2005, Chișinău, Proceedings, volume 2, p.1004-1007
- [7] D. Ioan, F. Enache, *Symbolic computation in electromagnetics*, Proceedings of 7th International Workshop Symbolic Methods and Applications to Circuit Design (SMACD 2002), Sinaia, Romania, October 10-11, 2002, pp.145-150
- [8] K. M. Heal, M. L. Hansen, K. M. Richard, *Maple V – Learning Guide*, Ed. Springer Verlag, 1996