THE COMPUTATION OF THE POWER FACTOR IN NETWORKS RUNNING IN DISTORTING AND NONSYMMETRICAL STATES

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Abstract - In networks running in distorting and nonsymmetrical states, beside the active power on fundamental and on positive sequence flow other powers injected by the distorting receivers and nonsymmetrical ones. The two act as generators of harmonic powers, respectively, nonsymmetrical powers. In a previous paper [7] it was presented the active powers flow.
This paper analyzes a network that supplies equivalent receivers grouped in three main categories, symmetrical receivers, nonsymmetrical and distorting receivers and points a way to calculate the power factor at every type of consumer. This way of splitting into terms the general expression of the power factor has a good merit, showing the influence of the distorting state, of the nonsymmetrical one and the reciprocal influence of the two running states.

***Keywords:** distorting state, nonsymmetrical state, power factor*

1. INTRODUCTION

Strictly speaking, the running states of a complex three phase network is nonsymmetrical and nonsinusoidal. It is nonsymmetrical because it is almost impossible to maintain the three phases balanced, speaking about the consumers connected at them and it is nonsinusoidal because it is impossible not to exist at least one nonlinear consumer (distorting receiver), which introduces harmonics in the network. Of course, if proper actions are taken the practical running state can be considered sinusoidal and in many cases symmetrical, too.
It is considered a complex three phase distribution network that consists of an equivalent generator, a supplying line through which different types of consumers are supplied with electric energy. The consumers are grouped in three equivalent ones, a symmetrical, a nonsymmetrical and a distorting one.
The working hypothesis is:
- The generator is symmetric and of very large power (the voltages on the bars in the distribution point are always symmetric).
- The supplying line is symmetric; the coupling reactances between phases are neglected.
From the same bars from the distribution point there are supplied three kinds of equivalent receivers connected in star with the neutral insulated:
- A symmetrical one, without coupling between the three phases.
- A distorting receiver, which introduces harmonics of current in the whole network.
- A nonsymmetrical one, without coupling between the three phases. His impedances are modelled through the following quantities:
$\underline{Z}_1 = \alpha(R + jX) = \alpha\underline{Z}$, $\underline{Z}_2 = \alpha^{-1}\underline{Z}$, $\underline{Z}_3 = e^{\alpha-1}\underline{Z}$.
The usage of this model presents the following advantages: the real positive number $\alpha \in [0, \infty)$, named coefficient of nonsymmetry of the receiver, through its variation, models the dynamic behaviour of the active and reactive loads from every phase of the receiver. For example: for $\alpha = 0$ can be modelled the short circuit on a phase, together with the interruption of an other one and the single phase supply on the third phase; for $\alpha = 1$ can be modelled the three phase symmetric receiver; for α taking very low values, two phases are very charged, one being very discharged; for α taking very high values, two phases are very discharged and one, very charged, a.s.o.
The following notations, superscripts and subscripts are used:
P, *Q*: active, respectively, reactive power; *1*: the fundamental (harmonic of rank 1); *h*: harmonics; the plus (+) sign: the component of positive sequence; the minus (-) sign: the component of negative sequence; g: generator; *l*: line; *s*: symmetrical receiver; *n*: nonsymmetrical receiver; *d*: distorting receiver.
The generator supplies active power only on fundamental and on positive sequence ($P_{g1}^{+} > 0$). This is absorbed, in different proportions, by line ($P_{l1}^{+} > 0$), by the symmetrical receiver ($P_{s1}^{+} > 0$), by the nonsymmetrical one ($P_{n1}^{+} > 0$) and by the distorting receiver ($P_{d1}^{+} > 0$).
The distorting receiver uses a big part of the power on fundamental and on positive sequence received from the generator, P_{d}^{+} the other being converted into active powers on harmonics of positive sequence, P_{dh}^{+}, which are spread in the whole network: to the line, P_{lh}^{+}, to the symmetrical receiver, P_{sh}^{+}, and to the nonsymmetrical one, P_{nh}^{+}.

The nonsymmetrical element receives active power on fundamental and on positive sequence from the generator ($P_{n1}^{+} > 0$) and active power on harmonics of positive sequence from the distorting element ($P_{nh}^{+} > 0$). An important part of P_{n1}^{+} is used by the receiver on fundamental ($P_{n1} > 0$) and the other part is converted into active power on fundamental of negative sequence, P_{n1}^{-}. Please remember that all the components of zero sequence are null in our case.

The same with the power received on harmonics, $P_{nh}^{+} > 0$: a part remains at the receiver, $P_{nh} > 0$ and the other part is converted into active power on harmonics of negative sequence, P_{nh}^{-}. The two powers of negative sequence are injected into network and flow through the line ($P_{l1}^{-} > 0$, $P_{lh}^{-} > 0$), to the symmetrical element ($P_{s1}^{-} > 0$, $P_{sh}^{-} > 0$) and to the distorting one ($P_{d1}^{-} > 0$, $P_{dh}^{-} > 0$), being, all of them, received powers.

With regard to the line and to the symmetrical receiver, both of them are acting the same. In all the situations, they act as receivers for all the active powers injected to them.

With reference to the notations from above, the terms P_{dh}^{+}, P_{n1}^{-}, P_{nh}^{-} are, at their origin, powers injected by an element, the distorting one (*d*) for the first power, respectively, the nonsymmetrical one (*n*) for the next two. They are powers transformed from one kind into another, from P_{d1}^{+} for the first term, from P_{n1}^{+} for the second and from P_{nh}^{+} for the last one. For the places where they manifest they represent additional active power losses, for the distorting element *d* in the case of the first term and for the nonsymmetrical one *n* in the case of the next two-expression.

If there is no distorting consumer, all the terms having the second subscript *h* are null:

$P_{sh}^{+} = 0$, $P_{sh}^{-} = 0$, $P_{dh}^{+} = 0$, $P_{dh}^{-} = 0$, $P_{nh} = 0$, $P_{nh}^{-} = 0$.

If there is no nonsymmetrical consumer all the terms having the superscript minus (-) are null:

$P_{s1}^{-} = 0$, $P_{d1}^{-} = 0$, $P_{n1}^{-} = 0$.

The terms from above are null because the cause disappeared.

The terms that show that the nonsymmetrical element injects some kinds of power but which are not produced by him, are also null in this case:

$P_{sh}^{-} = 0$, $P_{dh}^{-} = 0$, $P_{nh}^{-} = 0$

If there are no distorting and nonsymmetrical consumers, the operating state is sinusoidal and symmetrical and all the additional active powers losses are null.

2. THE POWER FACTOR AT THE CONSUMERS

The power factor at a receiver is defined by the expression $K_P = P/S$, where *P* represents the amount of active power successfully transformed into another kind of power and *S* represents the apparent power received by the consumer.

Regarding the distorting receiver:

If assume that the distorting receiver successfully transforms into another kind of power only the quantity P_d^{+} , the other being additional loss and that there are no other sources of reactive power in network other than the generator itself (and thus the apparent power received by him is S_{d1}^{+}), the power factor is:

$$K_{Pd} = \frac{P_d^{+}}{S_{d1}^{+}} = \frac{P_d - P_{d1}^{-} - \left|P_{dh}^{+}\right| - P_{dh}^{-}}{S_{d1}^{+}} = \frac{P_d - \Delta P_{d\,nn} - \Delta P_{d\,dd} - \Delta P_{d\,nd}}{S_{d1}^{+}} = K_{P\,d\,sym.sin.} - K_{P\,d\,nonsym} - K_{P\,d\,dis} - K_{P\,d\;either} \quad (1)$$

Where, for the distorting consumer, the terms represent:

- $K_{P\,d\,sym.sin.} = P_d / S_{d1}^{+}$: the power factor of the symmetrical (abbreviation *sym.*) and sinusoidal (abbreviation *sin.*) state;
- $K_{P\,d\,nonsym} = \Delta P_{d\,nn} / S_{d1}^{+}$: the power factor of nonsymmetry (abbreviation *nonsym*);
- $K_{P\,d\,dis} = \Delta P_{d\,dd} / S_{d1}^{+}$: the power factor of distortion (abbreviation *dis*);
- $K_{P\,d\;either} = \Delta P_{d\,nd} / S_{d1}^{+}$: the power factor *either* of distortion or nonsymmetry.

All the terms from expression (1) are positive. The power factor K_{Pd} takes his highest value in the sinusoidal and symmetrical state, being $K_{P\,d\,sym.sin.}$, which, in this case, has the expression $K_{P\,d\,sym.sin.} = P_{d1}^{+} / S_{d1}^{+}$. Any other operating state reduces the power factor with some quantities: $K_{P\,d\,nonsym}$ for the sinusoidal and nonsymmetrical state, $K_{P\,d\,dis}$ for the nonsinusoidal and symmetrical state and with $K_{P\,d\,dis}$, $K_{P\,d\,nonsym}$ and $K_{P\,d\;either}$ for the nonsinusoidal and nonsymmetrical state.

This decomposition of the power factor into certain quantities has more a didactic purpose and it shows very well:

- the influence of the distorting state (through the term $K_{P\,d\,dis}$) and the nonsymmetrical one (through the term $K_{P\,d\,nonsym}$);

- the reciprocal influence of the two running states (the distorting and the nonsymmetrical) through the term $K_{P\,d\ either}$.
- for the sinusoidal and symmetrical state the formula becomes the general one, $K_{P\,d\,sym.sin.} = P_{d1}^{+} / S_{d1}^{+}$.
- it shows the methods to improve the power factor, meaning the minimization of the magnitude of the nonsinusoidal and nonsymmetrical state, bringing it closer to the sinusoidal and symmetrical state and the minimization of the reactive power delivered by the generator to every consumer.

In particular cases, some terms can be null, such as:
- for the nonsinusoidal and symmetrical state:

$$K_{P\,d\,nonsym} = 0\ ,\ K_{P\,d\ either} = 0\,. \tag{2}$$

- for the sinusoidal and nonsymmetrical state:

$$K_{P\,d\,dis} = 0\,,\ K_{P\,d\ either} = 0\,. \tag{3}$$

- for the sinusoidal and symmetrical state:

$$K_{P\,d\,dis} = 0\,, K_{P\,d\,nonsym} = 0\,,$$
$$K_{P\,d\ either} = 0\,, K_{P\,d} = K_{P\,d\,sym.sin.}\,. \tag{4}$$

Regarding the nonsymmetrical receiver:
If assume that the nonsymmetrical receiver successfully transforms into another kind of power only the quantity P_{n1}, the other being additional loss and that there are no other sources of reactive power in network other than the generator itself, so the apparent power received by him being S_{n1}^{+}, the power factor is:

$$K_{P\,n} = \frac{P_{n1}}{S_{n1}^{+}} = \frac{P_n - \left|P_{n1}^{-}\right| - P_{nh} - \left|P_{nh}^{-}\right|}{S_{n1}^{+}} =$$
$$= \frac{P_n - \Delta P_{n\,nn} - \Delta P_{n\,dd} - \Delta P_{n\,nd}}{S_{n1}^{+}} = \tag{5}$$
$$= K_{P\,n\,sym.sin.} - K_{P\,n\,nonsym} - K_{P\,n\,dis} - K_{P\,n\ either}$$

Where, for the nonsymmetrical consumer, the terms represent:
- $K_{P\,n\,sym.sin.} = P_n / S_{n1}^{+}$: the power factor of the symmetrical and sinusoidal state;
- $K_{P\,n\,nonsym} = \Delta P_{n\,nn} / S_{n1}^{+}$: the power factor of nonsymmetry;
- $K_{P\,n\,dis} = \Delta P_{n\,dd} / S_{n1}^{+}$: the power factor of distortion;
- $K_{P\,n\ either} = \Delta P_{n\,nd} / S_{n1}^{+}$: the power factor *either* of distortion or nonsymmetry.

The biggest of all is, of course, the power factor of the symmetrical and sinusoidal state, which in this case has the expression $K_{P\,n\,sym.sin.} = P_{n1}^{+} / S_{n1}^{+}$.

The same remarks concerning the power factors mentioned related to the distorting element can be made related to the nonsymmetrical receiver. Same expressions as (2) – (4) can be written changing the subscript *d* into *n*.

Regarding the symmetrical receiver:
If assume that the symmetrical receiver successfully transforms into another kind of power only the quantity P_{s1}^{+}, the other being additional loss and that there are no other sources of reactive power in network other than the generator itself, so the apparent power received by him being S_{s1}^{+}, the power factor is:

$$K_{P\,s} = \frac{P_{s1}^{+}}{S_{s1}^{+}} = \frac{P_s - P_{l1}^{-} - P_{sh}^{+} - P_{sh}^{-}}{S_{s1}^{+}} =$$
$$= \frac{P_s - \Delta P_{s\,nn} - \Delta P_{s\,dd} - \Delta P_{s\,nd}}{S_{s1}^{+}} = \tag{6}$$
$$= K_{P\,s\,sym.sin.} - K_{P\,s\,nonsym} - K_{P\,s\,dis} - K_{P\,s\ either}$$

Where, for the symmetrical consumer, the terms represent:
- $K_{P\,s\,sym.sin.} = P_s / S_{s1}^{+}$: the power factor of the symmetrical and sinusoidal state;
- $K_{P\,s\,nonsym} = \Delta P_{s\,nn} / S_{s1}^{+}$: the power factor of nonsymmetry;
- $K_{P\,s\,dis} = \Delta P_{s\,dd} / S_{s1}^{+}$: the power factor of distortion;
- $K_{P\,s\ either} = \Delta P_{s\,nd} / S_{s1}^{+}$: the power factor *either* of distortion or nonsymmetry.

The biggest of all is, of course, the power factor of the symmetrical and sinusoidal state, which in this case has the expression $K_{P\,s\,sym.sin.} = P_{s1}^{+} / S_{s1}^{+}$.

The same remarks concerning the power factors mentioned related to the distorting element can be made related to the symmetrical receiver. Same expressions as (2) – (4) can be written changing the subscript *d* into *s*.

The proportion with which every of the three terms reduce the power factor depends on the magnitude of the nonsymmetry and of the installed power of the distorting receivers. The nonsymmetry is appreciated by the coefficients of dissymmetry for voltage and current (respectively by the coefficients of asymmetry, which in our case is null) that are: $K_{du} = U^{-} / U^{+}$, $K_{di} = I^{-} / I^{+}$. The magnitude of the distorting state is appreciated with the help of the coefficients of distortion on every harmonic (for voltage and current), $\gamma_{uh} = U_h / U_1$, $\gamma_{ih} = I_h / I_1$ and globally with the help of Total Harmonic Distortion (*THD*) for voltage (*u*) and current (*i*), which are computed with $THD_u = \sqrt{\sum_{h=2}^{\infty} U_h^2} \Big/ U_1$, $THD_i = \sqrt{\sum_{h=2}^{\infty} I_h^2} \Big/ I_1$.

In the considered case, for the coefficient of nonsymmetry $\alpha \in (0,\ 5)$, the magnitude of the distorting state is:

$K_{du} \in (0,\ 0.0587)$, $K_{di} \in (0,\ 0.67)$, $THD_u = 0.0327$, $THD_i = 0.0997$.

The three power factors were computed, being:

$K_{P\ sym.sin.} \in (0.57,\ 0.707)$, $K_{P\ nonsym} \in (0,\ 0.03)$,

$K_{P\ dis} = 2\% K_{P\,sym.sin.}$, $K_{P\ either} \in (0,\ 0.1)\% K_{P\,sym.sin.}$.

As it can be observed, the proportion with which the distorting and nonsymmetrical state reduces the power factor is not very significant, the most undesirable effect of operating in these kinds of states being the reducibility of the quality of the energy delivered.

3. CONCLUSIONS

- In a distorting and nonsymmetrical running state, in a network that has symmetrical, nonsymmetrical and distorting receivers, at every element of the network, beside the active power on fundamental and on positive sequence, can be found active powers on harmonics of positive and negative sequences.
- The general expression of the power factor at any kind of the consumers can be splitting into terms that show the influence of the distorting state, of the nonsymmetrical one and the reciprocal influence of the two running states.

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