ON THE SENSITIVITY ANALYSIS OF ANALOG CIRCUITS

Mihai IORDACHE, Lucia DUMITRIU, Dragos NICULAE

Authors' Politehnica" University of Bucharest, Electrical Engineering Department,
iordache@elth.pub.ro, dlucia@elth.pub.ro, dragosn@elth.pub.ro

Abstract – **The paper presents three procedures for the sensitivity analysis using auxiliary circuits: the Bykhovsky Perkins Cruz's method, the incremental-circuit approach and the adjoint-circuit approach. All methods, based on the auxiliary circuits, have needed an efficient partially-symbolic method for the circuit analysis. For this, it was elaborated a very good procedure for the partially-symbolic analysis of the circuits based on the modified nodal equations and on the semi-state equations. This method was implemented in a program - SYCIAN – Symbolic Circuit Analysis. SYCIAN program can analysis together the original circuit and the auxiliary circuit for the complex linear time-invariant analog circuits. In this paper we will show that the methods of the sensitivity computation, based on the auxiliary circuits, become very competitive ones, if it is used the SYCIAN program. It is also underlined the advantages and disadvantages of the three procedures. Some illustrative examples are presented.**

Keywords: Network functions, Lumped linear circuits, Sensitivity network, Symbolic analysis.

1. INTRODUCTION

In the design of any system (circuit), it is important to know the effect on the system performance due to the variations of some system (circuit) parameters. In the case of lumped, linear, time-invariant circuits, a precise measure of this effect can be expressed in terms of the *sensitivity function* to be defined next [1-4]. From a practical point of view, it is not sufficient for the design specification to be satisfied for a fixed set of nominal parameter values. A circuit designer should known the circuit performance is affected by changes in one or more parameter values. Any effect of the circuit function or any other circuit characteristic caused by a change in one or more circuit parameters is referred to as *circuit sensitivity* [1, 2, 4, 7, and 8]. Let $F(s)$ be any circuit function of interest (driving-point immittances, transfer immittances, voltage gains, or current gains). At any particular frequency F is in general a complex number. The value of F of course varies with the frequency ($s = j\omega$) in general, except for resistive circuits. Let x be any parameter associated with some circuit element, x may be the element value (such as the impedance or transadmittance) or some physical parameter (such as temperature or pressure) that affects the element value. The *relative sensitivity*, or simply the *sensitivity of a circuit function F*, with respect to a parameter x, denote by S_x^F, is defined as

$$S_x^F \stackrel{\mathrm{d}}{=} \frac{\partial F}{\partial x}\cdot\frac{x}{F} = \frac{\dfrac{\partial F}{F}}{\dfrac{\partial x}{x}} = \frac{\partial \ln F}{\partial \ln x}. \tag{1}$$

According to equation (1), we may interpret the sensitivity S_x^F as the ration of the fractional change in the circuit function F to the fractional change in the parameter x, provided that all changes are sufficiently small (approaching zero theoretically). Sometimes we refer to S_x^F as the *normalized sensitivity*, in contrast with the *unnormalized sensitivity*, which is simply the partial derivative $\partial F / \partial x$.

A circuit parameter x is called critical if the circuit sensitivity with respect to this parameter is very large. By using computer programs, the effects of parameter changes on the circuit performance can be predicted, allowing the circuit designer to select a circuit of low sensitivity in performance to one with higher sensitivity, while preserving the desired upper and lower limits of the performance function [1, 2, and 12].

The most important three reasons why parameter changes should be taken into account in circuit design are:

1. Parameter values of physical devices are not known exactly before-hand. There is always some discrepancy between the parameter values in a circuit model, which represents the physical circuit for computational purposes, and the exact parameter values. This is the problem of accuracy in circuit modeling and analysis.
2. During the lifetime of a manufactured circuit, parameters are subject to change through ageing and various environmental effects, such as ambient temperature and humidity. A sensitivity analysis is therefore required to find out which circuit parameter is critical.
3. The great spread of parameter values resulting from the circuit-manufacturing process requires the knowledge of the circuit performance in a certain range of parameter values, known as the tolerance range. This generates the need for *tolerance analysis* [1-4, 7-9].

There are several reasons for the great importance of network sensitivity in analog circuit design:

1. The study of circuit sensitivity enhances insight into circuit behavior, when changes in circuit parameters are involved. By dividing the circuit parameters into critical

and non-critical parameters, an effective method is provided to simplify circuit models for the purpose of more efficient circuit analysis.
2. Knowledge of the circuit sensitivity can be used as a basis for comparing different electric circuits. It helps the circuit designer in selecting the proper circuit for a specified application.
3. The effect of the manufacturing tolerance inherent in circuit elements can be investigated by sensitivity analysis. The notion of circuit sensitivity facilitates the development of methods for tolerance analysis.
4. Network sensitivity plays an important part in the design and optimization of reliable circuits.
In present, there are several computer-oriented methods for the calculation of sensitivities. Methods for the computation of multiparameter sensitivity are divided into three classes: 1. *Methods based on feedback theory* or the *bilinear theorem* for linear circuits; 2. *Direct methods*, by which the first-order derivatives of interest (and any desired higher-order derivatives) are computed directly from the circuit matrix, usually the nodal admittance matrix (or modified nodal matrix); 3. *Indirect method*s, which require an auxiliary circuit associated with the circuit under consideration.
The most of the methods based on the auxiliary circuits have needed an efficient partially-symbolic method for the circuit analysis. We have elaborated a very good procedure for the partially-symbolic analysis of the circuits based on the modified nodal equations and on the semi-state equations.[4-6, 9-12]. This method was implemented in a program - **SYCIAN** – **Sy**mbolic **Ci**rcuit **An**alysis [9]. SYCIAN program can analysis together the original circuit and the auxiliary circuit. In this paper we will show that the methods of the sensitivity computation, based on the auxiliary circuits, become very competitive ones, if it is used the SYCIAN program.

2. SENSITIVITY ANALYSIS USING AUXILIARY CIRCUITS

2.1. Bykhovski Perkins Cruz's method

One significant method, first enunciated by Bykhovski [1,7], was described by Perkins and Cruz [7] – called *Bykhovsky-Perkins-Cruz's method*. Let be a linear and time-invariant circuit *C*. An auxiliary circuit can be constructed by replacing independent voltage sources by short-circuits, removing independent current sources and applying a controlled source in the branch of the element *x* with respect to which the network sensitivity is desired. The auxiliary circuit C_a has the same graph as the original circuit *C*. In the auxiliary circuit the branch currents and branch voltages represent the derivatives $\partial \boldsymbol{i}_b / \partial x$ and $\partial \boldsymbol{v}_b / \partial x$ respectively. The auxiliary circuit thus obtained will be called the *sensitivity circuit*. Let us consider an *RLC* network containing n_R resistors, n_C capacitors and n_L inductors. The following branch relations hold:

$$u_{R_k} = R_k i_{R_k},\ k = \overline{1, n_R};\ i_{C_j} = C_j \frac{du_{C_j}}{dt},$$
$$j = \overline{1, n_C};\ u_{L_p} = L_p \frac{di_{L_p}}{dt},\ p = \overline{1, n_L} \tag{2}$$

If we assume that the parameter x be the resistance R_k of a resistor, we can write

$$\frac{\partial u_{R_k}}{\partial R_k} = i_{R_k} + R_k \frac{\partial i_{R_k}}{\partial R_k},\ i_{C_j} = C_j \frac{d}{dt}\left(\frac{\partial u_{C_j}}{\partial R_k}\right),$$
$$u_{L_p} = L_p \frac{d}{dt}\left(\frac{\partial i_{L_p}}{\partial R_k}\right) \tag{3}$$

According to the relations (3) all the above partial derivatives of voltages and currents with respect to R_k are regarded as voltages and currents of a new circuit, the sensitivity circuit, its branch relations will be similar to those of the original controlled voltage, except in the Branch R_k which is augmented with a controlled voltage source in series (as in Table 1).

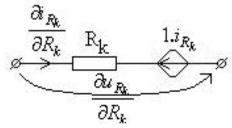
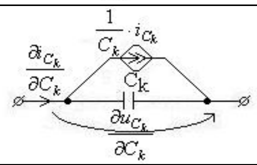
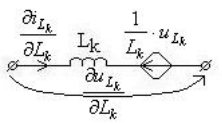
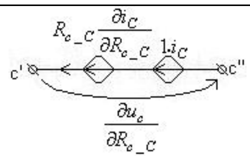
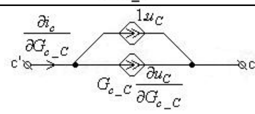
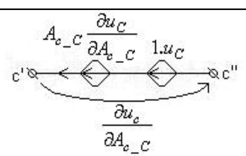
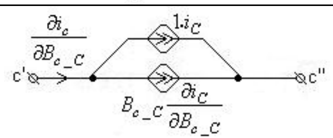
The circuit element *x*.	Branch structure in C_a.
Linear resistor liniar, R_k	
Linear capacitor, C_k	
Linear inductor, L_k	
CCVS $e_c(i_C)$, R_{c_C}	
VCCS $j_c(u_C)$, G_{c_C}	
VCVS $e_c(u_C)$, A_{c_C}	
CCCS $j_c(i_C)$, B_{c_C}	

Table 1: Construction of the auxiliary circuit.

The two circuits (the original circuit and auxiliary circuit) will be analyzed together having the same datum node (see the example).

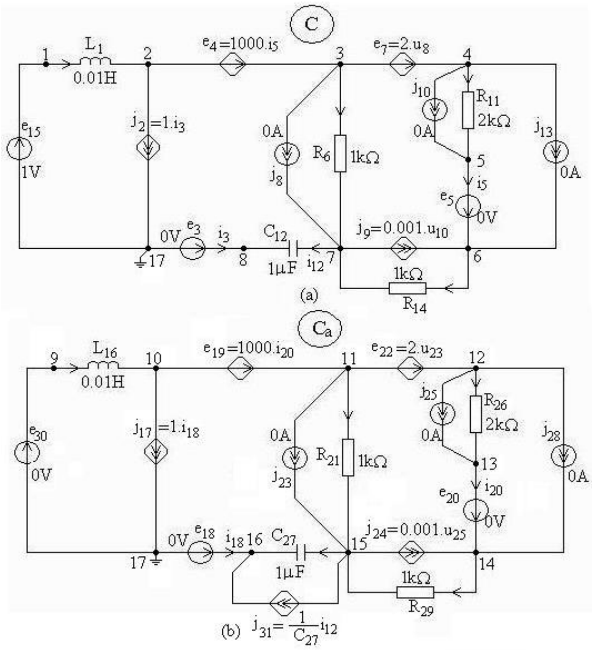


Figure 1. a) Original circuit C; b) Auxiliary circuit C_a.

Let the circuit of Fig. 1, a be given. It is required to find the first-order sensitivity $S_d(u_6, C_2)$. The sensitivity circuit is shown in Fig. 1, b.

Running the SYCIAN program [4, 9] we obtain the following results:

```
SA4,6_1,17_C12=U28 = .1200e11*s/(4000.+s)^2,
```

The desired sensitivity $\partial u_{13} / \partial C_{12} = S_d(u_{13}, C_{12})$ is equal to the voltage u_{28} from the circuit in Fig.1. b

$$\frac{\partial U_{13}}{\partial C_{12}} = u_{28} = \frac{\partial A_{4,6\_1,17}}{\partial C_2} = \frac{1.2 \cdot 10^{10} s}{(s+4000.0)^2} \tag{4}$$

2.2. Incremental circuit approach

Let us consider a linear circuit C consisting of a number of standard branches. We shall vary the impedance of each branch by a slight amount and obtain a perturbed circuit C_p. For C, we can write

$$\textbf{KCL: } \boldsymbol{A}\boldsymbol{I}_b = \mathbf{0}; \quad \textbf{KVL: } \boldsymbol{B}\boldsymbol{U}_b = \mathbf{0}, \tag{5}$$

where $\boldsymbol{A}$ and $\boldsymbol{B}$ is the reduced incidence matrix and fundamental loop matrix, respectively, and $\boldsymbol{I}_b$ and $\boldsymbol{U}_b$ are the branch current vector and branch voltage vector, respectively. Since the perturbed circuit Cp has the same topology as C, for C_p we have

$$\textbf{KCL: } \boldsymbol{A}(\boldsymbol{I}_b + \Delta\boldsymbol{I}_b) = \mathbf{0}; \quad \textbf{KVL: } \boldsymbol{B}(\boldsymbol{U}_b + \Delta\boldsymbol{U}_b) = \mathbf{0}. \tag{6}$$

From equations (5) – (6), we immediately have

$$\boldsymbol{A}\Delta\boldsymbol{I}_b = \mathbf{0}; \quad \boldsymbol{B}\Delta\boldsymbol{U}_b = \mathbf{0}, \tag{7}$$

which indicate that the incremental currents $\Delta\boldsymbol{I}_b$ and incremental voltages $\Delta\boldsymbol{U}_b$ have the same constraints as $\boldsymbol{I}_b$ and $\boldsymbol{U}_b$. Therefore, $\Delta\boldsymbol{I}_b$ and $\Delta\boldsymbol{U}_b$ could possibly be the branch currents and voltages of some circuit C_i having the same topology as C, provided that the branch characteristics of C_i are properly defined. We shall investigate how branch characteristics of C_i are to be defined for the purpose of producing $\Delta\boldsymbol{I}_b$ and $\Delta\boldsymbol{U}_b$. Consider an element in C with impedance Z, then

$$U = ZI. \tag{8}$$

For the perturbed circuit, the same element is described by

$$\begin{aligned} U + \Delta U &= (Z + \Delta Z)(I + \Delta I) = \\ &= ZI + Z\Delta I + I\Delta Z + \Delta Z\Delta I \end{aligned}. \tag{9}$$

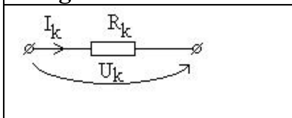
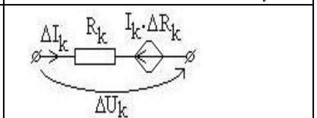
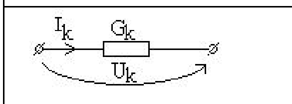
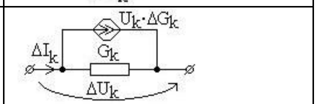
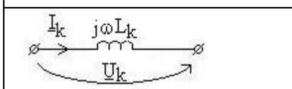
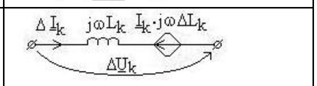
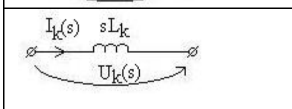
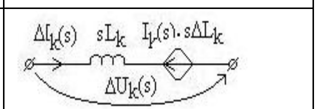
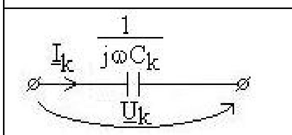
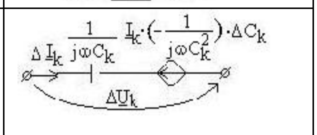
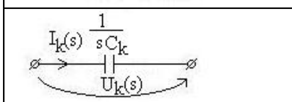
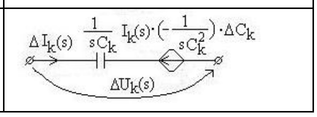
From equations (8) and (9), we have

$$\Delta U = Z\Delta I + I\Delta Z + \Delta Z\Delta I. \tag{10}$$

Provided that ΔZ (and hence ΔI and ΔU) is infinitesimally small, we can neglect higher order terms and rewrite equation (10) as

$$\Delta U = Z\Delta I + I\Delta Z \tag{11}$$

which indicates that in C_i, the branch having ($\Delta U, \Delta I$) consist of an impedance Z (the original impedance in C) in series with a current-controlled voltage sources $I\,\Delta Z$. This is illustrated in Table 2.

Original circuit C	**Incremental circuit C_i**
	
	
	
	
	
	

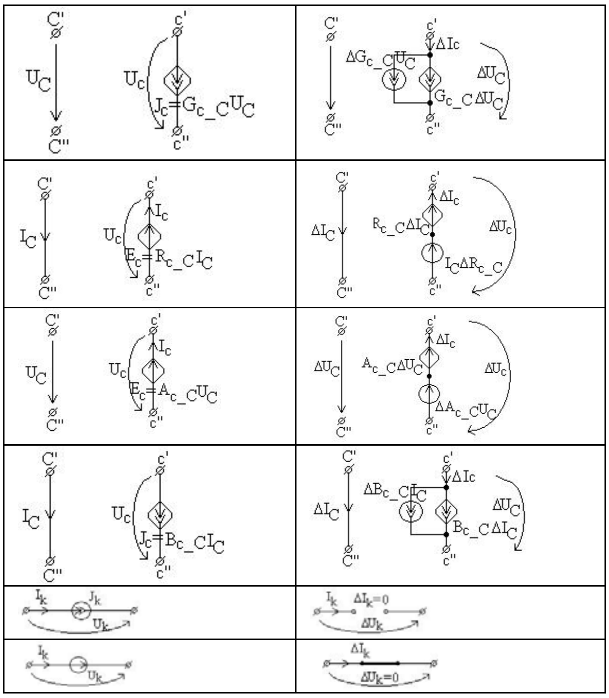


Table 2: Construction of the incremental circuit.

The circuit C_i derived from C according to the rules of Table 1.2. is an incremental circuit, since branch currents and voltages in C_i are ΔI 's and ΔU 's. It should be emphasized that the rules in Table 2 for the construction of the *incremental circuit are derived under the assumption that all parameter changes are very small* and that second-order effects can be neglected. Because all the controlled sources from the incremental circuit are controlling by the quantities from the original circuits, the two circuits must be analyzed together (having the same datum-node) [4, 9 – 12]. The active circuit C shown in Fig. 1, a has nominal values represented in this figure. The incremental circuit C_i is shown in Fig. 2. Find the partial derivates of the voltage gain $A_{2,3\_1,11}$ with respect to the circuit parameters.

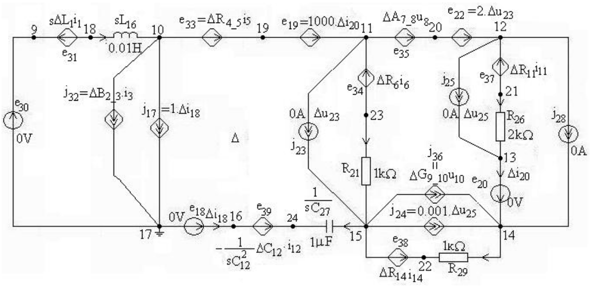


Figure 2. Sensitivity calculation by the incremental-circuit method.

Because, the input voltage E_i = 1 V then $A_{4,6\_1,17} = U_{13}$ and $\mathrm{d}A_{4,6\_1,17} = \mathrm{d}u_{28}$ (u_{28}).
Running the SYCIAN program [9] we obtain:

```
dA4,6_1,17 := s*(-4.5*dR14+7.5*dR6+3.0*dR11+
2500.0*dA7_82500000.0*dG9_10+.12e11*dC12+
.12e-3*dB2_3*s^2+.45e-2*s*dR4_5+2.5*dA7_8*s-
.45e-2*dR14*s-2500.000000*dG9_10*s-
.15e-2*dR11*s)/(s+4000.)^2;
```

and

$$\frac{\partial u_{13}}{\partial C_{12}} = \frac{\partial A_{3,7\_1,7}}{\partial C_2} = \frac{1.2 \cdot 10^{10}\, s}{(s+4000.0)^2}.$$

The sensitivity $\partial A_{4,6\_1,17} / \partial C_{12}$ is identical to the one obtained by the Bykhovsky-Perkins-Cruz's method.

2.3. Adjoint circuit approach

Two linear time-invariant circuits C and $\hat{C}$ are adjoint circuits of each other if the following three conditions are satisfied:

1. Both circuits have the same topology; i.e., $\boldsymbol{A} = \hat{\boldsymbol{A}}$, and $\boldsymbol{B} = \hat{\boldsymbol{B}}$. For controlled sources, we consider a controlling voltage as that across an open-branch (an ideal independent current source with j = 0.0A), and a controlling current as that through a short-circuit branch (an ideal independent voltage source with e = 0.0V).
2. If the no independent-source branches of C and $\hat{C}$ possess branch impedance matrices $\boldsymbol{Z}_b$, and $\hat{\boldsymbol{Z}}_b$, respectively, then

$$\boldsymbol{Z}_b^{\mathrm{t}} = \hat{\boldsymbol{Z}}_b. \tag{12}$$

On the other hand, if branch-admittance matrices $\boldsymbol{Y}_b$, and $\hat{\boldsymbol{Y}}_b$ exist, then

$$\boldsymbol{Y}_b^{\mathrm{t}} = \hat{\boldsymbol{Y}}_b. \tag{13}$$

In general case, the no source branches of C and $\hat{C}$ can always be characterized by hybrid matrices $\boldsymbol{H}_b$, and $\hat{\boldsymbol{H}}_b$ thus:

$$\begin{bmatrix} \boldsymbol{I}_{b1} \\ \boldsymbol{U}_{b2} \end{bmatrix} = \begin{bmatrix} \boldsymbol{Y}_{b11} & \boldsymbol{B}_{b12} \\ \boldsymbol{A}_{b21} & \boldsymbol{Z}_{b22} \end{bmatrix} \begin{bmatrix} \boldsymbol{U}_{b1} \\ \boldsymbol{I}_{b2} \end{bmatrix} = \boldsymbol{H}_b \cdot \begin{bmatrix} \boldsymbol{U}_{b1} \\ \boldsymbol{I}_{b2} \end{bmatrix} \tag{14}$$

and

$$\begin{bmatrix} \hat{\boldsymbol{I}}_{b1} \\ \hat{\boldsymbol{U}}_{b2} \end{bmatrix} = \begin{bmatrix} \hat{\boldsymbol{Y}}_{b11} & \hat{\boldsymbol{B}}_{b12} \\ \hat{\boldsymbol{A}}_{b21} & \hat{\boldsymbol{Z}}_{b22} \end{bmatrix} \begin{bmatrix} \hat{\boldsymbol{U}}_{b1} \\ \hat{\boldsymbol{I}}_{b2} \end{bmatrix} = \hat{\boldsymbol{H}}_b \cdot \begin{bmatrix} \hat{\boldsymbol{U}}_{b1} \\ \hat{\boldsymbol{I}}_{b2} \end{bmatrix}. \tag{15}$$

For C and $\hat{C}$ to be adjoint circuits of each other, we require that

$$\begin{bmatrix} \hat{\boldsymbol{Y}}_{b11} & \hat{\boldsymbol{B}}_{b12} \\ \hat{\boldsymbol{A}}_{b21} & \hat{\boldsymbol{Z}}_{b22} \end{bmatrix} = \begin{bmatrix} \boldsymbol{Y}_{b11}^{\mathrm{t}} & -\boldsymbol{A}_{b21}^{\mathrm{t}} \\ -\boldsymbol{B}_{b12}^{\mathrm{t}} & \boldsymbol{Z}_{b22}^{\mathrm{t}} \end{bmatrix}. \tag{16}$$

3. Corresponding independent sources in both circuits are the same in nature (current or voltage sources), but need not have the same values.

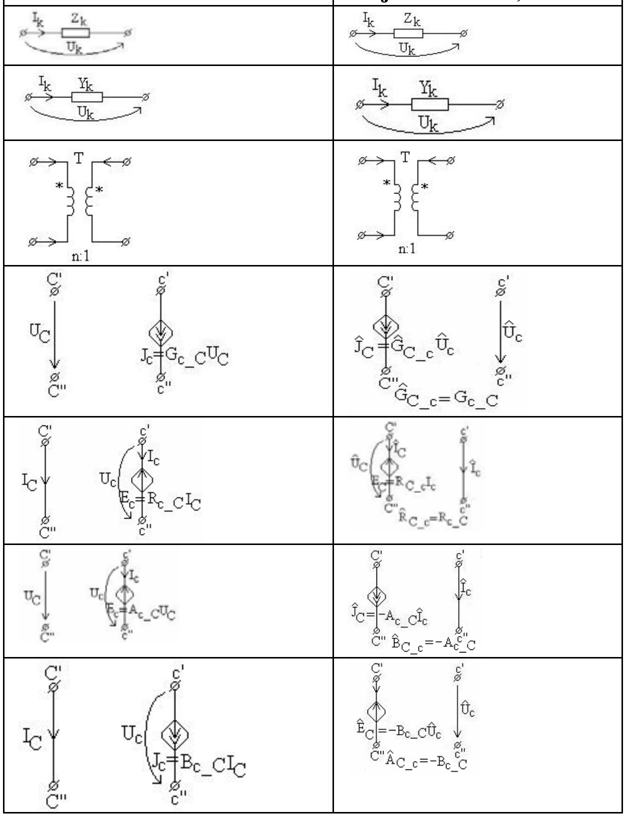
In many applications of adjoint circuits, it is sometimes convenient to extract all independent sources a multiport. We denote the port currents and voltages by $\boldsymbol{I}_p$ and $\boldsymbol{V}_p$, respectively, and the no source branch currents and voltages by $\boldsymbol{I}_b$ and $\boldsymbol{V}_b$, respectively. In [1, 2] it is proved that if the open-circuit impedance matrices $\boldsymbol{Z}_{oc}$, and $\hat{\boldsymbol{Z}}_{oc}$ (the short-circuit admittance matrices $\boldsymbol{Y}_{sc}$, and $\hat{\boldsymbol{Y}}_{sc}$) exist for the multiport created from C and its adjoint $\hat{C}$, then

$$\boldsymbol{Z}_{oc}^{\mathrm{t}} = \hat{\boldsymbol{Z}}_{oc}\,, \quad \boldsymbol{Y}_{sc}^{\mathrm{t}} = \hat{\boldsymbol{Y}}_{sc}\,. \tag{17}$$

It is more convenient to construct the adjoint circuit $\hat{C}$ from any given circuit C by use of Table 3.

Table 3: Construction of the adjoint circuit.

Original circuit, *C*	**Adjoint circuit, $\hat{C}$**



The multiport create from C and $\hat{C}$ by extracting all independent sources can be characterized by hybrid matrices $\boldsymbol{H}$ and $\hat{\boldsymbol{H}}$, respectively, as follows:

$$\begin{bmatrix} \boldsymbol{I}_E \\ \boldsymbol{U}_J \end{bmatrix} = \begin{bmatrix} \boldsymbol{Y}_{EE} & \boldsymbol{B}_{EJ} \\ \boldsymbol{A}_{JE} & \boldsymbol{Z}_{EE} \end{bmatrix} \begin{bmatrix} \boldsymbol{U}_E \\ \boldsymbol{I}_J \end{bmatrix} = \boldsymbol{H} \begin{bmatrix} \boldsymbol{U}_E \\ \boldsymbol{I}_J \end{bmatrix} \tag{18}$$

and

$$\begin{bmatrix} \hat{\boldsymbol{I}}_E \\ \hat{\boldsymbol{U}}_J \end{bmatrix} = \begin{bmatrix} \hat{\boldsymbol{Y}}_{EE} & \hat{\boldsymbol{B}}_{EJ} \\ \hat{\boldsymbol{A}}_{JE} & \hat{\boldsymbol{Z}}_{EE} \end{bmatrix} \begin{bmatrix} \hat{\boldsymbol{U}}_E \\ \hat{\boldsymbol{I}}_J \end{bmatrix} = \hat{\boldsymbol{H}} \begin{bmatrix} \hat{\boldsymbol{U}}_E \\ \hat{\boldsymbol{I}}_J \end{bmatrix}, \tag{19}$$

where the subscript E indicates independent voltage sources and J indicates independent current sources. It can be shown that matrices $\boldsymbol{H}$ and $\hat{\boldsymbol{H}}$ are related in the following manner:

$$\begin{bmatrix} \hat{\boldsymbol{Y}}_{EE} & \hat{\boldsymbol{B}}_{EJ} \\ \hat{\boldsymbol{A}}_{JE} & \hat{\boldsymbol{Z}}_{EE} \end{bmatrix} = \begin{bmatrix} \boldsymbol{Y}_{EE}^{\mathrm{t}} & -\boldsymbol{A}_{JE}^{\mathrm{t}} \\ -\boldsymbol{B}_{EJ}^{\mathrm{t}} & \boldsymbol{Z}_{EE}^{\mathrm{t}} \end{bmatrix}. \tag{20}$$

An equation relating the changes in $\boldsymbol{H}$ to the changes in $\boldsymbol{H}_b$ can be derived [1, 2]. This relationship has the following form:

$$\begin{aligned} & -\hat{\boldsymbol{I}}_J^{\mathrm{t}} \Delta \boldsymbol{U}_J + \hat{U}_E^{\mathrm{t}} \Delta \boldsymbol{I}_E = \\ & \begin{bmatrix} \hat{\boldsymbol{U}}_E^{\mathrm{t}} & -\hat{\boldsymbol{I}}_J^{\mathrm{t}} \end{bmatrix} \begin{bmatrix} \Delta \boldsymbol{Y}_{EE} & \Delta \boldsymbol{B}_{EJ} \\ \Delta \boldsymbol{A}_{JE} & \Delta \boldsymbol{Z}_{JJ} \end{bmatrix} \begin{bmatrix} \boldsymbol{U}_E \\ \boldsymbol{I}_J \end{bmatrix} = \\ & = \begin{bmatrix} -\hat{\boldsymbol{U}}_{l1}^{\mathrm{t}} & \hat{\boldsymbol{I}}_{l2}^{\mathrm{t}} \end{bmatrix} \begin{bmatrix} \Delta \boldsymbol{Y}_{11l} & \Delta \boldsymbol{B}_{12l} \\ \Delta \boldsymbol{A}_{21l} & \Delta \boldsymbol{Z}_{22l} \end{bmatrix} \begin{bmatrix} \boldsymbol{U}_{l1} \\ \boldsymbol{I}_{l2} \end{bmatrix}. \end{aligned} \tag{21}$$

For different types of elements in C and $\hat{C}$, the right side of equation (21) may be evaluated separately [1 – 9].

The procedure for calculating $\partial V_o / \partial x_k, \partial I_o / \partial x_k$, or $\partial h_{kj} / \partial x_k$ may by summarized as follows>

Step 1. Perform an analysis of the circuit C to obtain $\boldsymbol{V}_{b1}$ and $\boldsymbol{I}_{b2}$.

Step 2. Select the excitations for $\hat{C}$ such that one side of equation (20) yields only one term, which is $\Delta V_o, \Delta I_o$, or Δh_{kj}, of interest. Perform an analysis of the adjoint circuit $\hat{C}$ to obtain $\hat{\boldsymbol{V}}_{b1}$ and $\hat{\boldsymbol{I}}_{b2}$.

Step 3. Evaluate the right side of equation (21), either directly from matrix multiplications or with the aid to Table 4. From the resultant expression, obtain the desired partial derivatives.

Consider the circuit shown in Fig. 3, a. Find the partial derivatives of ΔA_{JE} with respect to all parameters of the circuit in Fig. 3. An examination of equation (20) shows that to have ΔA_{JE} only, we may choose $U_E = E_{15} = 1$ V, $I_J = J_{10} = 0$ A and $\hat{U}_E = \hat{E}_{15} = 0\,\mathrm{V}$, $\hat{I}_J = \hat{J}_{13} = -1$ A.

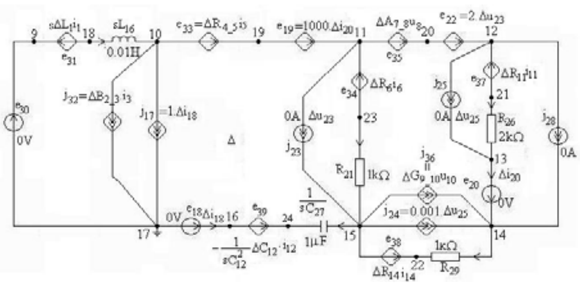


Figure 3. Sensitivity calculation by the adjoint-circuit method.

For example, the partial derivate of the voltage gain $A_{3,4\_1,5}$ with respect the parameter B_{2_3} has the following expression:

$$S_{B_{2\_3}}^{A_{4,6\_1,9}} = \frac{\partial A_{4,6\_1,17}}{\partial B_{2\_3}} = \frac{0.00012 * s^3}{(s + 4000.0)^2}.$$

The correctness of these results may be verified for this circuit by evaluating the following expression at the nominal element values:

```
A4,6_1,9 := -
1.*s*C12*R6*(A7_8+1.)*R11/((B2_3*C12*L1*R14+B2_3*
C12*L1*G9_10*R14*R11+B2_3*C12*L1*R6-
1.*C12*L1*R14-1.*C12*R6*L1-1.*C12*R6*L1*A7_8-
1.*C12*L1*G9_10*R14*R11-
1.*C12*L1*R11+B2_3*C12*L1*R11+B2_3*C12*L1*A7_8*R
6)*s^2+(C12*R6*R4_5*A7_8-1.*C12*R6*R14-
1.*C12*R6*R11-
1.*R14*C12*R6*G9_10*R11+C12*R6*R4_5)*s-1.*R14-
1.*R11-1.*G9_10*R14*R11-1.*A7_8*R6-1.*R6);
```

Evaluating the partial derivative with respect to the parameter B_{2_3} and substituting the nominal element values we obtain the same value for the $S_{B_{2\_3}}^{A_{4,6\_1,9}}$.

The adjoint-network approach can be used to derive second-order derivatives of linear, time-invariant circuits [1].The adjoint-network concept has been used to perform circuit analysis when large parameter changes occur in the circuit [2, 8]. It can also used for locating faults and for selecting test point with the purpose of locating faults in linear and nonlinear analog circuits [1 - 4].

3. CONCLUSION

The three methods for the sensitivity calculations, presented in this paper, can be applied to a large class of linear circuit containing: all four types of linear controlled sources, resistors, inductors, capacitors, nullors, and any multiterminal or multiport circuit element having an equivalent scheme made up only by two-terminal elements and controlled sources.

All sensitivity methods based on the auxiliary circuits have need of a very good procedure for partial-symbolic analysis. Our symbolic analysis technique, based on the modified nodal equations and implemented into a computing program, is suitable to analysis the original circuit together the auxiliary circuit. In this way, the sensitivity computation becomes more efficient. Higher-order derivatives can be easily be computed by attaching new higher-order sensitivity circuits to lower-order sensitivity networks. One of the disadvantages of the B - P - C's method is that in implementing of this method, the sensitivity network required for each circuit parameter of interest is coupled with the original circuit by inserting an appropriate controlled source. A large number of parameters x would make the size of the circuit that could be handled prohibitive. However, this method allows all frequency and time responses of both currents and voltages and their sensitivities to parameter changes to be evaluated in one computer run. From point of view of the computational effort, the incremental-circuit approach is less efficient than the adjoint-circuit approach. However, the incremental-circuit method has the following advantages: it provides better insight the effect of parameter variations, and in the analysis of the incremental network, the incremental currents and voltages of all branches are available from the calculations. Such incremental quantities are themselves of interest in some applications.

References

[1] A. E. Schwarz, Computer-aided design of microelectronic circuits and systems, Academic Press, London, 1987.

[2] L. O. Chua, and P. M. Lin, Computer-Aided Analysis of Electronic Circuits: Algorithms and Computational Techniques, Englewood cliffs, NJ:Prentice-Hall, 1975.

[3] W. J. McCalla, *Fundamentals of computer-aided circuit simulation*, Kluwer Academic Publishers, Boston, 1988.

[4] M. Iordache, Lucia Dumitriu, Modern theory of electrical circuits – Fundamentals, Applications, Algorithms, and Computing Programs, - Vol. II (in Romanian), All Educational Publishing, Bucharest 2000.

[5] M. Iordache, Lucia Dumitriu**,** "The Generalized Topological Formula for transfer Functions' Generation by Two-Graph tree enumeration", Analog Circuits and Signal Processing, Vol. 47, No. 1, April 2006, Kluwer Academic Publishers, pp. 85-100.

[6] M. Iordache, Lucia Dumitriu, "Time Domain Diakoptic Analysis Based on Reduced-Order State Equations", International Journal of Bifurcation and Chaos (IJBC), Volume: 17, Issue: 10 (October 2007, pp. 3625- 3631.

[7] W. R. Perkins, J. B. Cruz, *Engineering of Dynamic Systems*, John Wiley, New York, 1969.

[8] S. W. Director, R. A. Rohrer, "The generalized network and network sensitivities", IEEE Trans. CT, 18, Aug. 1969, pp. 318-323.

[9] M. Iordache, Lucia Dumitriu, I. Matei, **SYCIAN** – **Sy**mbolic **Ci**rcuit **An**alysis, User's Guide, Library of Electrical Department, Politehnica University of Bucharest, 2001.

[10] M. Iordache**,** Lucia Dumitriu**,** "Efficient Decomposition Techniques for Symbolic Analysis of Large – Scale Analog Circuits by State Variable Method", Analog Circuits and Signal Processing, Kluwer, Sep. 2004, pp. 234-256.

[11] Angela M. Hodge, R. W. Newcomb, "Semistate Theory and Analog VLSI Design", IEEE Circuit and Systems Magazine, Vol. 2, No. 2, Second Quarter 2002, pp.30-49.

[12] M. Iordache, Lucia Dumitriu, I. Matei, **SYTFUG** – **Sy**mbolic **T**ransfer **Fu**nction **G**eneration, User's Guide, Library of Electrical Department, Politehnica University of Bucharest, 2000.