EVALUATION OF SOLAR IRRADINACE TO A FLAT SURFACE ARBITRAY ORIENTED

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Abstract **– Design a photovoltaic or thermal system requires the irradiance available to collector surface.**
The paper proposes a methodology and software to evaluate the solar irradiance in any city of Romania, at any hour and any day of the year.

Keywords: irradiance, optimal angle, angle of incidence.

1. INTRODUCTION

Design a photovoltaic or thermal system requires the irradiance available to the collector surface [3].
Collector or PV module efficiency is influenced by the angle of solar radiation that falls on the aperture surface.

2. ASTRONOMY

The Earth revolves around the Sun to an appreciatively circular orbit (deviance is 1,7%).
The earth's rotation about its polar axis is done in 24 hours, and around the sun in 365,25 days.
The radiation intensity on the surface of the sun is approximately $6{,}33\times10^7$ W/m^2.

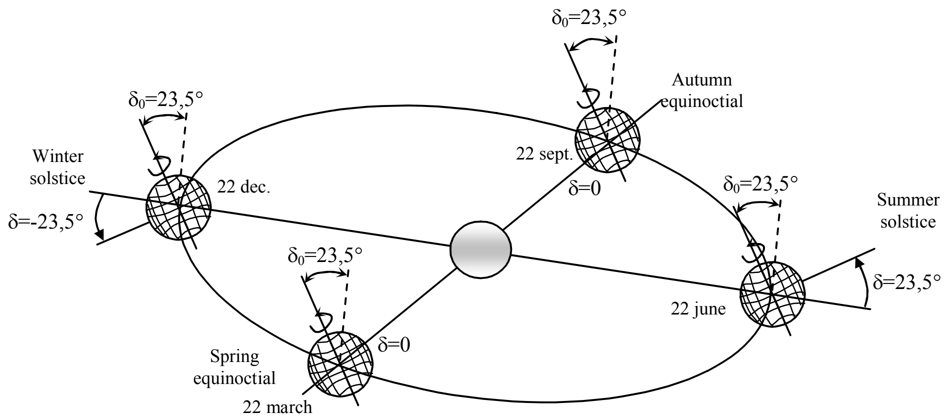

Figure 1. Earth orbit and declination angle, δ

Since radiation spreads out as the distance squared, by the time is travels to the earth ($1{,}496\times10^{11}$ m or 1AU), the radiant energy falling on 1 m^2 of surface area is reduced to 1367 W. The intensity of the radiation leaving the sun is relatively constant. Therefore, the intensity of solar radiation at a distance of 1AU is called the solar constant E_0 [1].
Irradiance is measured in W/m^2 and represents the density of instantaneous power of solar radiation.
Irradiation is measured in kWh/m^2 (or MJ/m^2) and represents the density of energy radiation [3].
Evaluation of extraterrestrial solar irradiance outside the earth's atmosphere is approximated by equation [4]:

$$E_{0,z} = E_0 \cdot \left[1 + 0{,}034 \cdot \cos\left(\frac{360\cdot}{365{,}25}(z-1)\right)\right] \quad \left[\frac{W}{m^2}\right] \qquad (1)$$

z – the day number (z=1 for 1 January).
The plane that includes the earth's equator is called the equatorial plane. If a line is drawn between the center of the earth and the sun, the angle between this line and the earth's equatorial plane is called the declination angle δ.
The declination angle in any day of the year is calculated with Cooper equation:

$$\delta = 23{,}45 \cdot \sin\left(360^\circ \frac{284+z}{365}\right) \quad [^\circ] \qquad (2)$$

The hour angle is the angular distance between the meridian of the observer and the meridian whose plane contains the sun. The hour angle is zero at solar noon (when the sun reaches its highest point in the sky) and increases by 15° every hour.

$$\omega = 15 \cdot (t_s - 12) \quad (3)$$

The hour angle of sun rise may be obtained form the equation:

$$\omega_s = \cos^{-1}(-\tan\phi \cdot \tan\delta) \quad [°] \quad (4)$$

The hour angle of sunset:

$$\omega_s' = -\cos^{-1}(-\tan\phi \cdot \tan\delta) \quad [°] \quad (5)$$

The hour angle is positive toward East and negative toward West.

$$\text{Hour of daylight} = \frac{2 \cdot \omega_s}{15} \quad (6)$$

The solar altitude angle (α_s) is defined as the angle between the central ray from the sun, and a horizontal plane containing the observer.

$$\sin\alpha_s = \sin\phi \cdot \sin\delta + \cos\phi \cdot \cos\delta \cdot \cos\omega = \cos\theta_z \quad (7)$$

As an alternative, the sun's altitude may be described in terms of the solar zenith angle (θ_z), which is simply the complement of the solar altitude angle.

$$\theta_z = 90° - \alpha_s \quad (8)$$

The azimuth angle γ is the angle between the normal projection to a horizontal plan and the local meridian; to north is zero and clockwise angles are positive.

The solar azimuth angle (γ_s), is the angle measured clockwise on the horizontal plane, from the north-pointing coordinate axis to the projection of the sun's central ray; in south – east direction the angels are negative and toward the west are positive.

$$\cos\gamma_s = \frac{\sin\alpha_s \cdot \sin\phi - \sin\delta}{\cos\alpha_s \cdot \cos\phi} \quad (9)$$

or:

$$\cos\gamma_s' = \frac{\sin\delta \cdot \cos\phi - \cos\delta \cdot \cos\omega \cdot \sin\phi}{\cos\alpha_s} \quad (10)$$

if sin ω>0, then $\gamma_s = 360° - \gamma_s'$, and if sin ω≤0 then $\gamma_s = \gamma_s'$.

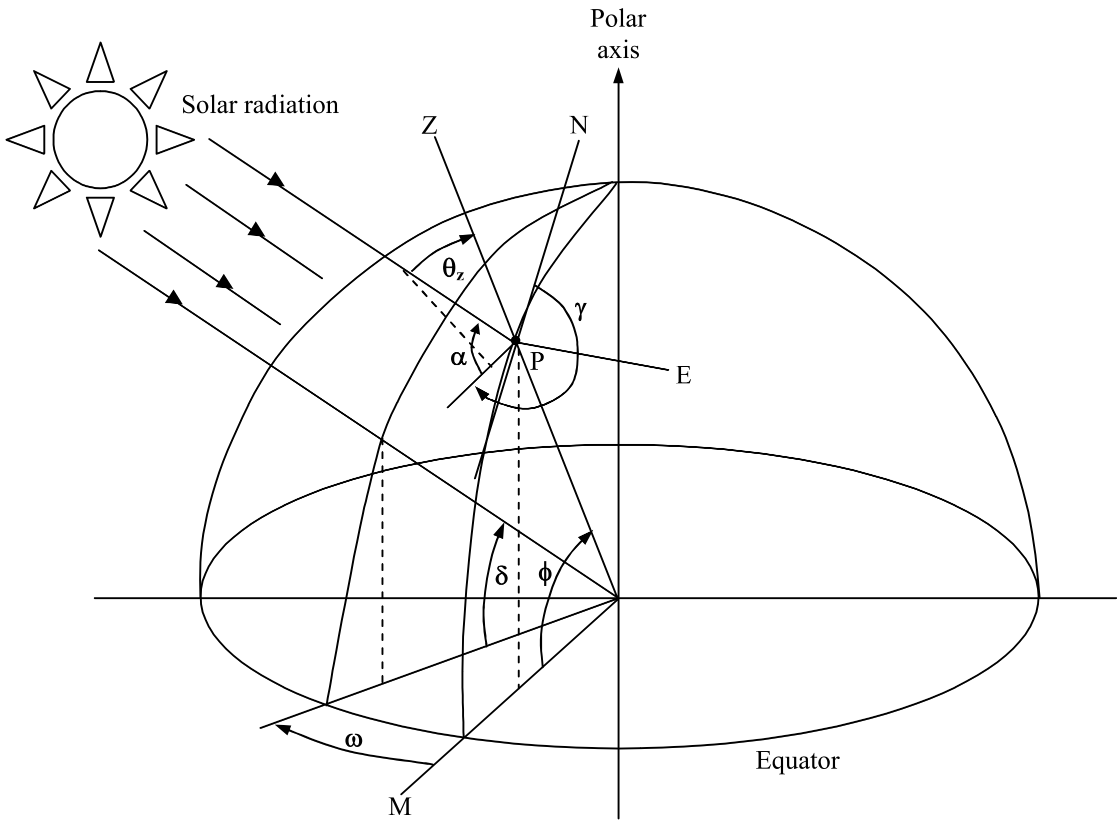

Figure 2. Earth's angles – Sun and Observer (P) - Sun

3. ANALYTICAL MODELS OF SOLAR IRRADINACE

As solar radiation passes through the earth's atmosphere, it is absorbed (the reason for some atmospheric heating), reflected (the reason astronauts can see the earth from the outer space), scattered (the reason one can read in the shade under a tree), and

transmitted directly (the reason there are shadows) [4].
On the surface on the earth, we perceive a beam or direct solar irradiance that comes directly from the disc of the sun and a diffuse or scattered solar irradiance that appear to come for all direction over the entire sky.
The sum of direct and diffuse solar irradiance is called the global or total solar irradiance.
Without atmosphere, the solar irradiance on a horizontal surface is [2]:

$$E_{aa} = E_{0,z} \cdot \sin\alpha_s = E_{0,z} \cdot \cos\theta_z \quad (11)$$

In the literature are presented many models in order to evaluate the solar irradiance in presence of the atmosphere:
a. A simple half sine model [4]
Often, a simple analytical model of clear-day solar irradiance is all that is needed to predict phenomena related to solar energy system design.
The only input required is the times sunrise, sunset and the peak, noontime solar irradiance level.

$$E_{pa} = E_{amaiza} \cdot \sin\left[\frac{180 \cdot (t - t_{rasarire})}{t_{apunere} - t_{rasarire}}\right] \left[W/m^2\right] \quad (12)$$

b. Hottel'as clear-day model (1976) [4]
Hottel's clear-day model of direct normal solar irradiance is based on atmospheric transmittance calculation:

$$E_{dir} = E_{0,z} \cdot \left(a_1 + a_2 \cdot e^{-\frac{a_3}{\cos\theta_z}}\right) \left[W/m^2\right] \quad (13)$$

For the urban 6 km visibility haze model, the parameters are:

$$a_1 = 0{,}4327 - 0{,}00821 \cdot (6 - A)^2$$
$$a_2 = 0{,}5055 - 0{,}00595 \cdot (6{,}5 - A)^2 \quad (14)$$
$$a_3 = 0{,}2711 - 0{,}01858 \cdot (2{,}5 - A)^2$$

A – local elevation [km].
Solar diffuse irradiance on a horizontal surface is:

$$E_{dif} = E_{0,z} \cdot \cos\theta_z \cdot \left[0{,}2710 - 0{,}2939 \cdot \left(a_0 + a_1 \cdot e^{-a_3 \cdot \cos\theta_z}\right)\right]\left[W/m^2\right] \quad (15)$$

c. Adnot model (1979), [2]
Global solar irradiance in a clear-day sky is:

$$E_{pa} = 951{,}39 \cdot (\cos\theta_z)^{1,15} \quad (16)$$

d. Empirical Irradiance Model – EIM (Paulescu and Schlett, 2004), [2]:

$$E_{pa} = E_{0,z} \cdot \left[1 - 0{,}4645 \cdot e^{-0,69 \cdot \cos\theta_z}\right] \cdot e^{\frac{0,05211}{\cos\theta_z}} \cdot \cos\theta_z \quad (16)$$

Base on the studies of Bădescu (1997), Paulescu and Schlett (2004) had concluded that for Romania can be applied Adnot model and EIM model.

For an uncertain sky Paulescu and Schlett, (2004) recommends a model that used the nebulosity.

$$E_{cv} = c(N) \cdot E_{pa} \quad (17)$$

where:

$$c(N) = 1 - 0{,}6586 \cdot (N)^{2,7545} \quad (18)$$

4. EVALUATION OF GLOBAL SOLAR IRRADIANCE ON A SURFACE ARBITRARY ORIENTED [2]

There are few meteorological stations, which measures the global irradiance on an inclined surface.
The global irradiance (in case on uncertain sky) is equal with the sum between the direct and diffuse irradiation.

$$E_{cv} = E_{dir} + E_{dif} \quad (19)$$

The tilt angle $\theta_î$ is the angle between the aperture surface and the horizontal surface; $0 \le \theta_î \le 180$. In practice the maximum tilt angle is 90°.
The optimal angle of aperture surface is calculated with the equation:

$$\theta_î = \varphi - \delta \quad (20)$$

For the analytical models can be calculated the clearness index:

$$k_T = \frac{E_{cv}}{E_{0,z}} \quad (21)$$

E_{cv} – global solar irradiation calculated (measured) for an uncertain sky per hour, day, month, and year.
Diffuse irradiation for an uncertain sky is determined with the equation:

$$\frac{E_{dif}}{E_{cv}} = 1 - 1{,}13 \cdot k_T \Rightarrow E_{dif} = E_{cv} \cdot (1 - 1{,}13 \cdot k_T) \quad (22)$$

and:

$$E_{dir} = E_{cv} - E_{dif} \quad (23)$$

Between the direct solar irradiance on a tilt surface and the direct solar irradiance on a horizontal surface we can use a dependency:

$$E_{dir}(\theta_î) = \frac{\cos\theta_i}{\cos\theta_z} \cdot E_{dir} \quad (24)$$

θ_z – zenith angle;
θ_i – incidence angle.
The incidence angle is given by equation:

$$\cos\theta_i = \sin\alpha_s \cdot \cos\theta_î + \cos\alpha_s \cdot \sin\theta_î \cdot \cos(\gamma_s - \gamma) \quad (25)$$

Special case:
- horizontal surface, $\theta_î$=0:

$$\cos\theta_i = \sin\alpha_s \qquad (26)$$

- vertical surface:

$$\cos\theta_i = \cos\alpha_s \cdot \cos(\gamma - \gamma_s) \qquad (27)$$

- horizontal surface, south oriented:

$$\cos\theta_i = \sin\alpha_s \cdot \cos\theta_î - \cos\alpha_s \cdot \sin\theta_î \cdot \cos\gamma_s \qquad (28)$$

Between the diffuse solar irradiance on a tilt surface and the diffuse solar irradiance on a horizontal surface it was established the equation:

$$E_{dif}(\theta_î) = \frac{1+\cos\theta_î}{2} \cdot E_{dif} \qquad (26)$$

Taking into account the tilt surface the reflected irradiance is:

$$E_r(\theta_î) = \frac{1-\cos\theta_î}{2} \cdot E \cdot \rho \qquad (27)$$

ρ –albedou, or reflection.

Ground	Albedo, ρ
Tilled soil	0,2
Green soil	0,3
Snow	0,5÷0,8

Table 1. Albedou

5. VISUAL BASIC SOFTWARE

The methodology presented is implemented using Visual Basic application.

Visual Basic is a branch of Visual Studio from Microsoft, which allows to create some standard interface like in Windows (windows, buttons, list, etc) without writing the codes.

After the installation of the software, we open the application. It appears three interface (one active, and two inactive). In the white textboxes the input data are written, and when we press the buttons the values are calculated.

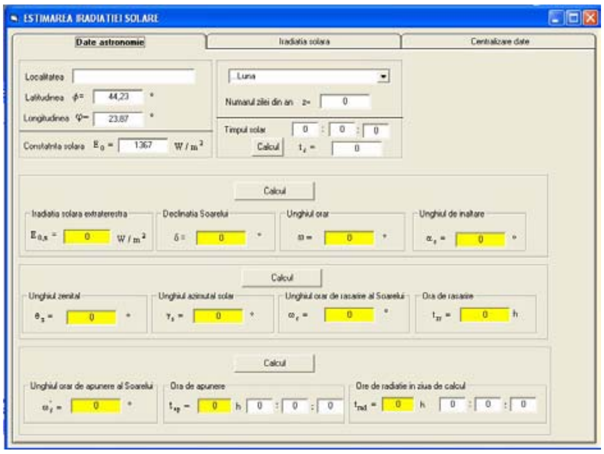

Figure 3. Nr. 1 Interface

For Craiova latitude is ϕ = 44,23°, and the longitude is φ = 23,87°. The evaluation of direct solar irradiance is done for 22 June and 22 December, hour by hour.

Day number is z=174, and z=356.

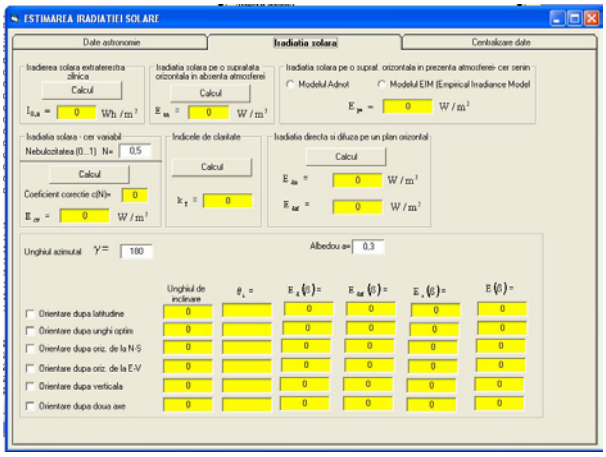

Figure 4. Nr. 2 interface

The calculated values are presented in table 2 and 3.

Nr. crt.	Ora	$E_{0,z}$ [W/m^2]	δ [°]	ω [°]	α_s [°]	θ_z [°]	E_{aa} [W/m^2]	E_{pa} Model EIM [W/m^2]	N	E_{cv} [W/m^2]	E_{dir} [W/m^2]	$\theta_î$ [°]	$E_{dir}(\theta_î)$ [W/m^2])
1.	6:00	1321,03	23,44	-90	16,10	73,89	366,51	272,61	0	272,61	63,57	20,79	36,27
2.	7:00	1321,03	23,44	-75	26,59	63,41	591,29	437,72	0	437,72	163,89	20,79	137,75
3.	8:00	1321,03	23,44	-60	37,31	52,68	800,76	605,85	0	605,85	313,98	20,79	299,98
4.	9:00	1321,03	23,44	-45	47,93	42,07	980,63	759,17	0	759,17	493,00	20,79	500,38
5.	10:00	1321,03	23,44	-30	57,86	32,13	1118,64	881,57	0	881,57	664,79	20,79	696,55
6.	11:00	1321,03	23,44	-15	65,85	24,15	1205,41	960,39	0	960,39	788,98	20,79	839,88
7.	12:00	1321,03	23,44	0	69,20	20,79	1234,99	987,58	0	987,58	834,28	20,79	891,85
8.	13:00	1321,03	23,44	15	68,85	24,15	1205,41	960,39	0	960,39	788,98	20,79	839,88
9.	14:00	1321,03	23,44	30	57,86	32,13	1118,64	881,57	0	881,57	664,79	20,79	696,55
10.	15:00	1321,03	23,44	45	47,93	42,07	980,63	759,17	0	759,17	493,00	20,79	500,46
11.	16:00	1321,03	23,44	60	37,31	52,68	800,76	605,85	0	605,85	313,98	20,79	299,98

Nr. crt.	Ora	$E_{0,z}$ [W/m^2]	δ [°]	ω [°]	α_s [°]	θ_z [°]	E_{aa} [W/m^2]	E_{pa} Model EIM [W/m^2]	N	E_{cv} [W/m^2]	E_{dir} [W/m^2]	$\theta_î$ [°]	$E_{dir}(\theta_î)$ [W/m^2])
12.	17:00	1321,03	23,44	75	26,59	63,41	591,29	437,72	0	437,72	163,89	20,79	137,75
13.	18:00	1321,03	23,44	90	16,10	73,89	366,51	272,61	0	272,61	63,57	20,79	36,27
Total											**5810,7**	-	**5913,55**

Table 2. Direct solar irradiance on a tilt surface on 22 June

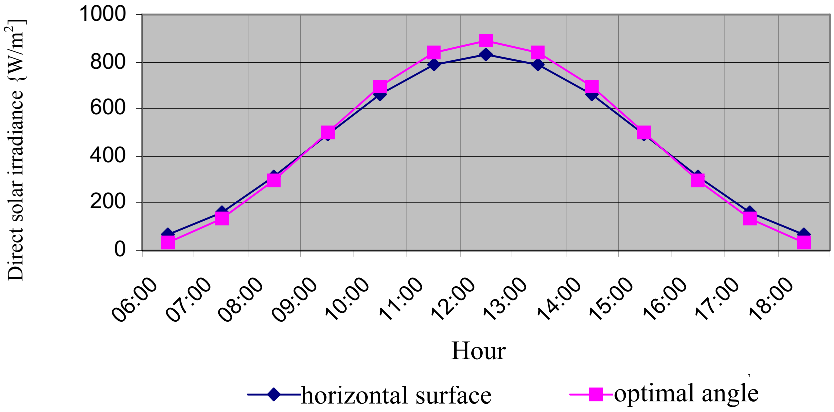

Figure 5. Direct solar irradiance on a tilt surface on 22 June

Nr. crt.	Ora	$E_{0,z}$ [W/m^2]	δ [°]	ω [°]	α_s [°]	θ_z [°]	E_{aa} [W/m^2]	E_{pa} Model EIM [W/m^2]	N	E_{cv} [W/m^2]	E_{dir} [W/m^2]	$\theta_î$ [°]	$E_{dir}(\theta_î)$ [W/m^2])
1.	8:00	1412,89	-23,44	-60	2,93	87,06	172,38	110,41	0	110,41	9,75	67,67	110,21
2.	9:00	1412,89	-23,44	-45	10,80	79,20	264,74	206,92	0	206,92	34,24	67,67	137,69
3.	10:00	1412,89	-23,44	-30	16,96	73,03	412,35	305,74	0	305,74	74,76	67,67	227,27
4.	11:00	1412,89	-23,44	-15	20,94	60,05	505,16	372,30	0	372,30	110,85	67,67	301,16
5.	12:00	1412,89	-23,44	0	22,32	67,67	536,79	395,66	0	395,66	125,20	67,67	324,54
6.	13:00	1412,89	-23,44	15	20,94	60,05	505,16	372,30	0	372,30	110,85	67,67	301,17
7.	14:00	1412,89	-23,44	30	16,96	73,03	412,35	305,74	0	305,74	74,76	67,67	227,29
8.	15:00	1412,89	-23,44	45	10,80	79,20	264,74	206,92	0	206,92	34,24	67,67	137,71
9.	16:00	1412,89	-23,44	-60	2,93	87,06	172,38	110,41	0	110,41	9,75	67,67	110,24
Total											**584,4**	-	**1877,3**

Table 3. Direct solar irradiance on a tilt surface on 22 December

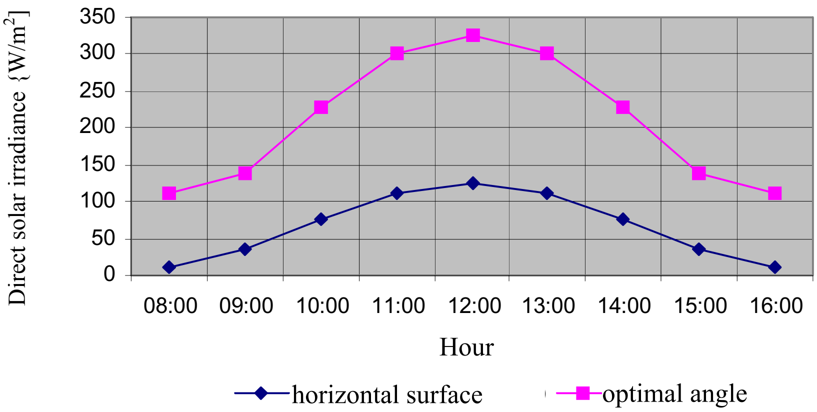

Figure 6. Direct solar irradiance on a tilt surface on 22 December

6. CONCLUSIONS

The software presented can be used to evaluate the direct solar irradiance in any city on Romania at any hour, and day being a practical and easy to use.

We can conclude:

- on a 22 June are 13 hours of daylight and 22 December are only 9 hours;
- on 22 June the amount of radiation is with 2% bigger when the surface is tilt with optimal angle toward horizontal surface;
- on 22 December the amount of radiation is with 69% bigger when the surface is tilt with optimal angle toward horizontal surface;

In general the collectors are south oriented and with an annual optimal angle.

However is important also to study the cases: single and two-axis tracking apertures for the collectors.

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