7TH INTERNATIONAL CONFERENCE ON ELECTROMECHANICAL AND POWER SYSTEMS
October 8-9, 2009 - Iaşi, Romania

SOME ASPECTS ABOUT NUMERICAL SIMULATION OF PLATE HEAT EXCHANGER USING FINITE ELEMENT METHOD

Roxana GRIGORE, Sorin POPA

University of Bacau, Romania, rgrigore@ub.ro

Abstract – **Heat exchangers are thermal equipments very important for thermal and electrical power generation. Specialty literature emphasizes that more than half of primary energy consumed in a country goes to the final form of energy through a chain of transformations involving the average of 2-3 heat exchangers. The paper presents a 3D model of a counter flow plate heat exchanger and the numerical simulation of thermal regime of this heat exchanger, using finite element method.**

Keywords: *heat transfer, counter-flow heat exchanger, governing equation, finite element method, mesh.*

1. COUNTERFLOW PLATE HEAT EXCHANGER

The studied heat exchanger is a pack of three stainless steel plates with gaskets. The hot water flows are in one direction in alternating chambers while the cold water flows are in counter flow in the other alternating chambers. The number of passes is 1 and the number of channels is n_c= 2, like in figure 1. The width of channel is 5 mm.The thermal agents are directed into their proper chambers either by a suitable gasket made from rubber EPDM.The stainless steel plate thickness is δ_p = 2 mm.

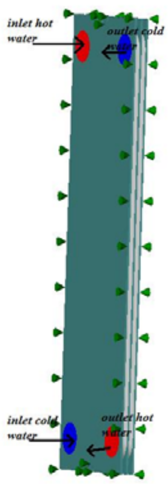

Figure1: Schematic presentation of the studied heat exchanger.

The geometric dimensions of the plate are represented in figure 2.

2. MODELING AND SIMULATION OF HEAT EXCHANGER

Modeling of a process or equipment involves the execution of the next steps [1]:

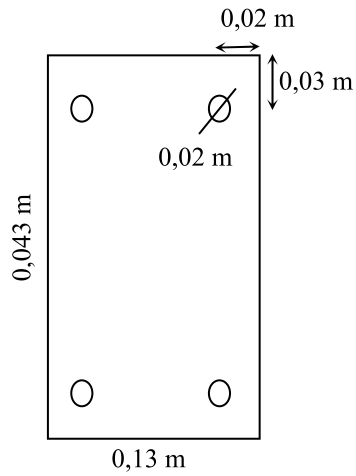

Figure 2 - Geometric dimensions of the plate.

- Physical modeling;
- Mathematical modeling;
- Simulation (numerical modeling);
- Visualization;
- Validation.

Mathematical modeling includes assignation of governing equations. The partial differential equations governing fluid flow and heat transfer include the continuity equation, the Navier-Stokes equations and the energy equation. These equations are intimately coupled and non-linear making a general analytic solution impossible except for a limited number of special problems, where the equations can be reduced to yield analytic solutions. Because most practical problems of interest do not fall into this limited category, approximate methods are used to determine

the solution to these equations. There are numerous methods available for doing so. In this paper is used the Cosmos/Flow program.
Cosmos/Flow solves the mathematical equations which represent heat and momentum transfer in a moving fluid. The finite element method is used to *discretize* the flow domain, thereby transforming the governing partial differential equations into a set of algebraic equations whose solution represents an approximation to the exact analytical solution. It is used the iterative resolving technique: Newton – Raphson scheme, which assure a great rate of convergence.

2.1.Governing equations

The governing equations for fluid flow and heat transfer are the Navier-Stokes or momentum equations and the First Law of Thermodynamics or energy equation. The governing pdes can be written as [2]:
Continuity equation:

$$\frac{\partial\rho}{\partial t}+\frac{\partial\rho u}{\partial x}+\frac{\partial\rho v}{\partial y}+\frac{\partial\rho w}{\partial z}=0 \qquad (1)$$

x-,y-,z-momentum equations:

$$\rho\frac{\partial u}{\partial t}+\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial y}+\rho w\frac{\partial u}{\partial z}=$$
$$=\rho g_x-\frac{\partial p}{\partial x}+\frac{\partial}{\partial x}\left[2\eta\frac{\partial u}{\partial x}\right]+\frac{\partial}{\partial y}\left[\eta\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right]+\frac{\partial}{\partial z}\left[\eta\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)\right]+S_\omega+S_{DR}$$

$$\rho\frac{\partial v}{\partial t}+\rho u\frac{\partial v}{\partial x}+\rho v\frac{\partial v}{\partial y}+\rho w\frac{\partial v}{\partial z}=$$
$$=\rho g_y-\frac{\partial p}{\partial y}+\frac{\partial}{\partial y}\left[2\eta\frac{\partial v}{\partial y}\right]+\frac{\partial}{\partial x}\left[\eta\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right]+\frac{\partial}{\partial z}\left[\eta\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)\right]+S_\omega+S_{DR}$$

$$\rho\frac{\partial w}{\partial t}+\rho u\frac{\partial w}{\partial x}+\rho v\frac{\partial w}{\partial y}+\rho w\frac{\partial w}{\partial z}=$$
$$=\rho g_z-\frac{\partial p}{\partial z}+\frac{\partial}{\partial z}\left[2\eta\frac{\partial w}{\partial z}\right]+\frac{\partial}{\partial x}\left[\eta\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)\right]+\frac{\partial}{\partial y}\left[\eta\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)\right]+S_\omega+S_{DR} \qquad (2)$$

The two source terms in the momentum equations, S_ω and S_{DR}, are for distributed resistances and rotating coordinates, respectively.The distributed resistance term can be written in general as:

$$S_{DR}=-\left(K_i+\frac{f}{d}\right)\frac{\rho V_i^2}{2}-C\eta V_i \qquad (3)$$

where i refer to the global coordinate direction (*u, v, w* momentum equation), *f*- friction factor, *d*- hydraulic diameter, *C* – permeability and the other factors are descripted in table 1. Note that the *K*-factor term can operate on a single momentum equation at a time because each direction has its own unique *K*-factor. The other two resistance types operate equally on each momentum equation.
The other source term is for rotating flow. This term can be written in general as:

$$S_\omega=-2\rho\omega_i\times V_i-\rho\omega_i\times\omega_i\times r_i \qquad (4)$$

where *i* refer to the global coordinate direction, ω is the rotational speed and r is the distance from the axis of rotation.
For incompressible and subsonic compressible flow, the energy equation is written in terms of static temperature:

$$\rho c_p\frac{\partial T}{\partial t}+\rho c_p u\frac{\partial T}{\partial x}+\rho c_p v\frac{\partial T}{\partial y}+\rho c_p w\frac{\partial T}{\partial z}=$$
$$=\frac{\partial}{\partial x}\left[k\frac{\partial T}{\partial x}\right]+\frac{\partial}{\partial y}\left[k\frac{\partial T}{\partial y}\right]+\frac{\partial}{\partial z}\left[k\frac{\partial T}{\partial z}\right]+q_V \qquad (5)$$

The variables from these equations are defined in Table 1.

Variable	Description
c_p	specific heat at constant pressure
k	thermal conductivity
p	pressure
q_V	volumetric heat source
T	temperature
t	time
u	velocity component in x-direction
v	velocity component in y-direction
w	velocity component in z-direction
ρ	density
η	dynamic viscosity

Table 1 - Variables of the governing equations.

The equations describe the fluid flow and heat transfer under steady-state conditions for Cartesian geometries. For the turbulent flow, the solution of these equations would require a great deal of finite elements (on the order of 10^6 – 10^8) even for a simple geometry as well as near infinitesimal time steps. COSMOS/Flow solves the *time-averaged* governing equations.
The time-averaged equations are obtained by assuming that the dependent variables can be represented as a superposition of a mean value and a *fluctuating value*, where the fluctuation is about the mean[2]. For example, the velocity componenet in y-direction can be written:

$$V = V + v', \text{[m/s]} \qquad (6)$$

where V [m/s] – the mean velicity, v' [m/s] – the fluctuation about the mean. This representation is introduced into the governing equations and the equations themselves are averaged over time.

2.2. Discretization method

In COSMOS/Flow, the finite element method is used to reduce the governing partial differential equations (pdes) to a set of algebraic equations.The role of finite element method in numerical simulation is shown in figure 3.
The dependent variables are represented by polynomial shape functions over a small area or volume (element). These representations are substituted into the governing pdes and then the weighted integral of these equations over the element is taken where the weight function is chosen to be the same as the shape function. The result is a set of algebraic equations for the dependent variable at discrete points or nodes on every element.

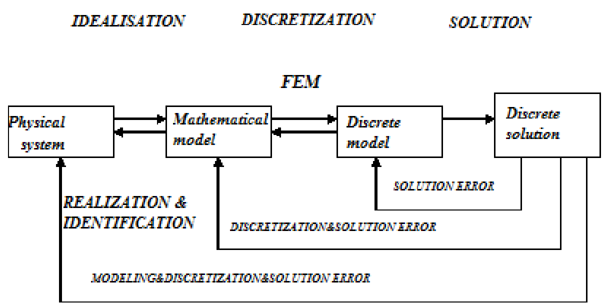

Figure 3 - Simplified structure of the simulation.

2.3. Boundary conditions and simplified hypothesis for modelling and simulation plate heat exchanger

The 3D model is realized in SolidWork. There are considered several simplified hypothesis:

- Hot water and cold water are Newtonian fluids;
- Turbulent flow is fully developed;
- Working fluids are incompressible;
- Steady state conditions.

The volumetric heat source term in the energy equation (5) is zero in this model.
The next boundary conditions are proposed:

No.	Fluid	Description	U.M.	Value
1	Hot Water inlet	Temperature – T_1'	^oC	53
2		Flow volume – V_1'	m^3/s	$9,524\times10^{-5}$
3		Heat load – Q_1	W	20828
4	Cold Water inlet	Temperature – T_2'	^oC	8
5		Flow volume – V_2'	m^3/s	$8,163\times10^{-5}$
6		Heat load – Q_2	W	2741,7
7	Air	Film coefficient- α	W/m^2K	5
8		Bulk Temperature	^oC	30

Table 2 - Inlet data.

It calculates the heat flow of the hot water entering the heat exchanger, Q_1, with the next formula:

$$Q_1 = \rho_1 \cdot c_{p1} \cdot V_1 \cdot T_1' \quad [W] \tag{7}$$

where: ρ_1 = 988,611 m^3/kg, c_{p1}= 4182,46 J/kgK – thermo physical proprieties of water at temperature T_1'=53°C.
In the same mode, it calculates the heat flow of the cold water entering the heat exchanger, Q_2, with the relationship (8):

$$Q_2 = \rho_2 \cdot c_{p2} \cdot V_2 \cdot T_2' \quad [W] \tag{8}$$

where: ρ_2= 999,8 m^3/kg, c_{p2}= 4199,07 J/kgK – thermo physical proprieties of water at temperature T_2'=8°C.

3. RESULTS DISPLAY

After the analysis was processed it can be visualized the results, under graphical form or numerical value. Analyze run for 100 iterations, in turbulence conditions. In figure 4 is presented the distribution of the nodal temperature on the heat exchange surface.
Average temperatures are calculated at the outlet of hot water and cold water and it obtained the next values:

No.	Fluid	Description	U.M.	Value
1	Hot Water outlet	Temperature – T_1''	^oC	17,8
2	Cold Water outlet	Temperature – T_2''	^oC	39,6

The hot and cold fluid temperature distributions in the counter flow heat exchanger are shown in the figure 5.

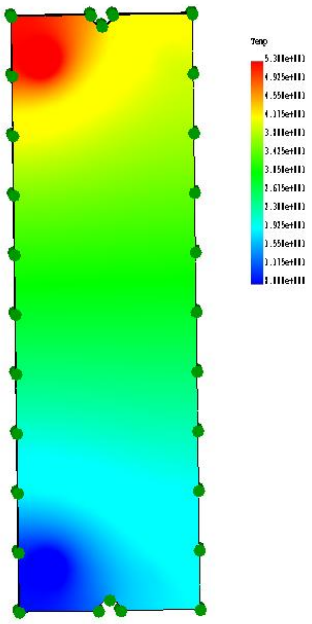

Figure 4 - Distribution of the temperature of heat exchange surface.

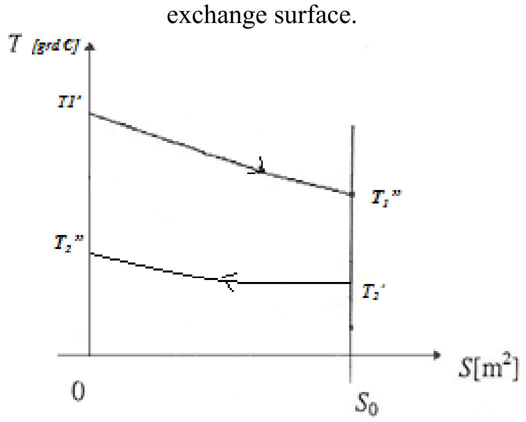

Figure 5 - The hot and cold fluid temperature distributions in the counter flow heat exchanger.

LMTD – log mean temperature difference is computed under assumption of counter flow condition [3]:

$$LMTD = \frac{[T_1' - T_2''] - (T_1'' - T_2')}{\ln \frac{T_1' - T_2''}{T_1'' - T_2'}} \quad [^\circ C] \tag{9}$$

In this case *LMTD* = 11,46 °C.

The overall heat transfer coefficient *U* is determined from heat transfer equation:

$$U = 4742\ W/m^2K. \tag{10}$$

The distribution of resultant heat flux, q_s in W/m², are shown in figure 6.

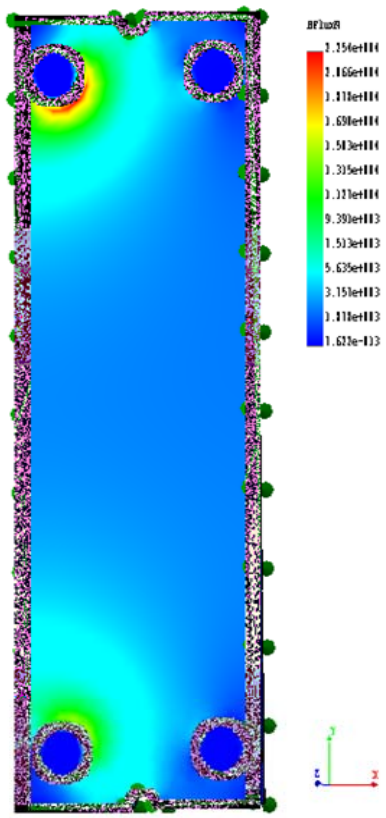

Figure 6 – Distributions of resultant heat flux.

4. CONCLUSIONS

It is very important to realize the mesh with great accurately. Values obtained for the temperature and heat flux are in according with technical possibilities of studied heat exchanger.

The model developed using the program COSMOS/ Flow proves a very important tool for the study of heat transfer in a plate heat exchanger.

References

[1] R. Grigore, V.Dragusanu, G.Lazaroiu, S.Popa, *Study Regarding Simulation and Modelling of Steam Condenser,* MOCM -14, Volume 1, 2008, pp. 121-126.

[2] *** COSMOS/Flow Technical Reference

[3] A. Badea, H. Necula, M. Stan, L. Ionescu, P. Blaga., G.Darie, *Echipamente şi instalaţii termice,* Editura Tehnică, Bucureşti, 2003

[4] S.V Patanker, *Numerical Heat Transfer and Fluid Flow*, Hemisphere Publishing, New York, 1980.

[5] F. B. Incropera, D. P. Dewitt, *Fundamentals of Heat and Mass Transfer*, John Wiley and Sons, 1985.

[6] G. Lazaroiu, *Sisteme de programare pentru modelare si simulare*, Editura Politehnica Press, Bucuresti, 2005.