ELECTROMAGNETIC FIELD DISTRIBUTION IN VICINITY OF POWER LINES ABOVE REAL EARTH

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Abstract – **A method for electromagnetic field components calculation in vicinity of power lines is presented. The electromagnetic field distribution of 2x110 kV power line, which is commonly used in Serbian electric power system, is analysed. For determination of power line conductors influence on electromagnetic field, the complex ground return plane approach is applied. In that approach a return current flow in homogeneous earth was modelled by a perfect conducting plane located at a complex depth below the earth surface. An integral equation for potential is formed also, taken into account influences of currents and line charges induced on the power line tower. Solving that equation, the induced line charges on the power line tower can be determined and electromagnetic field distribution calculated. All results will be presented graphically. Especially will be considered influences of different parameters values on induced charge distribution along the tower. The charges and currents induced in tower have more relevant influence on the electric field than on the magnetic field distribution. That influence is noticed in close vicinity of power line. That distance is about 1/10 of tower height. In other areas that influence can be neglected. Obtained results will be compared with finite element method results.**

Keywords: *Power line, Complex ground return plane approach, Integral equation, Finite element method.*

1. INTRODUCTION

Our living and daily environment is full of generated electric and magnetic fields, collectively called "electromagnetic fields" hereafter. As result, people are always exposed to those electromagnetic fields composed of various frequency components.

The typical sources of extremely low-frequency (ELF) electromagnetic fields in Serbia are electric power transmission and distribution lines and other appliances of 50 Hz power frequency. The strength of these fields decreases rapidly with increasing distance from the source. At power line frequencies the electric and magnetic fields are decoupled. They do not generate each other and can be analysed independently. Magnetic field components produced by power lines have more relevant influence to possible health effects than the electric field components [1]. The magnetic fields are difficult to shield and they can penetrate surrounding buildings and into human bodies compared to electric fields.

In this paper a procedure for electromagnetic field components calculation in vicinity of power line [2] is presented. It is considered that TEM mode exists on the power line.

Influences of power line conductors and tower are examined. Induced charges and currents on the tower are determined as a solution of integral equation for potential. This equation can be numerically solved using the point matching method and polynomial approximation for unknown function of induced line charges on tower.

Numerical results are given for the power line of 2x110 kV, which is commonly used in Serbian electric power system [3, 4].

Some obtained results will be compared with finite element method results [5].

2. THEORETICAL BACKGROUND

A power line consists of N conductors placed parallel to the ground, Fig.1. It is considered that conductors are straight and have infinite lengths.

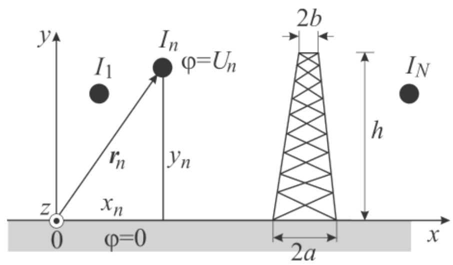

Figure 1: Power line cross-section.

The voltages between the power line conductors and the ground, so called phase voltages, $\underline{U}_n$, $n = 1, 2, ..., N$, and line charges, $\underline{q}'_n$, of conductors are connected with system of equations:

$$\begin{aligned}
\underline{U}_1 &= a_{11}\underline{q}'_1 + a_{12}\underline{q}'_2 + ... + a_{1N}\underline{q}'_N \\
\underline{U}_2 &= a_{21}\underline{q}'_1 + a_{22}\underline{q}'_2 + ... + a_{2N}\underline{q}'_N \\
&... \\
\underline{U}_N &= a_{N1}\underline{q}'_1 + a_{N2}\underline{q}'_2 + ... + a_{NN}\underline{q}'_N
\end{aligned} \quad (1)$$

where a_{nm} are coefficients calculated using:

$$a_{nm} = \frac{1}{2\pi\varepsilon_0} \ln \frac{d_{n'm}}{d_{nm}},\ n \neq m$$
$$\text{and} \qquad (2)$$
$$a_{nn} = \frac{1}{2\pi\varepsilon_0} \ln \frac{2y_n}{R_{0n}},\ n = m,$$

where: R_{0n} is the n^{th} conductor radius, y_n is a height of n^{th} conductor, d_{nm} is a distance between the n^{th} and m^{th} conductor axis, $d_{n'm}$ is a distance between the n^{th} conductor image axis and m^{th} conductor axis and ε_0 is dielectric permittivity of air ($\varepsilon_0 = 8.85 \cdot 10^{-12}$ [F/m]).

Equation (1) can be written in the matrix form

$$\underline{\overleftrightarrow{U}} = \overleftrightarrow{a}\underline{\overleftrightarrow{q}}',\ \text{where}$$

$$\underline{\overleftrightarrow{U}} = \begin{bmatrix} \underline{U}_1 \\ \underline{U}_2 \\ \vdots \\ \underline{U}_N \end{bmatrix},\ \underline{\overleftrightarrow{q}}' = \begin{bmatrix} \underline{q}'_1 \\ \underline{q}'_2 \\ \vdots \\ \underline{q}'_N \end{bmatrix} \ \text{and}$$

$$\overleftrightarrow{a} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1N} \\ a_{21} & a_{22} & \cdots & a_{2N} \\ \vdots & & & \\ a_{N1} & a_{N2} & \cdots & a_{NN} \end{bmatrix}$$

The inverse matrix of matrix $\overleftrightarrow{a}$ is

$$\overleftrightarrow{b} = \overleftrightarrow{a}^{-1} = \operatorname{adj}\overleftrightarrow{a} / \det\overleftrightarrow{a},\ \det\overleftrightarrow{a} \neq 0.$$

So the expression (1) can be written in the form

$$\underline{\overleftrightarrow{q}}' = \overleftrightarrow{b}\underline{\overleftrightarrow{U}}. \qquad (3)$$

This expression has more convenient form for practical applications because in the power lines the potentials of conductors are known. Using this expression it is possible to calculate conductor's line charges.

The conductor's currents, $\underline{I}_n$, can be determined using

$$i = \frac{dq}{dt}, \qquad (4)$$

i.e.

$$\underline{I}_n = c\underline{q}'_n,\ n = 1, 2, ..., N, \qquad (5)$$

where $c = 3 \cdot 10^8$ m/s.

2.1. Complex ground return plane approach

This concept is proposed by Dubanton at Electricité de France and can be found in [6]. A return current flow in homogeneous earth was modelled by a perfect conducting plane located at a complex depth p below the earth surface, Fig. 2. The parameter p is given by:

$$\underline{p} = \frac{1}{\sqrt{\mathrm{j}\omega\mu_0\sigma}}. \qquad (6)$$

This new-formed plane can be considered as a mirroring surface, so the image theorem in the plane mirror can be applied.

The two-dimension simplification, which is applied here, over three dimensions, assumes that the lines are straight and go for a sufficient distance toward infinity to make two-dimensional analysis valid [7].

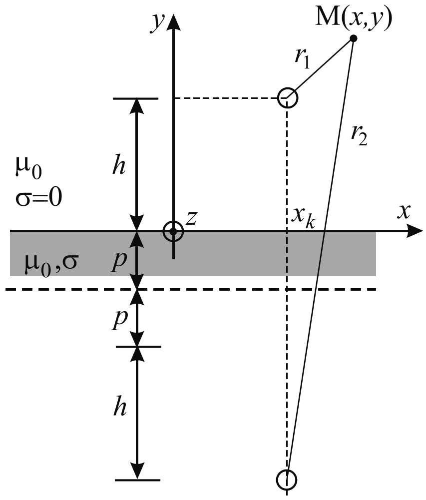

Figure 2: Complex ground return plane approach.

The magnetic flux density components are:

$$\underline{B}_x(x,y) = \sum_{n=1}^{N} \frac{\mu_0 \underline{I}_n}{2\pi} \left[-\frac{y - y_n}{(x - x_n)^2 + (y - y_n)^2} - \right.$$
$$\left. - \frac{y + y_n + 2\underline{p}}{(x - x_n)^2 + (y + y_n + 2\underline{p})^2} \right] \qquad (7)$$

$$\underline{B}_y(x,y) = \sum_{n=1}^{N} \frac{\mu_0 \underline{I}_n}{2\pi} \left[\frac{x - x_n}{(x - x_n)^2 + (y - y_n)^2} + \right.$$
$$\left. + \frac{x - x_n}{(x - x_n)^2 + (y + y_n + 2\underline{p})^2} \right]. \qquad (8)$$

The total magnetic flux density from power line conductors is

$$\underline{\boldsymbol{B}}_{\text{pl}} = \underline{B}_x \hat{x} + \underline{B}_y \hat{y}$$

$$B_{\text{pl}}(x,y) = \sqrt{\underline{B}_x \underline{B}_x^* + \underline{B}_y \underline{B}_y^*}, \qquad (9)$$

and total magnetic field is

$$H_{\text{p1}}(x,y) = B_{\text{p1}}(x,y) / \mu_0. \qquad (10)$$

2.2. Electric field calculation from power line conductors

The electric field components from power line, Fig. 3, can be determined using following expressions [2]

$$\underline{E}_x = -\partial\underline{\varphi}/\partial x,\ \underline{E}_y = -\partial\underline{\varphi}/\partial y, \tag{11}$$

where

$$\underline{\varphi} = -\frac{1}{2\pi\varepsilon_0}\sum_{n=1}^{N}\underline{q}'_n\left(\ln R_{1n} - \frac{\underline{\varepsilon}-\varepsilon_0}{\underline{\varepsilon}+\varepsilon_0}\ln R_{2n}\right) \tag{12}$$

is the electric scalar potential. $\underline{\varepsilon} = \varepsilon - \mathrm{j}\sigma/\omega$, $\underline{q}'_n$ is the charge per unit length of n^{th} conductor.

At power line frequencies, the earth can be considered as a paramagnetic conducting area, so the expression (12) can be simplified using that $\mu \approx \mu_0$, $3 \le \varepsilon_r \le 11$ and 10^{-3} S/m $\le \sigma \le 10^{-5}$ S/m. Thus,

$$\underline{\varphi} = -\frac{1}{2\pi\varepsilon_0}\sum_{n=1}^{N}\underline{q}'_n \ln\frac{R_{1n}}{R_{2n}} \tag{13}$$

where $R_{1n} = \sqrt{(x-x_n)^2 + (y-y_n)^2}$,

$R_{2n} = \sqrt{(x-x_n)^2 + (y+y_n)^2}$.

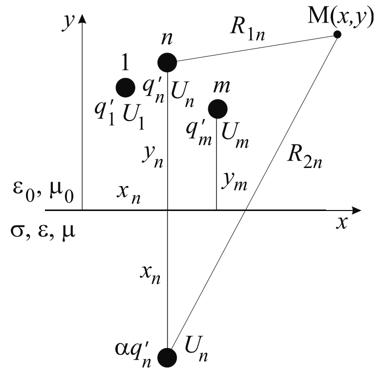

Figure 3: Image theorem.

Using expression (11) it is possible to calculate the electric field from power line conductors

$$\underline{\boldsymbol{E}}_{\text{pl}} = \underline{E}_x\hat{x} + \underline{E}_y\hat{y},$$

$$E_{\text{pl}} = \sqrt{\underline{E}_x\underline{E}_x^* + \underline{E}_y\underline{E}_y^*}\,. \tag{14}$$

3. INTEGRAL EQUATION FOR POWER LINE TOWERS

The power line towers can be considered as conductors having extended prolate rounded cup of height h and circular cross section, due to the practical solution for power line tower, as well as to the fact that observed frequencies are industrial. The radii of tower cross section at the bottom and of the top are noted with a and b ($a > b$), respectively, Fig.1. It is assumed that power line route is straight in direction of $z-$axis and infinite number of towers is equidistantly arranged with distance d. Using boundary condition that the tower potential is equal to zero, the following integral equation for determination of induced line charges along the tower can be derived:

$$\underline{\varphi}_{\text{pl}}\Big|_{\substack{x=0\\ z=r_t}} + \sum_{n=-\infty}^{\infty}\frac{1}{4\pi\varepsilon}\int_0^h \underline{q}'(y')\left(\frac{1}{R_n} - \frac{1}{R'_n}\right)dy' = 0\,,$$

$$0 \le y \le h\,, \tag{15}$$

where tower axis is defined by $x = 0$, $z = 0$, $r_t = a + (b-a)y/h$ is a radius of the tower,

$$R_n = \sqrt{x^2 + (y-y')^2 + (z-nd)^2}\,,$$

$$R'_n = \sqrt{x^2 + (y+y')^2 + (z-nd)^2}$$

and $\underline{q}'(y')$ is the induced line charge.

Integral equation (15) is automatically satisfied on the ground ($y = 0$). The ground influence on the tower is taken into account using image theorem. The mutual influence of the towers is also taken into account.

This integral equation cannot be exactly solved, so the point matching method with polynomial approximation for line charge distribution on tower sections is applied. Now, this equation has a form:

$$\underline{\varphi}_{\text{pl}}\Big|_{\substack{x=0\\ z=r_t}} + \sum_{m=1}^{M}\sum_{n=-\infty}^{\infty}\frac{1}{4\pi\varepsilon}\int_{h_{m-1}}^{h_m}\underline{q}'_m(y')\left(\frac{1}{R_n} - \frac{1}{R'_n}\right)dy' = 0$$

$$0 \le y \le h \tag{16}$$

where M is the number of tower sections, h_m is bottom and h_{m-1} is height of upper basis in m^{th} section and

$$\underline{q}'_m(y') = \sum_{k=0}^{K_m}\underline{Q}_{mk}\,y'^k\,,\ h_{m-1} \le y' \le h_m\,,$$

$$m = 1, 2, ..., M \tag{17}$$

is the line charge density on the m^{th} section ($h_0 = 0$ and $h_M = h$), and K_m is polynomial degree in line charge approximation. $\underline{Q}_{mk}$ are unknown coefficients, which should be determined. $\underline{Q}_{10} = 0$ is obtained from condition $\underline{q}'(0) = 0$. The number of coefficients $\underline{Q}_{mk}$ is:

$$J = \sum_{m=1}^{M}K_m - M + 1\,. \tag{18}$$

They can be determined by matching the integral equations (16) in J points placed at tower surface. The matching points are chosen equidistantly so that on each section are placed $K_m - 1$ points, except on the last section where should be placed K_m points. The matching points overlapping should be avoided.
Satisfying these conditions, the system of J integral equations is formed. Solving this system, the unknown coefficients $\underline{Q}_{mk}$ can be determined.
The current along the tower is

$$\underline{I}_{\mathrm{t}}(y) = \mathrm{j}\omega \int_{y}^{h} \underline{q}'(y')\,\mathrm{d}\,y' . \qquad (19)$$

The electric field component can be calculated as:

$$\underline{\boldsymbol{E}} = \underline{\boldsymbol{E}}_{\mathrm{pl}} + \underline{\boldsymbol{E}}_{\mathrm{t}} \text{, where} \qquad (20)$$

$$\underline{\boldsymbol{E}}_{\mathrm{t}} = \sum_{n=-\infty}^{\infty} \frac{1}{4\pi\varepsilon_0} \int_{0}^{h} \underline{q}'(y') \left(\frac{\boldsymbol{R}_n}{R_n^3} - \frac{\boldsymbol{R}'_n}{R'^3_n} \right) \mathrm{d}\,y' , \qquad (21)$$

is the electric field component caused by induced line charges on the tower.
The magnetic field component can be calculated as:

$$\underline{\boldsymbol{H}} = \underline{\boldsymbol{H}}_{\mathrm{pl}} + \underline{\boldsymbol{H}}_{\mathrm{t}} \text{, where} \qquad (22)$$

$$\underline{\boldsymbol{H}}_{\mathrm{t}} = \frac{\mathrm{rot}(\underline{A}_{\mathrm{t}}\hat{y})}{\mu_0} , \qquad (23)$$

is the magnetic field component caused by currents induced on tower. $\underline{\boldsymbol{A}}_{\mathrm{t}}$ is the magnetic vector potential of currents along the tower:

$$\underline{A}_{\mathrm{t}} = \sum_{n=-\infty}^{\infty} \frac{\mu_0}{4\pi} \int_{0}^{h} \underline{I}_{\mathrm{t}}(y') \left(\frac{1}{R_n} - \frac{1}{R'_n} \right) \mathrm{d}\,y' . \qquad (24)$$

4. NUMERICAL RESULTS

The power line of 2x110kV with two grounding wires is considered, Fig. 4. This power line is commonly used in Serbian power electric system. The used parameters are shown in Table 1. The height of conductors 1 and 2 is $h_1 = 13\,\mathrm{m}$.
The power line is placed above a real earth with parameters $\varepsilon_{\mathrm{r}} = 5$ and $\sigma = 10^{-5}$ S/m.
Using the expression (5) the conductor currents are calculated:

$$I_1 = I_2 = (-161 - \mathrm{j}16.4)\,\mathrm{A} ,$$
$$I_3 = I_4 = (89.4 + \mathrm{j}152)\,\mathrm{A} ,$$
$$I_5 = I_6 = (71 - \mathrm{j}153)\,\mathrm{A} ,\ I_7 = I_8 = (-25.1 + \mathrm{j}27.5)\,\mathrm{A} .$$

Tower radius and height are $r_{\mathrm{t}} = 0.25\,\mathrm{m}\,(a = b)$ and $h = 25\,\mathrm{m}$, respectively.

A distance between two towers is $d = 100\,\mathrm{m}$.

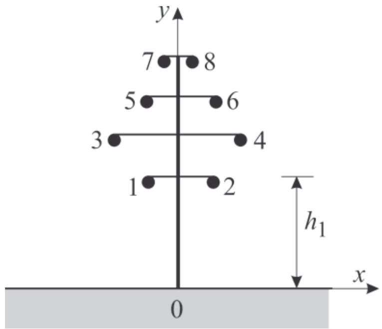

Figure 4: 2x110kV power line cross-section.

	j	x_j [m]	y_j [m]	r_j [mm]
Phase I	1	-3.8	h_1	11
	2	3.8		
Phase II	3	-4.6	$h_1 + 4.7$	11
	4	4.6		
Phase III	5	-3	$h_1 + 9.2$	11
	6	3		
Grounding wires	7	-2	$h_1 + 11.2$	4.5
	8	2		

Table 1: Parameters and their values.

In Figs. 5 and 6, the induced charge distribution along the tower for different parameters are shown.

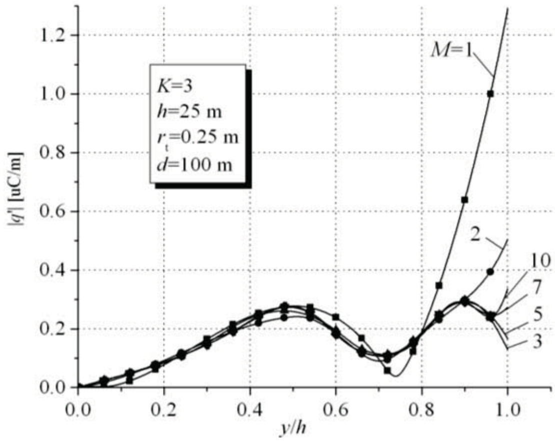

Figure 5: Induced charge distribution along the tower for different tower section numbers.

From those figures is evident that good results convergence is obtained for $M \geq 3$ and $K \geq 2$. In further calculations the values of these two parameters will be: $M = 5$ and polynomial degree in the line charge approximation $K_1 = \ldots = K_5 = K = 3$. Using those parameters the electromagnetic field components can be calculated.

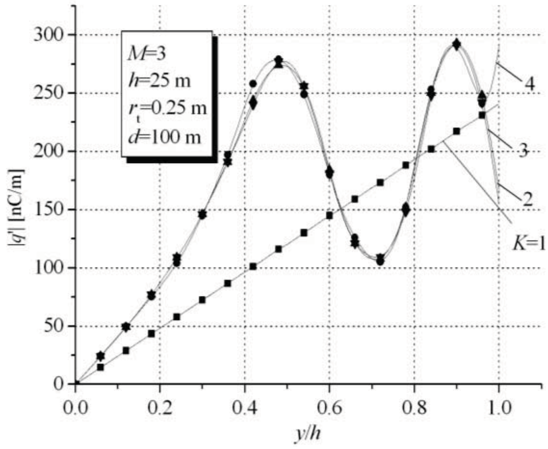

Figure 6: Induced charge distribution along the tower for different polynomial degrees in line charge approximation.

The electromagnetic field components caused by the power line conductors are calculated using expressions (10) and (14). Those electric and magnetic field distributions are shown in Figs. 7 and 8, respectively.

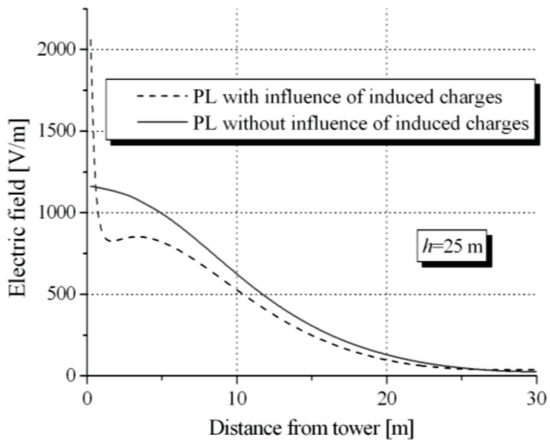

Figure 7: Electric field distribution in vicinity of 2x110 kV power line.

The charges and currents induced in tower have more relevant influence on the electric field than on the magnetic field distribution. The currents influences on the magnetic field distribution can be neglected, Fig. 9. Fig. 7 gives a comparison of electric field distribution with and without influence of induced charges. The influence of induced charges on the electric field distribution near the tower is evident. That value is around 2.25 kV/m and it is still less than values proposed in the ICNIRP recommendation [8]. It should be less than 10 kV/m in working and 5 kV/m in living environment. The calculated magnetic field values are below the boundary values from the ICNIRP recommendation. Those values should be less than 400 A/m in working and 80 A/m in living environment.

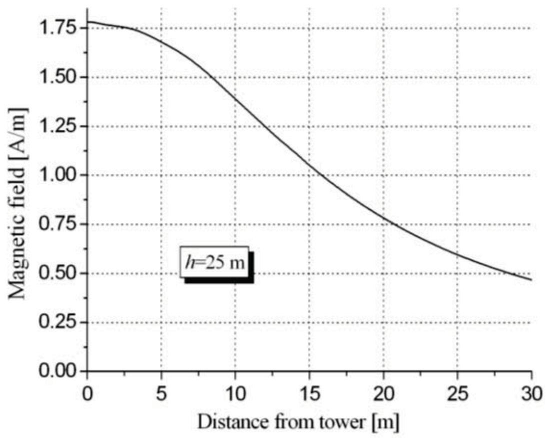

Figure 8: Magnetic field distribution in vicinity of 2x110 kV power line.

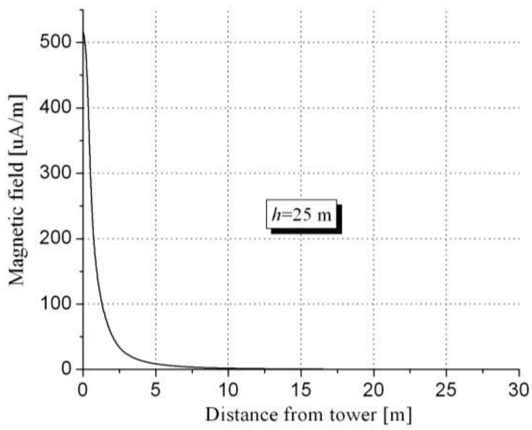

Figure 9: Magnetic field distribution from induced currents on power line tower.

In Fig. 10 the obtained results for magnetic field distribution are compared with FEM results [5]. Using this software the mesh with 179369 nodes was created. The power line of 2x110 kV is placed in the air above a real earth of parameters $\sigma = 10^{-5}$ S/m and $\mu_r = 1$. Through the conductors flow currents calculated using the expression (5).

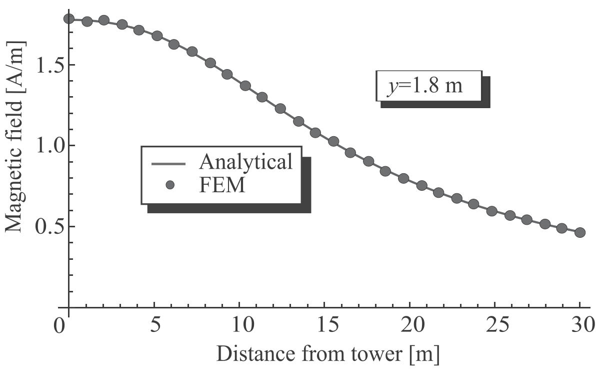

Figure 10: Results comparison.

An excellent result agreement is obtained. All calculations presented here are performed at height of 1.8m that correspond to an average human height.
The induced current distribution along the tower, given by expression (19), is shown in Fig. 11.

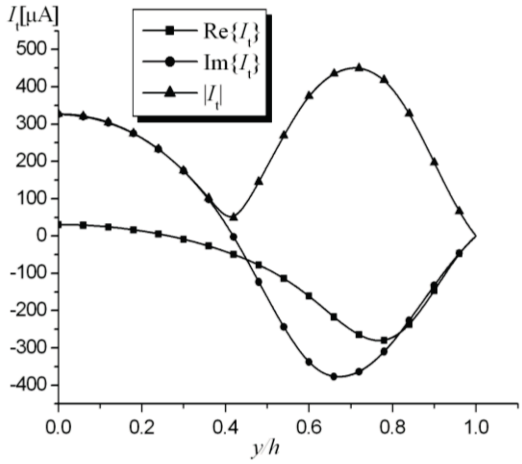

Figure 11: Current distribution along power line tower.

5. CONCLUSIONS

An analytical method for electromagnetic field calculation in vicinity of power line is presented. The interaction between power line conductors and towers are taken into account. Influences of currents and line charges, induced on the power line tower, on electric field distribution in close vicinity of the 2x110 kV power line are noticed during this research. That distance is about 1/10 of tower height. In other areas that influence can be neglected.
An influence of different parameters values on induced charge distribution along the tower is examined. The good result convergence is obtained when number of tower sections is $M \geq 3$ and polynomial degree in the line charge approximation is $K \geq 2$.
Further investigation will include conductor catenaries influence on electromagnetic field distribution as well as the change of line direction.

Acknowledgments

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