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STUDY OF UNEVENNESS HEIGHT DISTRIBUTION FOR GRAPHITE PANTOGRAPH CONTACT STRIP

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Abstract: **An assumption of data distribution is of great importance because in many cases it determines the method that ought to be used to estimate the unknown parameters in the model. In this paper we performed a study of unevenness height distribution for three graphite plates, with different wear degree, used as pantograph contact strips. Using roughness diagram we determined the height, number of asperities and statistical values. With Kolmogorov test we verify the unevenness height distribution for two cases: exponential and normal distribution. We assumed a theoretical repartition, which is compared with empirical repartition. The obtain values for random variable are compared to verify the hypothesis assumed. Then we verify the equality of variance with Fischer-Snedecor test and the equality of mean with Student test for two probes which have the same repartiton. The study results of unevenness for three graphite plates used as pantograph contact strips, based on roughness diagram, let to conclusion that the unevenness height distribution vary with degree of wear.**

Keywords: *Kolmogorov test, unevenness, statistical distribution, graphite, contact strip.*

1. INTRODUCTION

Many data analysis methods depend on the assumption that data were sampled from a normal distribution or at least from a distribution which is sufficiently close to a normal one[1]. An assumption is of great importance because in many cases it determines the method that ought to be used to estimate the unknown parameters in the model and also dictates the test procedures, which the analyst may apply.

The influence of unevenness height statistical distribution on contact pressure variation with displacement is pointed out in [2].

The Kolmogorov test is the most well known test for normality and it is applied in many studies [3].

In [1] is presented an improvement Kolmogorov test for normality where a sample is compared with a normal distribution. It proposes to select the mean and variance of the normal distribution that provide the closest fit to the data. If the result does not lead to an acceptable fit, the data is probably not normal.

In this paper we perform a study of unevenness height distribution for three graphite plates with different degree of wear, use as pantograph contact strip. Using roughness diagram we verify the unevenness height distribution with Kolmogorov test.

2. BASIC EQUATIONS OF KOLMOGOROV TEST

With a series of n independent observations of a random variable (a series of data), we construct the empirical distribution function $F_n(x)$ using the cumulative frequency in the predefined interval [4]:

$$x_0^* \le x_1^* \le ... \le x_n^* \Rightarrow F_n(x) = \begin{cases} 0 & x < x_0^* \\ \frac{1}{n}\sum_{i=1}^{k-1} n_i & x_{k-1}^* < x \le x_k^* \\ 1 & x \ge x_n^* \end{cases} \quad (1)$$

We assumed a theoretical repartition $F_\xi(x)$, which must be compared with empirical repartition. Let denote the maximum difference between the two distributions [4] as:

$$\mathrm{D}_n = \sup_x \left| F_\xi(x) - F_n(x) \right| \quad (2)$$

If the proposed repartition $F_\xi(x)$ is well, when $n \to \infty$ the empirical repartition is close to $F_n(x)$ (Glivenko, 1933):

$$P\left(\lim_{n\to\infty} \mathrm{D}_n = 0 \right) = 1 \quad (3)$$

Maximum difference is also a random variable and its reparations results from Kolmogorov theorem:

If $F_\xi(x)$ is absolute continuous then [4]:

$$\lim_{n\to\infty} P\left(\sqrt{n} \cdot \mathrm{D}_n < z \right) = \mathrm{K}(z) \quad (4)$$

$$\mathrm{K}(z) = \begin{cases} 0 & z \le 0 \\ \sum_{k=-\infty}^{+\infty} (-1)^k \cdot \exp(-2 \cdot k^2 \cdot z^2) & z > 0 \end{cases} \quad (5)$$

For large enough values of n, the function K(z) can be considered like a repartition function of random variable $\lambda_n = \sqrt{n} \cdot \mathrm{D}_n$ [4]:

$$P(\lambda_n < z) \approx \mathrm{K}(z) \quad \mathrm{K}(1) = 0.73 \quad \mathrm{K}(1.5) = 0.97 \qquad (6)$$

The values of λ_n can be utilized like a test. If $1 \le \lambda_n \le 5$ the hypothesis is suspect and it is recommended to increases the experience number. If $\lambda_n > 1.5$ an event almost impossible is happened and the assumption that the distribution function is $F_\xi(x)$ must be rejected. If $\lambda_n < 1$ there are no reasons to rejecte the assumption.

3. UNEVENNESS HEIGHT DISTRIBUTION EVALUATION

Figure 1 present the roughness diagram for three graphite plates with different wear degree.

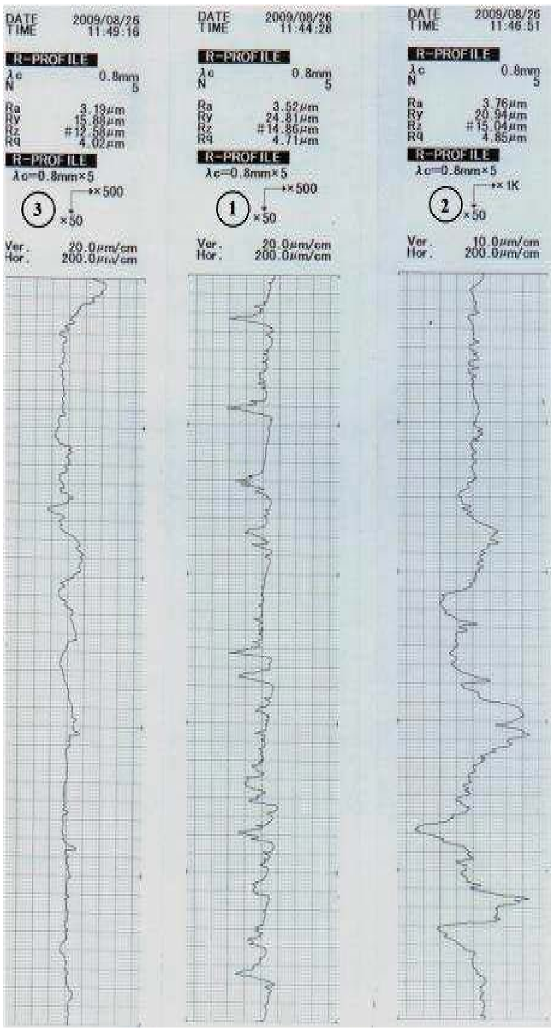

Figure 1 Roughness diagram for graphite plates

In table 2 the next notations are used:

- n is samples size;
- *m, M* are minimal and is maximal value;
- *mean* is mean value of sample;
- *Stdev, Var are* standard deviation and variance.

Table 1 give the experimental values for graphite plates obtained from roughness diagram.

h_1	N_1	h_2	N_2	h_3	N_3
1	14	0.5	11	1	5
3	9	1.5	8	3	5
5	6	2.5	3	5	9
7	6	3.5	2	7	1
9	3	4.5	4	9	0
11	2	5.5	2	11	1
13	3	6.5	3		
15	2	7.5	1		
17	1	8.5	2		
19	3	9.5	4		

Table 1: Experimental values for graphite plates

	n	*m*	*M*	*mean*	*Stdev*	*Var*
Sample 1	49	2	19	6.26	5.25	27.65
Sample 2	40	0.5	9.5	3.57	3.13	9.81
Sample 3	21	1	11	3.95	2.41	5.84

Table 2: Statistical values for graphite plates

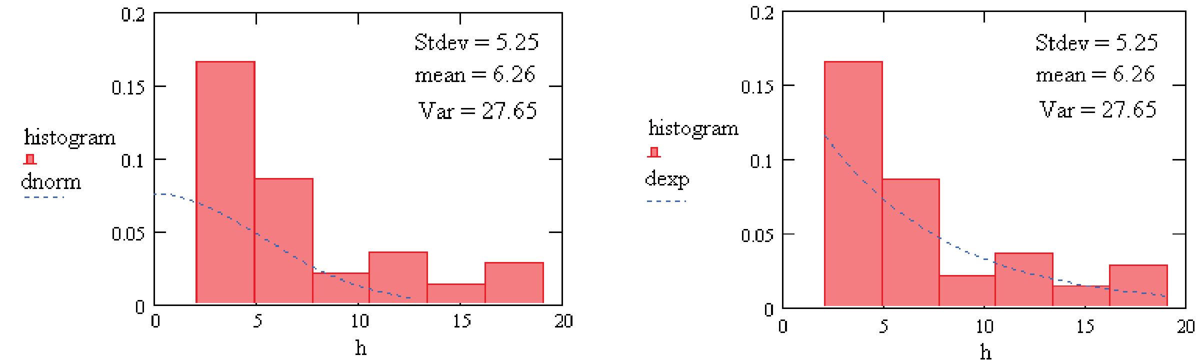

Figure 2: Histogram and density repartition for sample 1

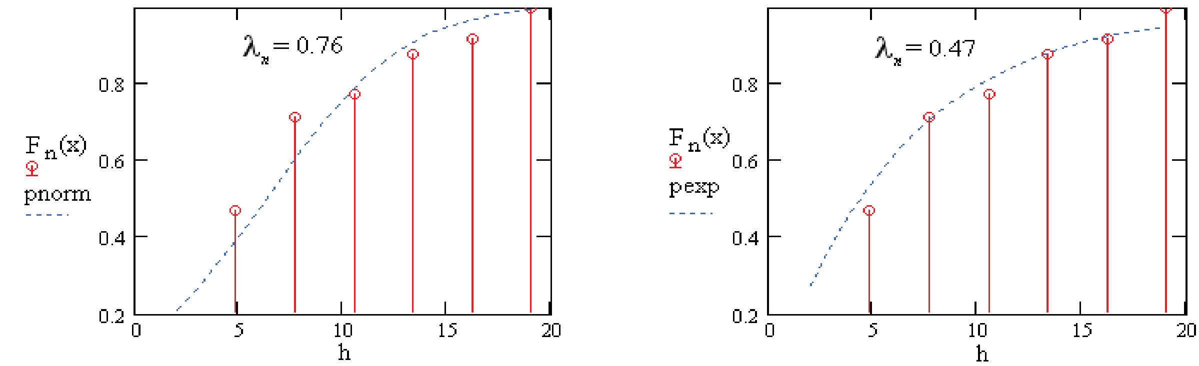

Figure 3: Empirical distribution function and repartition function for sample 1

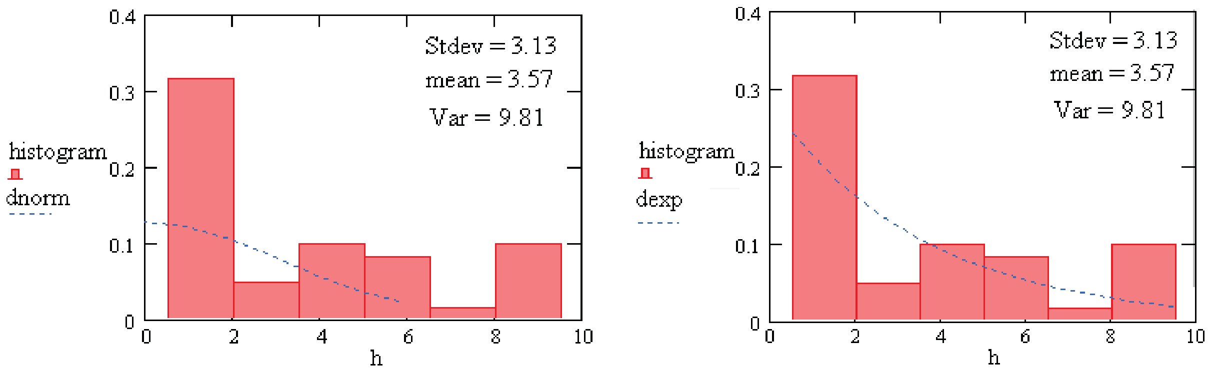

Figure 4: Histogram and density repartition for sample 2

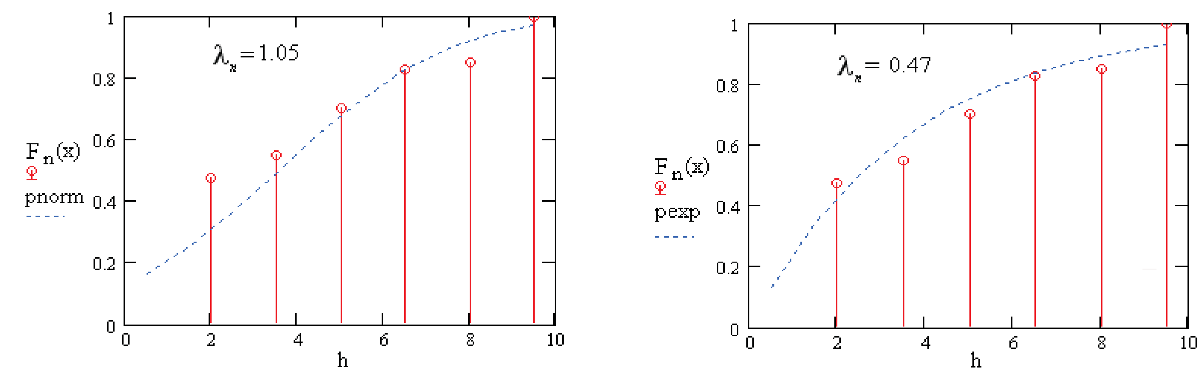

Figure 5: Empirical distribution function and repartition function for sample 2

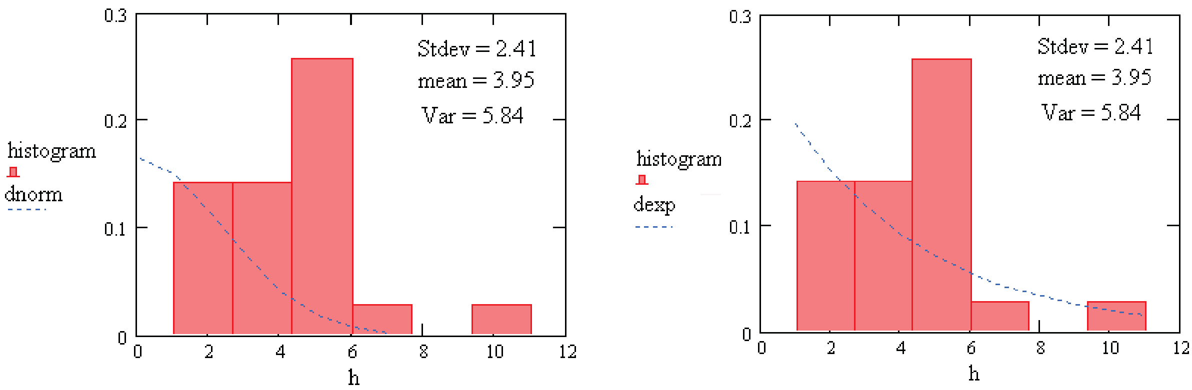

Figure 6: Histogram and density repartition for sample 3

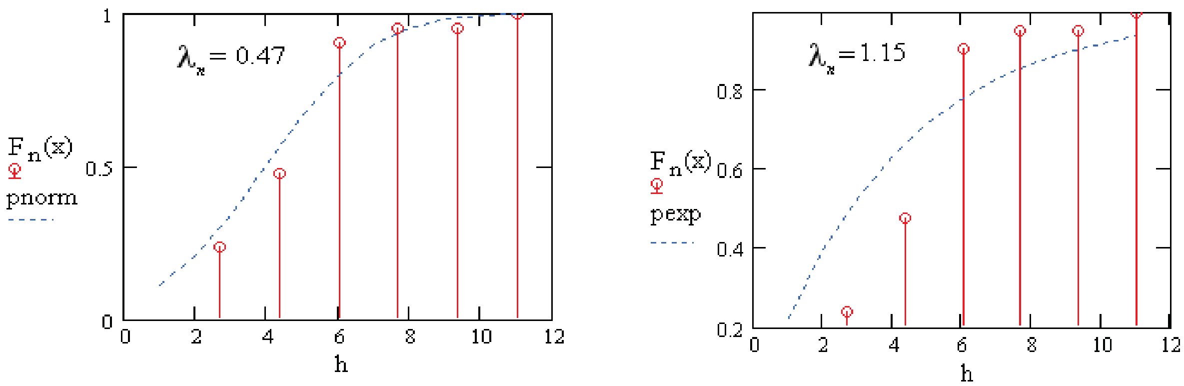

Figure 7: Empirical distribution function and repartition function for sample 3

Solving equations 1-5 we obtain λ_n (table 3).

Distribution		Sample1	Sample2	Sample3
normal	λ_n	0.76	1.05	0.47
exponential	λ_n	0.47	0.47	1.15

Table 3: Values for λ_n

Since λ_n have the same value for samples 1 and 2 we can verify the equality of variance with *Test F of Fischer-Snedecor* [4].

The random variable wich satisfies the density distribution *F-S* is:

$$\gamma = \frac{\frac{1}{m}\cdot\sum_{i=1}^{m}\xi_i^2}{\frac{1}{n}\cdot\sum_{i=1}^{n}\eta_i^2} \tag{7}$$

The density distribution *F-S* with *m* and *n* free degrees is given by relation (8).

$$f_\gamma(x) = \frac{m^{\frac{m}{2}}\cdot n^{\frac{n}{2}}\cdot\Gamma\left(\frac{m+n}{2}\right)}{\Gamma\left(\frac{m}{2}\right)\cdot\Gamma\left(\frac{n}{2}\right)}\cdot\frac{x^{\frac{m}{2}-1}}{(m\cdot x+n)^{\frac{m+n}{2}}}, x\geq 0 \tag{8}$$

The test value t is the ratio between the two samples variance. Critical value for a trust α is $q=1-\alpha/2$ of repartition with (m-1), (n-1) free degrees $x_q(m-1,n-1)$.

The hypothesis of equality variance is rejected if $t > x_q(m-1,n-1)$ or the repartition function $pF(t,m-1,n-1) > q$. The results of *Test F* obtained by solving equations (7-8) are given in table4.

$t = Var_1/Var_2$	qF	pF	q
2.83	1.85	0.99	0.975

Table 4: Results of *Test F*

Analyzing the results of table 4 we can conclude that the hypothesis of equality variance is rejected.

We can verify the mean equality for samples 1 and 2 with *Student test* . The density distribution with m free degrees is given by relation [10]:

$$f_t(x) = \frac{\Gamma\left(\frac{m+1}{2}\right)}{\Gamma\left(\frac{m}{2}\right)\cdot\sqrt{m\cdot\pi}}\cdot\left(1+\frac{x^2}{m}\right)^{-\frac{m+1}{2}} \tag{9}$$

The value of test t for two samples with dimensions m and n is:

$$t = \frac{\left(\bar{x}-\bar{y}\right)\cdot\sqrt{\frac{n_x\cdot n_y}{n_x+n_y}}}{\sqrt{\frac{\left(n_x-1\right)\cdot\sigma_x+\left(n_y-1\right)\cdot\sigma_y}{n_x+n_y-2}}} \tag{10}$$

where:

- $\bar{x}$, $\bar{y}$ are statistical means;
- σ_x, σ_y are variances.

Critical value for a trust α is $q=1-\alpha/2$ of repartition with $m=n_x+n_y-2$, $x_q(m)$.

The hypothesis of equality mean is rejected if the next conditions are satisfied:

- $|t| > x_q(m)$;

- the repartition function $pt(t,m) > q$.

The results of Student test for sample 1 and 2 are presented in table 5.

t	qt	pt	q
2.87	1.98	0.99	0.975

Table 5: Results of *Student test*

Analyzing the results presented in table 5 we can conclude that the hypothesis of mean equality is rejected.

4. CONCLUSIONS

In this paper we perform a study of unevenness height distribution for three graphite plates with different degree of wear, use as pantograph contact strip.

Using roughness diagram we verify the unevenness height distribution with Kolmogorov test.

From Kolmogorov, Fischer-Snedecor and Student tests we can conclude that:

- sample 2 is more worn, polished with smaller roughness and dispersion than sample 1;
- samples 1 and 2 could have exponential distribution;
- more worn is sample 3 wich distribution became normal;
- after large roughness disappear by wear, the distribution approaches to the normal;
- the unevenness height distribution vary with degree of wear.

References

[1] Z. Drezner, O. Turel, D. Zerom, *A modified Kolmogorov-Smirnov test for normality*, Munich Personal RePEc Archive, MPRA 2008.

[2]C. F. Ocoleanu, Ghe. Manolea, G. Cividjian, *Experimental study of contact resistance variations for pantograph-contact line contact*, Proceedings of WSEAS International conference on Risk Management, Assessement and Mitigation, Bucharest, Romania, 2010, pp. 101-105.

[3]L. D. Hall, D. Mba, R. H. Bannister, *Acoustic emission signal classification in condition monitoring using the Kolmogorov-Smirnov statistic*, Journal of Acoustic emission, Vol. 19, 2001, pp. 209-228.

[4] G. Cividjian, A. G. Cividjian, *Modeles Statisques et Fiabilite*, Edition University of Craiova, 2003.

[5] Cividjian G.A., Pascu I., Bunescu A., Matei D.,M., Dolan A., *Experimental study of the* resistance *of flat Cu-W contacts*, XII-th International Symposium on Electrical Apparatus and Technologies, SIELA, Procedingss, Vol.1, 31 May-1 june, Bulgaria, pp.58-65, 2001.

[6] Chang W.R., Etsion I., Bogy D.B., *An elastic-plastic model for the contact of rough surfaces*, ASME J. Tribology, 109, pp. 257-263, 1987.

[7]H. Ventsel, *Theories des probabilites*, Ed. Mir, Moscou, 1973.

[8]Collina Andrea, Bucca Giuseppe, *Procedure for the wear prediction of collector strip and contact wire in pantograph-catenary system,* Wear 266, 2008, pp. 59-64.

[9] Tu Chuan-jun, Chen Zhen-hua, Chen Ding, Yan Hong-ge, He Feng-yi, *Tribological behavior and wear mechanism of resin-matrix contact strip against copper with electrical current,* Transactions of nonferrous metals society of China, 18, pp. 1157-1163, 2008.

[10]G. Mihoc, A. Muja, E. Diatcu, *Bazele matematice ale teoriei fiabilității*, Ed. Dacia, Cluj-Napoca, 1976.

[11]V. Panaite, R. Munteanu, *Control statistic şi fiabilitate,* EDP Bucureşti, 1982.