CONCEPTS FOR ERROR MODELING OF MINIATURE ACCELEROMETERS USED IN INERTIAL NAVIGATION SYSTEMS

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Abstract – **The paper is a study of the error models for the acceleration sensors used in the strap-down inertial navigation systems. Two categories of errors models, presented in the literature, are described: IEEE standardized models, and un-standardized derived models. Generally the models contains the scale factor error, the asymmetric scale factor error, the nonlinear scale factor error, the misalignment errors, the non-orthogonality errors, noise and all the static errors terms. Also, a simplified model considering the parameters from the real sensors data sheets is analytically established and software implemented using Matlab/Simulink software. The model can be successfully used in the numerical simulation of any strap-down inertial navigation system, to create similar conditions with the real ones from the point of view of the distortions that affects the useful acceleration signal, when passing through any accelerometric detection device desired to be implemented in the sensing block of the navigator. Another advantage of this software implemented model is the possibility to have in the disturbed acceleration signal, used in numerical simulation, each type of the sensor's errors taken independently or any combination between these errors; the model allows the selection of any sensor's errors or combination between them to be superposed on the acceleration applied along of the accelerometer sensitivity axis.**

Keywords: *inertial navigation, acceleration sensors, sensors errors, error analytical model, error software model*

1. INTRODUCTION

Inertial navigation is the process of measuring the total acceleration of the vehicle and its integration to determine vehicle speed and position relative to a starting point. In addition to determining the position and vehicle speed, inertial navigation system must provide information on aircraft attitude in the horizontal plane, in terms of angles of pitch, roll and yaw. This information is vital for the pilot to fly the plane safely in any weather conditions, including the poor visibility or reference points missing situations [1], [2].

Generally, the three basic functions of a strap-down inertial navigation system (INS) are: sensing, calculation and output [3] (Fig. 1). The sensing function is fulfilled by the accelerometers and gyros, these measurements are transmitted to the navigation computer. The computer uses this data to generate speed, position, attitude, attitude rates, altitude and distance to destination. If true speed of a computer is provided by an aerodynamic system, INS can also calculate the wind speed and drift angle. Output function is restricted to the forward calculated data to flight control systems, weapons systems, sensors for recognition, or to display and control unit, as required for particular missions.

So, a strap-down inertial navigation includes: a navigation computer to perform mathematical calculations, a precision clock for timing the operations of integration, an accelerometric ensemble to measure specific force, a software gravity model resident in the navigation computer, used to determine the gravitational acceleration as a function of calculated position, and an attitude reference, which defines the angular orientation of the triad of accelerometers relative to inertial space. The attitude reference is provided by a numerical integration software resident in navigation computer, using as inputs the measurements obtained from a gyros triad with axes parallel to the accelerometers triad. For a system with medium accuracy are used gyros with drift in the range $0.3^o/h \div 10\ ^o/h$ and for one with very good accuracy, gyros with drift up to $0.3^o/h$ [4].

In general, a navigation system required accuracy is a nautical mile per hour of flight for most applications. For certain phases of flight some constraints may be placed on the accuracy of the system, so if the flight phase is close to the ground the accuracy must be of several meters. In this case it's necessary to assist the INS by another navigation system to update data in order to achieve the accuracy required. This situation requires the connection to the navigation computer of multiple navigation systems and the using of more complicated software implemented algorithms.

Position, speed and attitude determination error for an aircraft, using an inertial navigation system, result from the imperfect knowledge of initial conditions, from the errors due to numerical calculation, and from the inertial sensors errors (accelerometers and gyros). So, the inertial sensors performances play a key role in determining the accuracy of navigation system which incorporates them and, therefore, should be taken into account in its design.

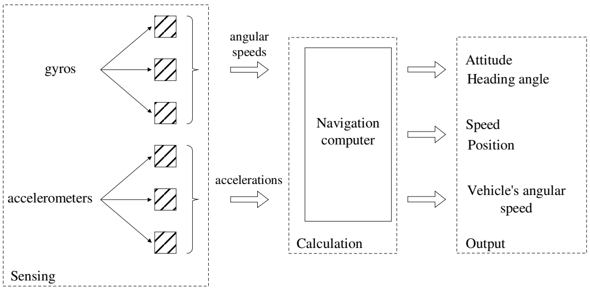

Figure 1: Simplified diagram of a strap-down INS.

From other point of view, the using of the new technologies, micro-electromechanical systems (MEMS) or nano-electromechanical systems (NEMS), play an important role in the reduction of the cost and of the volume of the inertial sensors. Apparently, the cost and volume decreasing offer an important advantage for the miniaturized inertial sensors, but, in fact, the fabrication processes of such sensors make these very sensitive to the changes of the environmental conditions: temperature, pressure, electric, and magnetic fields, vibrations. As a consequence, the sensors output can vary rapidly, widely and sometimes randomly, and is very hard to be modeled. In many cases, this sensitivity causes the decreasing of the sensor performances, adding more error types and possibly, higher errors than those of traditional non-miniaturized sensors [5]. So, all inertial sensors are subject to errors which limit the accuracy to which the applied specific force can be measured, causing in this way unacceptable drifts and bias [6]. Two categories of errors can be subtracted from here: deterministic errors, and stochastic errors. While the deterministic errors can be eliminated as a consequence of a relative easily calibration procedure, the stochastic errors estimation necessitates a more complex process. Some specific techniques were developed in this way, like Allan variance method or Power Spectrum Density (PSD) method [7], [8]. The calibration procedures can be also divided into conventional procedures, and modern procedures, the latest ones supposing the presence of a Kalman filtering scheme providing the optimal estimates of the calibration coefficients. The most used calibration procedures are the conventional ones.

The here presented work is a part of a research project first stage concerning the developpment of *high-precision strap-down inertial navigators, based on the connection and adaptive integration of the nano and micro inertial sensors in low cost networks, with a high degree of redundance*, financed by National Council for Scientific Research in Higher Education (CNCSIS) in Romania.

2. ANALYTICAL ACCELEROMETERS' ERROR MODELS

From an author to another, the error model of the accelerometer is more or less complicate. The considering or neglecting of an error term in the model depends by the sensor type and quality, but also by the possibility to be estimated through the adopted calibration procedure. The accelerometers can be individually tested and calibrated, but also as a part of an inertial measurement unit (IMU) of an inertial navigation system.

After Titerton ([6]), the measurement provided by an accelerometer (a_{ix}) may be expressed as the form

$$a_{ix} = (1+S_x)a_x + M_y a_y + M_z a_z + B_f + B_v a_x a_y + n_x, \quad (1)$$

where a_x is the acceleration applied in the direction of the sensitive axis, a_y and a_z are the accelerations applied perpendicular to the sensitive axis, S_x is the scale factor error, M_y and M_z are the cross axis coupling factors, B_f is the measurement bias, B_v is the vibro-pendulous error coefficient, and n_x is the random bias.

Reconsidering the error model for the full accelerometric triad of an IMU, after El-Diasty [9] we can have the following matriceal equation

$$f_{imu} \approx [I + S_a + \delta S_a]f + b_a + \delta b_a + w_a, \quad (2)$$

where I is the unit matrix 3x3, S_a is the matrix containing the scale factor errors (diagonal), and non-orthogonality errors (non-diagonal elements), b_a is the vector of accelerometers' biases, δS_a is the matrix containing the scale residual errors (diagonal elements) and residual non-orthogonality errors (non-diagonal elements), δb_a is the vector of the accelerometers' residual errors, and w_a are the accelerometers' zero mean white noises,

$$S_a = \begin{bmatrix} s_{xx} & s_{xy} & s_{xz} \\ s_{yx} & s_{yy} & s_{yz} \\ s_{zx} & s_{zy} & s_{zz} \end{bmatrix}, \ \delta S_a = \begin{bmatrix} \delta s_{xx} & \delta s_{xy} & \delta s_{xz} \\ \delta s_{yx} & \delta s_{yy} & \delta s_{yz} \\ \delta s_{zx} & \delta s_{zy} & \delta s_{zz} \end{bmatrix}, \quad (3)$$

$$b_a = [b_x \quad b_y \quad b_z]^T, \ \delta b_a = [\delta b_x \quad \delta b_y \quad \delta b_z]^T, \quad (4)$$

$$w_a = [w_x \quad w_y \quad w_z]^T. \quad (5)$$

The model contains both deterministic and stochastic errors. The biases and scale factor errors can be estimated through laboratory calibration, or by using of a Kalman filter, while the residual biases and residual scale errors are random errors and usually can be modeled by stochastic models inside a Kalman filter at each epoch and then removed [9].

An advanced error model of the accelerometers triad [10], [11] contains the scale factor error, the asymmetric scale factor error, the nonlinear scale factor error, the misalignment errors, the non-orthogonality errors and all the static errors terms,

$$\begin{aligned} a_{\ddot{x}} &= (1+SF_x)\ddot{x} + SFA_x|\ddot{x}| + SFN_x\ddot{x}^2 + \sin(\Delta_{Az}+\delta_{Az})\ddot{y} + \\ &\quad + \sin(\Delta_{Ay}-\delta_{Ay})\ddot{z} + c_{\ddot{x}} + b_{\ddot{x}} + w_x, \\ a_{\ddot{y}} &= (1+SF_y)\ddot{y} + SFA_y|\ddot{y}| + SFN_y\ddot{y}^2 + \sin(\Delta_{Az}-\delta_{Az})\ddot{x} + \\ &\quad + \sin(\Delta_{Ax}+\delta_{Ax})\ddot{z} + c_{\ddot{y}} + b_{\ddot{y}} + w_y, \\ a_{\ddot{z}} &= (1+SF_z)\ddot{z} + SFA_z|\ddot{z}| + SFN_z\ddot{z}^2 + \sin(\Delta_{Ay}+\delta_{Ay})\ddot{x} + \\ &\quad + \sin(\Delta_{Ax}-\delta_{Ax})\ddot{y} + c_{\ddot{z}} + b_{\ddot{z}} + w_z. \end{aligned} \tag{6}$$

$a_{\ddot{x}}, a_{\ddot{y}}, a_{\ddot{z}}$ are the measured accelerations, $\ddot{x}, \ddot{y}, \ddot{z}$ are the true accelerations, SF_x, SF_y, SF_z are the longitudinal, vertical, and lateral, scale factors errors, SFA_x, SFA_y, SFA_z are the longitudinal, vertical, and lateral, scale factors asymmetry, SFN_x, SFN_y, SFN_z are the longitudinal, vertical, and lateral, scale factors nonlinearity, $\delta_{Ax}, \delta_{Ay}, \delta_{Az}$ are the accelerometer triad misalignments about x, y and z axis, $\Delta_{Ax}, \Delta_{Ay}, \Delta_{Az}$ are the accelerometer triad non-orthogonality errors about x, y and z axis, $c_{\ddot{x}}, c_{\ddot{y}}, c_{\ddot{z}}$ are the longitudinal, vertical, and lateral constant biases, and $b_{\ddot{x}}, b_{\ddot{y}}, b_{\ddot{z}}$ are the longitudinal, vertical, and lateral random biases.

Starting from the parameters and errors of rotation and acceleration sensors the IEEE specialists conceived and standardized several models for various types of inertial sensors [12]-[15]. IEEE established models are used by manufacturers to calibrate sensors and achieve their data sheets. Also, these models have an important role in the clearing by the users of a large number of errors affecting the sensors.

The first model proposed by IEEE for accelerometer transducers was standardized in 1972 and was realized for „Linear, Single-Axis, Pendulous, Analog Torque Balance Accelerometers” [12]. It implies the presence of errors due to bias, scale nonlinearity, non-alignment between the axis of input and output, sensitivity to acceleration applied perpendicular to the entry level (cross-axis sensitivity), noise and calibration errors of scale factor. Such a model can be described by the equation [12]

$$\begin{aligned} A_{ind} = \frac{E}{K_1} &= K_o + a_i + K_2 a_i^2 + K_3 a_i^3 + \delta_o a_p - \delta_p a_o + \\ &\quad + K_{ip} a_i a_p + K_{io} a_i a_o, \end{aligned} \tag{7}$$

where A_{ind} is the acceleration indicated by the accelerometer, expressed in g units, E is the accelerometer output expressed in output units, a_i is the acceleration component along positive input axis (IA) (in g units), a_p is the acceleration component along pendulous axis (PA) (in g units), a_o is the acceleration component along output axis (OA) (in g units), K_o is the bias in g, K_1 is the scale factor, expressed in output units/g, K_2, K_3 are second-order, respectively third order, nonlinearity coefficients (in $1/g$, respectively $1/g^2$, units), δ_o, δ_p are the misalignment angles of the input axis with respect to the input reference axis about the output reference and pendulous reference axes, respectively, expressed in radians, and K_{ip}, K_{io} are constants of proportionality, which highlights the dependence of output from the product of accelerations applied normal and parallel to the input (cross-coupling), expressed in $g/g\cdot$(cross g). δ_o and δ_p include angular errors of the mounting fixture, dividing head errors, and errors in mounting as well as misalignment of the input axis with respect to the input reference axis [12]. The output axis is OA=IA$\times$PA.

The previous model was reedited in 1978, when a new standard was proposed for the „Linear, Single-Axis, Digital, Torque Balance Accelerometer” [13]. The model was conceived in two approaches: conventional higher order nonlinearity and cross coupling model (equation identical with (7)), and asymmetrical scale factor model, described by the expression [13]

$$A_{ind} = \frac{E}{K_1\zeta_+ + K_1'\zeta_-} = K_o + a_i + \delta_o a_p - \delta_p a_o, \tag{8}$$

where A_{ind} is the acceleration indicated by the accelerometer, expressed in g units, E is the accelerometer output expressed in pulses per second, a_i is the acceleration component along positive input axis (in g units), a_p is the acceleration component along pendulous axis (in g units), a_o is the acceleration component along output axis (in g units), K_o is the bias in g, K_1 is the scale factor for $a_i>0$, expressed in (pulse/s)/g, K_1' is the scale factor for $a_i<0$, expressed in (pulse/s)/g, δ_o, δ_p are the misalignment angles of the input axis with respect to the input reference axis about the output reference and pendulous reference axes, respectively, expressed in radians, and

$$\begin{cases} \zeta_+ = 1, \ and \ \zeta_- = 0 \ \ for \ \ a_i > 0, \\ \zeta_+ = 0, \ and \ \zeta_- = 1 \ \ for \ \ a_i < 0. \end{cases} \tag{9}$$

The model (8) was completed in 1992, 2001 and finally in 2009, with some terms, according to the „IEEE Recommended Practice for Precision Centrifuge Testing of Linear Accelerometers” [14],

$$\begin{aligned} a_s = E/K_1 &= K_o + a_i + K_2 a_i^2 + K_3 a_i^3 + K_{ip} a_i a_p + \\ &\quad + K_{io} a_i a_o + K_{op} a_o a_p + \delta_o a_p - \delta_p a_o + \\ &\quad + K_{pp} a_p^2 + K_{ppp} a_p^3 + K_{oo} a_o^2 + K_{ooo} a_o^3 + \dots . \end{aligned} \tag{10}$$

E is the sensor output (output units), a_s is the indicated sensor output (m/s^2), K_o is the bias (m/s^2), K_1 is the scale factor (output units/(m/s^2)), K_2 is the second-order coefficient (1/(m/s^2)), K_3 is the third-order coefficient (1/(m/s^2)^2), K_{ip}, K_{io}, and K_{op} are the cross-coupling coefficients (1/(m/s^2)), a_i is the acceleration component along positive input axis (IA) (m/s^2), a_p is the acceleration component along pendulous axis (PA) (m/s^2), a_o is the acceleration component along output axis (OA) (m/s^2), δ_o, δ_p are the misalignment angles of the input axis with respect to the input reference axis about the output reference and pendulous reference axes, respectively, expressed in radians, $K_{pp}a_p^2$ is the second-order PA coefficient, $K_{ppp}a_p^3$ is the third-order PA coefficient, $K_{oo}a_o^2$ is the second-order OA coefficient, and $K_{ooo}a_o^3$ is the third-order OA coefficient. From one sensor type to another several terms can be added in the model, i.e. the apparent scale factor asymmetry, or apparent bias asymmetry [14].

A more complex model was proposed and standardized in 1998, and after that completed in 2008, for „Linear, Single-axis, Non-gyroscopic Accelerometers" [15],

$$\begin{aligned} E = K_1\{ & K_o + K_o' \operatorname{sgn}(a_i)/2 + \left(1 + K_1' \operatorname{sgn}(a_i)/2\right)a_i + \\ & + K_{oq}a_i|a_i| + K_2a_i^2 + K_3a_i^3 + \sum_{n\ge 4} K_n a_i^n + \delta_o a_p - \\ & - \delta_p a_o + K_{ip}a_ia_p + K_{io}a_ia_o + K_{po}a_pa_o + K_{pp}a_p^2 + \\ & + K_{oo}a_o^2 + K_{\text{spin}}\omega_i\omega_p + K_{\text{ang.accel}}\dot{\omega}_o + \varepsilon\}, \end{aligned} \tag{11}$$

with E - accelerometer output in accelerometer output units, a_i, a_p, a_o, - applied acceleration components along input axis (IA), pendulous axis (PA), and output axis (OA) respectively, expressed in g units, K_o - bias expressed in g, K_o' - bias asymmetry (g), K_1 - scale factor, expressed in output units/g, K_1' - scale factor asymmetry (dimensionless), K_2, K_3 are second-order, respectively third order, nonlinearity coefficients (in 1/g, respectively 1/g^2, units), K_n (n=4, 5, 6…) - other higher order coefficients along IA (1/g^{n-1}), K_{oq} - odd quadratic coefficient (1/g), δ_o, δ_p are the misalignment of the IA with respect to the input reference axis about the OA and PA, respectively, expressed in radians, K_{ip}, K_{io}, and K_{op} are the cross-coupling coefficients (g/g·(cross g) or 1/g), K_{pp}, K_o - cross-axis nonlinearity coefficients (g/(cross g^2)), sgn(a_i) - sign function of the a_i component (is ambiguous in bias asymmetry model when a_i=0 and sgn(a_i)=0 [15]), ω_i, ω_p, ω_o - angular velocity components along the IA, PA and OA (rad/s), $\dot{\omega}_i, \dot{\omega}_p, \dot{\omega}_o$ - angular acceleration components along the IA, PA and OA (rad/s^2), ε - measurement and process noise and unmodeled errors (g), K_{spin}, $K_{\text{ang.accel}}$ - spin correction coefficient, and angular acceleration coefficient (converted to g/(rad/s^2));

$$K_{\text{spin}} = -[(J_i - J_p) - P r_c]/P, \tag{12}$$

$$K_{\text{ang.accel}} = [J_o - P r_c]/P, \tag{13}$$

where J_i, J_p, and J_o are pendulum principal moments of inertia relative to center of hinge (g·cm^2), P is the pendulum pendulosity i.e. pendulum mass times distance of center of mass from center of hinge (g·cm), r_c is the distance from the center of hinge to a point at the null position at the pendulum at which accelerations are assumed to act on the pendulum (cm) [15].

If a simple calibration procedure is performed, the scale factor K_1 can be expressed as [15]

$$K_1 = K_1^0(1 + k_1), \tag{14}$$

where K_1^0 is the nominal scale factor resulted from the calibration procedure, and k_1 is a scale factor correction. In fact, this correction factor acting only at the a_i component level because its product with the other terms in the equation model is ignorable small [15]. Thus, if the error equation model of an accelerometer is by the form

$$A_{ind} = E / K_1 = a_i + (\text{other.terms}), \tag{15}$$

after the calibration, the indicated acceleration can by expressed as follows

$$A_{ind} = E / K_1^0 = (1 + k_1)a_i + (\text{other.terms}). \tag{16}$$

From the other point of view, alternative expressions for the bias and scale factor asymmetry models can be used [15],

$$K_o^+ = K_o + K_o'/2,\ K_o^- = K_o - K_o'/2, \tag{17}$$

$$K_1^+ = K_1\left(1 + K_1'/2\right),\ K_1^- = K_1\left(1 - K_1'/2\right), \tag{18}$$

with K_0^+, K_1^+ bias and scale factor for positive a_i, and K_0^-, K_1^- bias and scale factor for negative a_i, respectively. So, the average bias and the bias asymmetry are

$$K_o = (K_o^+ + K_o^-)/2,\ K_o' = K_o^+ - K_o^-, \tag{19}$$

while, the average scale factor and scale factor asymmetry are given by

$$K_1 = (K_1^+ + K_1^-)/2,\ K_1' = (K_1^+ - K_1^-)/K_1. \tag{20}$$

The misalignment angles δ_o, δ_p in equation model (11) were defined by right-handed Euler angles. If the accelerometer is used in a strap-down inertial navigation system, it is indicated to use the direction cosines, so the term $(\delta_o a_p - \delta_p a_o)$ will be replaced by $(\delta_p a_p + \delta_o a_o)$, where δ_p and δ_o are the misalignment of the IA with respect to the input reference axis in the plane of the PA and OA, respectively, expressed in radians [15].

3. MATLAB/SIMULINK ACCELEROMETERS' MODEL

The present studies of the strap-down inertial navigation systems related to the involved sensors suppose analyze of the systems in order to estimate the sensors errors after that compensate them in order limit the influence of the inertial sensors error in the solution of navigation. In this way, based on the previous exposed models, many calibration techniques, off-line and on-line, were developed for the inertial sensors, coupled or un-coupled, with the mechanization model of the strap-down inertial navigation systems. Because, the most numerical simulations of strap-down inertial navigation systems performed in the design phase, presented in the literature, suppose the application of clean acceleration and rotation signals to the system input, without errors and noises, the study of the designed navigation systems errors is made without taking into account the inertial sensors errors. To put up a complex study for a navigation system, near by the real conditions, which will include the real errors of the used sensors, an equivalent model to the previously presented models can be analytically established and software implemented. What is important in this model is that it will consider the parameters from the real sensors data sheets. After a study related to the parameters given by producers in a series of accelerometers data sheets [16]-[24], and taking into account the correlations with previous accelerometers' analytical models, an equation describing a simplified model was chosen

$$a = (a_i + Na_i + B + k_c a_c + \nu)(1 + \Delta K / K). \quad (21)$$

The proposed model covers the main errors of the acceleration sensors: bias, scale factor error, sensitivity axis misalignment, cross axis sensitivity and noise. In equation (21) we have: a - sensors output acceleration (disturbed signal) expressed in m/s^2, a_i - applied acceleration (m/s2), N -sensitivity axis misalignment (radians), B - bias (expressed in percents of span), a_c - cross-axis acceleration (m/s^2), k_c - cross-axis sensitivity (expressed in percents of a_c), ν - sensor noise (given by its density ν_d expressed in $\mu g/\sqrt{\text{Hz}}$, K - scale factor (expressed in mV/g), and ΔK - scale factor error (percents of K).

Implementing software the analytical model (21), the Matlab/Simulink model in Fig. 2 was obtained. Based on the acceleration sensors parameters variation limits given in the data sheets, the obtained model is generally valid for all acceleration sensors, and, as a consequence, can be used to numerical simulate the influences of theirs errors on the navigation solution (position, speed and attitude channels).

Software obtained model architecture and design are based on the observation according with that a part of the parameters in the acceleration sensors data sheets do not have a fixed value and vary arbitrary in a certain interval. This situation appears especially at the miniaturized sensors for which the values of the parameters are offered for a large lot of sensors and not for each sensor separately. Thus, regarding the data sheets information, the cross-axis sensitivity is given through its maximum value k_c as percent of a_c, the bias is given through its maximum absolute value as percent of span, the scale factor error is given through its maximum absolute value ΔK as percent of K, while the sensor noise is given through its density maximum value ν_d. According to Fig. 2 and equation (21), by using the Matlab function „rand(1)" are generated the following parameters: 1) Bias - is a random value in the interval (-B, B); 2) Cross axis sensitivity - is a random value in the interval (0, k_c); 3) Scale factor error - is a random value in the interval (-ΔK, ΔK). The noise are generated by using the Simulink block "Band-Limited White Noise" and the „RandSeed" Matlab function, function that gives a random value of the noise density in the interval (80% ν_d, ν_d).

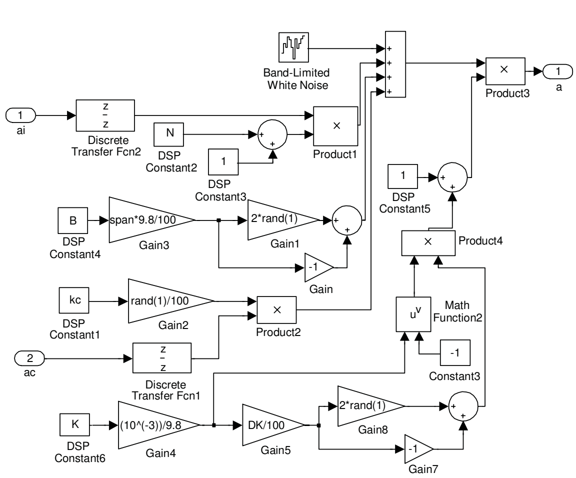

Figure 2: Matlab/Simulink accelerometers model.

By grouping the Matlab/Simulink schema in Fig. 2, the equivalent block in Fig. 3 is obtained. The "Accelerometer model" block in Fig. 3 has as inputs the acceleration a_i applied along of the sensitivity axis and the cross-axis acceleration a_c (acceleration applied in a perpendicular plane), and as output the disturbed acceleration a.

Ten acceleration sensors (ADXL321, ADXL78, IOA, FOA, QAF50, QA3000-030, A3022-050-P, S1210L-025, S1221x-025, CXL25LP1), produced by Analog Device, Phone-Or, Honeywell, MSI Sensors, Silicon Design, and Crossbow Technology, were already implemented in the model [16]-[18], [22]-[24]. The already implemented sensors are realized in MEMS, MOEMS or classical technology (see Table 1). The change of the sensor type is made by using the interface in Fig. 4, which allows also the sensors parameters setting in a custom variant by direct introduction of the parameters in the dedicated fields. Another advantage of this software implemented model is the possibility to have in the disturbed acceleration signal a, used in numerical simulation, each type of the sensor's errors taken independently or any combination between these errors. So, the interface allows the selection of any sensor's errors or combination between them to be superposed on the acceleration a_i applied along of the accelerometer sensitivity axis.

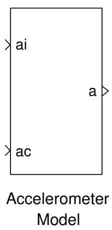

Figure 3: „Accelerometer Model" block.

The model can be successfully used in the numerical simulation of any strap-down inertial navigation system, to create similar conditions with the real ones from the point of view of the distortions that affects the useful acceleration signal a_i, when passing through any accelerometric detection device desired to be implemented in the sensing block of the navigator.

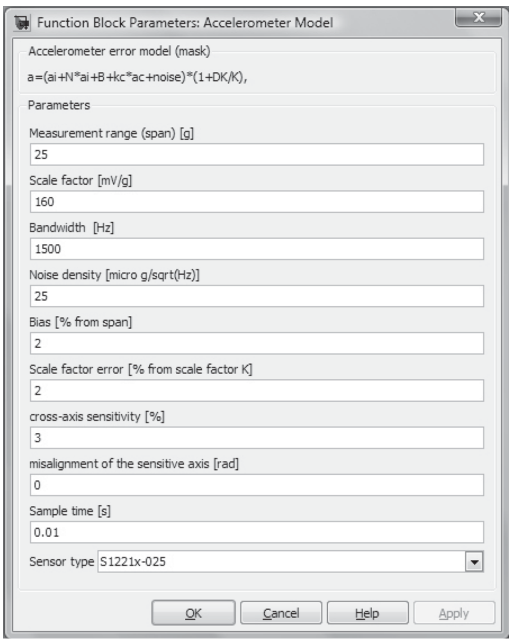

Figure 4: Interface of the accelerometers' Matlab/Simulink proposed model.

Analyzing the data in Table 1, on can observe that the classical sensors, un-miniaturized, frequently used in inertial navigators, have high performance relative to the others categories. On other hand, the MEMS and MOEMS technologies become strong competitors, having the advantages of the miniaturization, of the small energetic consumption, and of a very small price (of a few hundreds times smaller than the price for classical sensors). A high performance classical accelerometer can costs few k\$, while a MEMS or MOEMS accelerometer can cost less than one hundred \$, at a performance level that allows its using in a strap-down inertial system.

To see how the sensors' errors influence the applied acceleration a_i, a simulation was made using the proposed model for a_i=1g, and for a cross-axis acceleration a_c=1g. For the already implemented accelerometers described in Table 1, the disturbed signals in Fig. 5 were obtained. Observing the noise amplitudes on can conclude that the classical sensors are a few times noiseless than the miniaturized ones. Also, from this point of view, the MOEMS accelerometers are in a median position in the MEMS accelerometers category. The pattern of the disturbed signal obtained at the model output and its average values highlight the strong influence of the sensors' errors on the acceleration a_i applied along of an accelerometer sensitivity axis. Also, these create an image of the impact that these errors have it on the inertial navigator performances yet from the start of the procedure to estimate the navigation solution (the sensing phase).

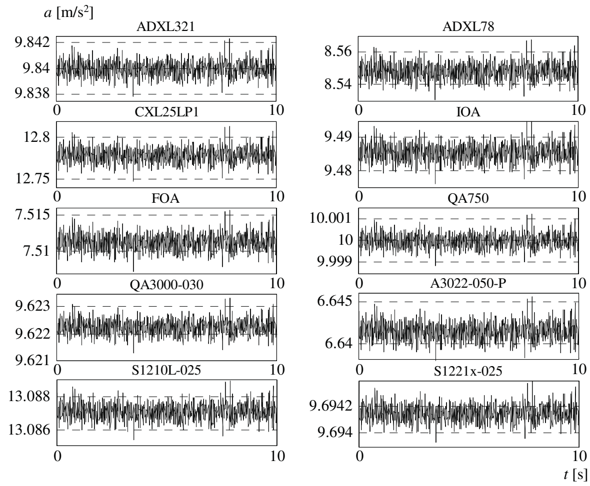

Figure 5: The output of the model for the already implemented accelerometers, for a_i=1g and a_c=1g.

The conceived model has the advantages to can work independently with each of the errors affecting the sensors and, in this way, to can study theirs influences on the inertial navigator.

Further, the model will be used to develop some sensors fusion algorithms, which would allow the improvement of the sensors' signals quality and an estimation of the inertial navigation systems' performances starting from the design stage.

No.	Acc. type	Input range [g]	Scale factor [mV/g]	Bandwidth [Hz]	Noise density [$\mu g / \sqrt{Hz}$]	Bias [% of span]	Scale factor error [% of scale factor]	Cross-axis sensitivity [%]	Sensitivity axis misalignment [radians]	Power [mW]	Operating principle	Producer
1	ADXL321	±18	57	2500	320	1.7	10	2	0.0175	0,84	MEMS Capacitive	Analog Device USA
2	ADXL78	±35	55	400	1100	2	5	5	0.0175	6,5	MEMS Capacitive	Analog Device USA
3	CXL25LP1	±25	80	100	1000	2	5	5	0.035	25	MEMS Capacitive	Crossbow Technology USA
4	IOA	±10	100	100	300	2	5	3	0.01	50	MOEMS Optic Integrated	Phone-Or Israel
5	FOA	±20	50	100	120	2	5	3	0.01	120	MOEMS Fiber Optics	Phone-Or Israel
6	QA750	±30	1,2	500	70	0.027	2	0	0.007	480	Classical Capacitive	Honeywell USA
7	QA3000-030	±60	1,2	500	70	0.0067	2	0	0.001	480	Classical Capacitive	Honeywell USA
8	A3022-050-P	±20	3	1000	350	4.2	5	3	0	7	MEMS Piezoresistive	MSI Sensors USA
9	S1210L-025	±25	160	1000	158	2	2	3	0	50	MEMS Capacitive	Silicon Design USA
10	S1221x-025	±25	160	1500	25	2	2	3	0	50	MEMS Capacitive	Silicon Design USA

Table 1: The characteristics of the model' already implemented accelerometers

3. CONCLUSIONS

The papers presented seven analytic accelerometer error models used in this moment to on-line or off-line calibration of acceleration sensors used in strap-down inertial navigation systems. Three of the described models are un-standardized, while the other four are IEEE standardized. From the models various errors affecting the acceleration signal clean detection were highlighted.
Starting from the idea that the most numerical simulations of strap-down inertial navigation systems performed in the design phase, presented in the literature, suppose the application of clean acceleration and rotation signals to the system input, without errors and noises, we proposed a simplified error model for accelerometers to allow the study of the designed navigation systems errors taking into account the inertial sensors errors. To have a complex study of the navigation systems, near by the real conditions, which includes the real errors of the used sensors, the proposed model was built starting from the parameters contained in the real sensors data sheets. The accelerometers' model was analytically defined, software implemented and numerical simulated for ten acceleration sensors (ADXL321, ADXL78, CXL25LP1, IOA, FOA, QAF50, QA3000-030, A3022-050-P, S1210L-025, S1221x-025), at an accelerometer sensitivity axis applied acceleration, and at a cross-axis acceleration of 1*g*.

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