THE STUDY OF THE AUTOMATIC SYSTEM FOR THE SEALED CABINS AIRFLOW REGULATION USING INDIRECTLY ACTION AIRFLOW REGULATORS

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Abstract – **The paper presents the mathematical model of an automatic system for the sealed cabin airflow regulation using indirectly action airflow regulator. The goal of this Automatic Regulation System (ARS) is to constantly provide a certain pressure in the cabin, accordingly to the altitude. After deducing the equations of the system and the state equations that characterize it, the authors of the paper have simulated the system in Matlab/Simulink medium in order to analyze the ARS's dynamic characteristics; thus, using a Matlab/Simulink model and a Matlab program, time characteristics representing the time variation of the non-dimensional cabin pressure, time variations of the non-dimensional displacement of the sensitive element and time variation of the non-dimensional regulator surface have been obtained. One highlights the disturbances that act on the system and the transfer functions which express the response of this system to these disturbances and one makes assessments regarding these disturbances' influence upon stabilization of the cabin pressure. The originality of this work is the obtaining of the system's mathematical model and the analysis of its dynamic behavior (the obtaining of the transfer functions, poles, zeros, stability's analysis, response of the system to one or two inputs and so on). The duration of the transient regime is about 18 seconds. In general this transient regime isn't a good one, but, in this case, this transient regime's duration is good because the system of pressure's regulation is a slow system (the pressure's regulation must not be done very quickly). The system is stable (the step responses of it proves this fact).**

Keywords: *airflow, cabin, regulator, debit.*

1. INTRODUCTION

The goal of the Automatic Regulation System (ARS) is to constantly provide a certain pressure in the cabin, accordingly to the altitude. The main methods of airflow automatic regulation are: compressor provided airflow control, control evacuation of the surplus of a constant airflow provided by the compre-ssor and the control on the cabin compressor [1], [2], [3], [4], [5]. The structure of the system is presented if figure 1 (1-variable valve; 2 – servo-piston; 3 – distributor; 4 – Venturi tube; 5 – manometric capsules; 6 – aneroid capsules; 7 - compressor). In the stationary regime the plate of the distributor 3 is on the mid position and prevents air to eject into atmosphere. If the airflow increases, the pressures are no more even $(p_k - p_v \neq 0)$; hence, the plate changes its position; thus, the hatch 1 rotates, opening the exhaust of air in the atmosphere. In the case of airflow decreasing, the system reacts in the same manner backwards.

2. SEALED CABIN'S EQUATIONS

Using notations from figure 2, the compressor airflow Q_k has two components: Q_t – the airflow transmitted to the cabin and Q_e – the atmosphere evacuated airflow.

$$Q_k = Q_t + Q_e. \tag{1}$$

The status equation for the sealed cabin can be written as

$$p_c V_c = m_c R T_c^0, \tag{2}$$

which leads to the differential form

$$\begin{aligned} V_c \frac{\mathrm{d}p_c}{\mathrm{d}t} &= R T_c^0 \frac{\mathrm{d}m_c}{\mathrm{d}t}, \\ \frac{\mathrm{d}m_c}{\mathrm{d}t} &= Q_k - Q_e - Q_r - Q_f, \end{aligned} \tag{3}$$

In dynamic regime, first equation (3) becomes

$$V_c \frac{\mathrm{d}}{\mathrm{d}t}\Delta p_c = R T_c^0 \left(\Delta Q_k - \Delta Q_e - \Delta Q_r - \Delta Q_f\right), \tag{4}$$

with

$$\begin{aligned} \Delta Q_k &= \left(\frac{\partial Q_k}{\partial p_k}\right)_0 \Delta p_k + \left(\frac{\partial Q_k}{\partial p_v}\right)_0 \Delta p_v + \left(\frac{\partial Q_k}{\partial T_k}\right)_0 \Delta T_k, \\ \Delta Q_e &= \left(\frac{\partial Q_e}{\partial p_k}\right)_0 \Delta p_k + \left(\frac{\partial Q_e}{\partial S_e}\right)_0 \Delta S_e + \left(\frac{\partial Q_k}{\partial T_k}\right)_0 \Delta T_k, \\ \Delta Q_r &= \left(\frac{\partial Q_r}{\partial p_c}\right)_0 \Delta p_c + \left(\frac{\partial Q_r}{\partial S_r}\right)_0 \Delta S_r, \\ \Delta Q_f &= \left(\frac{\partial Q_f}{\partial p_c}\right)_0 \Delta p_c. \end{aligned} \tag{5}$$

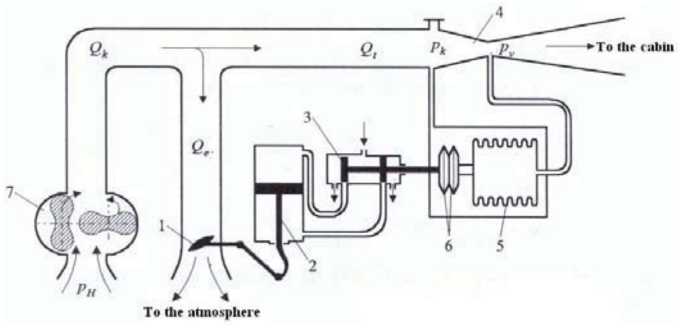

Figure 1: Automatic system for the airflow regulation using indirectly action airflow regulators

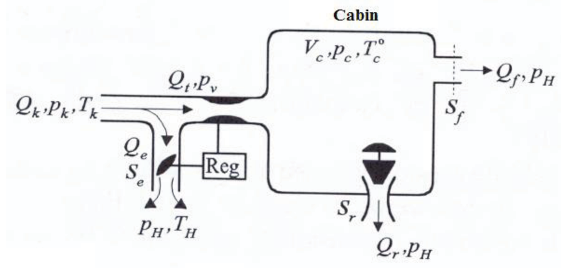

Figure 2: The sealed cabin –schematics structure

Replacing these equations in (4) and adopting dimensionless variables

$$\frac{\Delta p_c}{p_N}=\bar{p}_c,\ \frac{\Delta p_k}{p_N}=\bar{p}_k,\ \frac{\Delta p_v}{p_N}=\bar{p}_v;\ \frac{\Delta T_k}{T_{k_{\max}}}=\theta_k;$$
$$\frac{\Delta S_r}{S_{r_{\max}}}=\bar{S}_r,\ \frac{\Delta S_e}{S_{r_{\max}}}=\bar{S}_e;\ \frac{\Delta Q_k}{Q_{t_{\max}}}=q_k,\ \frac{\Delta Q_r}{Q_{t_{\max}}}=q_r, \quad (6)$$
$$\frac{\Delta Q_f}{Q_{t_{\max}}}=q_f,\ \frac{\Delta Q_e}{Q_{t_{\max}}}=q_e$$

equation (4) becomes

$$\tau_c \frac{\mathrm{d}}{\mathrm{d}t}\bar{p}_c + k\bar{p}_c + k_e\bar{S}_e + k_v\bar{p}_v = F(t), \quad (7)$$

where

$$\tau_c = \frac{V_c p_N}{RT_c^0 Q_{t_{\max}}} \quad (8)$$

is the cabin filling constant,

$$k=\left(\frac{\partial q_r}{\partial \bar{p}_c}-\frac{\partial q_f}{\partial \bar{p}_c}\right)_0,\ k_e=\left(\frac{\partial q_e}{\partial \bar{S}_r}\right)_0,\ k_v=-\left(\frac{\partial q_k}{\partial \bar{p}_v}\right); \quad (9)$$

k is the auto-equalization constant, k_e – the constant which characterize the position of the regulation element in the exhaust tube;

$$F(t)=\left(\frac{\partial q_k}{\partial \bar{p}_k}-\frac{\partial q_e}{\partial \bar{p}_k}\right)_0\bar{p}_k+\left(\frac{\partial q_k}{\partial \theta_k}-\frac{\partial q_e}{\partial \theta_k}\right)_0\theta_k-\left(\frac{\partial q_e}{\partial \bar{S}_r}\right)_0\bar{S}_r \quad (10)$$

is the time function which describes the perturbation.

3. ELASTIC SENSITIVE ELEMENT EQUATION

The elastic sensitive element and its characteristic parameters are presented in figure 3.

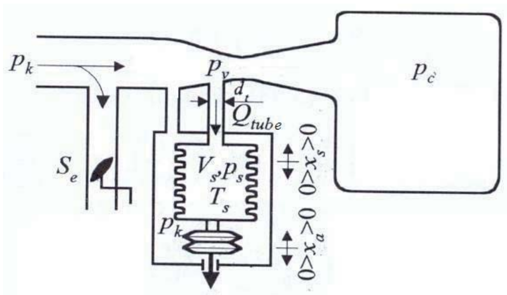

Figure 3: Characteristics of elastic sensitive element

From state equation, expressed for the volume of the gophred component, after derivation, one yields (one has assumed that $\Delta V_S = 0$ and $\frac{\mathrm{d}T_S}{\mathrm{d}t} \cong 0$)

$$V_S\frac{\mathrm{d}p_S}{\mathrm{d}t}=RT_S\frac{\mathrm{d}m_S}{\mathrm{d}t},$$
$$\frac{\mathrm{d}m_S}{\mathrm{d}t}=Q_{tube}. \quad (11)$$

The mass debit is [6]

$$Q_{tube}=\rho_v Q_{tube}^v \cong \rho_{v0}Q_{tube}^v =$$
$$=\rho_{v0}\frac{\pi}{128\eta}\frac{d_t^4}{l_t}\left(p_v-p_s\right)=N\varsigma\rho_{v0}\left(p_v-p_s\right), \quad (12)$$

where $\rho_v \cong \rho_{v0}$ is the air's density in the exhaust, η – the dynamic viscosity in the capillaries, d_t – and l_t – the diameter and length of the capillary tube; $N=\pi/128\eta$ and $\varsigma = d_t^4/l_t$.

From the previous two equations one gets the dynamic regime equation

$$\tau_s\frac{\mathrm{d}}{\mathrm{d}t}\Delta p_S+\Delta p_S=\Delta p_v, \quad (13)$$

with $\tau_s = V_s / RT_s^0 N\varsigma\rho_{v0}$ – the time constant of the gophred tube.

Noting with x_s – the movement of the rigid center of the gophred tube, one obtains the forces equilibrium equation [7]

$$S_s\left(p_k - p_s\right) = k_s x_s , \tag{14}$$

where S_s represents the effective surface of the gophred tube and k_s – the elasticity constant.

In dynamic regime

$$S_s\left(\Delta p_k - \Delta p_s\right) = k_s \Delta x_s . \tag{15}$$

Thus, if $\Delta p_k \neq 0 \Rightarrow \Delta p_s \neq 0$ and eventually $\Delta x_s \neq 0$.

In the same time, variation $\left(p_k - p_{k_0}\right)$ induces a deformation of the capsules, and consequently a supplementary moment of the rigid center of the gophred tube x_a

$$\begin{aligned} S_a\left(p_k - p_{k_0}\right) &= k_a x_a, \\ S_a \Delta p_k &= k_a \Delta x_a , \end{aligned} \tag{16}$$

where S_a represents the effective surface of the capsules and k_a – the elasticity constant of the capsules.

The total displacement of the sensitive element is

$$x = x_a + x_s = \frac{S_a}{k_a}\left(p_k - p_{k_0}\right) + \frac{S_s}{k_s}\left(p_k - p_s\right). \tag{17}$$

The dynamic regime is described by equation

$$\Delta p_s = \left(1 + \frac{k_s}{k_a}\frac{S_a}{S_s}\right)\Delta p_k - \frac{k_s}{S_s}\Delta x . \tag{18}$$

Substituting Δp_s in equations (13) and (18) one gets

$$\frac{k_s}{S_s}\left(\tau_s \frac{\mathrm{d}}{\mathrm{d}t}\Delta x + \Delta x\right) = \left(1 + \frac{k_s S_a}{k_a S_s}\right)\left(\tau_s \frac{\mathrm{d}}{\mathrm{d}t}\Delta p_k + \Delta p_k\right) - \Delta p_v \tag{19}$$

or

$$\tau_s \frac{\mathrm{d}\bar{x}}{\mathrm{d}t} + \bar{x} + k_v' \bar{p}_v = f(t), \tag{20}$$

where

$$\begin{aligned} \bar{x} &= \frac{\Delta x}{x_{\max}}, \\ k_v' &= \frac{S_s p_N}{k_s x_{\max}}, \\ f(t) &= \frac{S_s p_N}{k_s x_{\max}}\left(1 + \frac{k_s S_a}{k_a S_s}\right)\left(\tau_s \frac{\mathrm{d}}{\mathrm{d}t}\bar{p}_k + \bar{p}_k\right). \end{aligned} \tag{21}$$

4. THE SERVO-PISTON EQUATION

With sufficient precision one may describe the equation of the piston as

$$\begin{aligned} \tau_{sp}\frac{\mathrm{d}\bar{y}}{\mathrm{d}t} &= \bar{x}, \\ \bar{y} &= \frac{\Delta y}{y_{\max}}, \end{aligned} \tag{22}$$

with y – the piston moment, τ_{sp} – time constant (accordingly to the maximum travel of the distributor's plate). One yields [8]

$$\tau_e \frac{\mathrm{d}\bar{S}_{\mathrm{e}}}{\mathrm{d}t} = -\bar{x}, \tag{23}$$

with τ_e – the working element time constant.

The dependence between pressures p_v and p_c may be expressed using one of the two forms (p – the loss of pressure between minimum section of the Venturi tube and the section of the entrance of the air in the cabin)

$$\begin{aligned} p_v &= p_c + p, \\ \Delta p_v &= \Delta p_c + \Delta p \end{aligned} \tag{24}$$

or using non-dimensional variables

$$\bar{p}_v = \bar{p}_c + \bar{p}. \tag{25}$$

5. MATHEMATIC MODEL OF THE ARS

The mathematical model of the ARS is described by equations (7), (20), (23) and (25)

$$\begin{aligned} &\tau_c \frac{\mathrm{d}}{\mathrm{d}t}\bar{p}_c + k\bar{p}_c + k_e\bar{S}_e + k_v\bar{p}_v = F(t), \\ &\tau_s \frac{\mathrm{d}\bar{x}}{\mathrm{d}t} + \bar{x} + k_v'\bar{p}_v = f(t), \\ &\tau_e \frac{\mathrm{d}\bar{S}_{\mathrm{e}}}{\mathrm{d}t} = -\bar{x}, \\ &\bar{p}_v = \bar{p}_c + \bar{p}. \end{aligned} \tag{26}$$

Substituting $\bar{p}_v$ in the second and the fourth equation (26), one obtains the equations in the complex domain

$$\begin{aligned} &k_c\left(\tau_k \mathrm{s} + 1\right)\bar{p}_c + k_e\bar{S}_e = G(\mathrm{s}), \\ &\left(\tau_s \mathrm{s} + 1\right)\bar{x} + k_v'\bar{p}_c = g(\mathrm{s}), \\ &\tau_e \mathrm{s}\bar{S}_e = -\bar{x}, \end{aligned} \tag{27}$$

where

$$\begin{aligned} k_c &= k + k_v, \\ \tau_k &= \tau_c / k_c, \\ G(\mathrm{s}) &= F(\mathrm{s}) - k_v\bar{p}(\mathrm{s}), \\ g(\mathrm{s}) &= f(\mathrm{s}) - k_v'\bar{p}(\mathrm{s}). \end{aligned} \tag{28}$$

The block diagram of the system described by equations (26) is presented in figure 4.

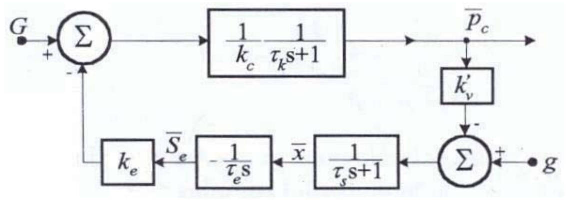

Figure 4: The block diagram of the ARS

The system from figure 4 may have another form if instead of two inputs (the two disturbances) one uses an equivalent disturbance

$$
\begin{aligned}
h(s) &= G(s) + \cdot g(s) \cdot \frac{k_e}{\tau_e s(\tau_s s + 1)}; \\
h(s) &= \frac{\tau_e s(\tau_s s + 1) \cdot G(s) + k_e \cdot g(s)}{\tau_e s(\tau_s s + 1)} \cong \frac{k_e}{\tau_e s(\tau_s s + 1)} g(s).
\end{aligned}
\tag{29}
$$

One may conclude that the influence of disturbance $g(s)$ on the non-dimensional cabin pressure $\overline{p}_c$ is higher than the influence of disturbances $G(s)$.

Because the system has two inputs and one output, one may obtain two transfer functions

$$
\begin{aligned}
H_1(s) &= \frac{\overline{p}_c(s)}{G(s)} = \frac{\tau_e \tau_s s^2 + \tau_e s}{A \cdot s^3 + B \cdot s^2 + C \cdot s + D}, \\
H_2(s) &= \frac{\overline{p}_c(s)}{g(s)} = \frac{k_e}{A \cdot s^3 + B \cdot s^2 + C \cdot s + D}
\end{aligned}
\tag{30}
$$

where

$$
\begin{aligned}
&A = k_c \tau_e \tau_k \tau_s ; B = k_c \tau_e (\tau_k + \tau_s); \\
&C = k_c \tau_e ; D = k_e k'_v .
\end{aligned}
\tag{31}
$$

Thus,

$$\overline{p}_C = H_1(s) \cdot G(s) + H_2(s) \cdot g(s). \tag{32}$$

Taylor series expansions of the two transfer functions $H_i(s), i = \overline{1,2}$ is made using the formula

$$H_i(s) \cong H_i(0) + \left(\frac{\partial H_i(s)}{\partial s}\right)_0 s; \tag{33}$$

one yields

$$H_1(s) \cong \frac{\tau_e}{k_e k'_v} s ; H_2(s) \cong \frac{1}{k'_v} - \frac{k_c \tau_e}{k_e k'_v} s. \tag{34}$$

Thus,

$$\overline{p}_c = \frac{\tau_e}{k_e k'_v} s \cdot G(s) + \left(\frac{1}{k'_v} - \frac{k_c \tau_e}{k_e k'_v} s\right) \cdot g(s). \tag{35}$$

In the stationary regime $s \cdot G(s) \cong 0$ and the above equation becomes

$$\overline{p}_c = \frac{1}{k'_v} \cdot g(s). \tag{36}$$

The equations (26) or (27) may be written in the matriceal form. If one chooses the state vector $z = [\overline{p}_c \ \overline{x} \ \overline{S}_e]^T$, the input vector $w = [G \ g]^T$, the state equation has the form

$$\dot{z} = Az + Bw, \tag{37}$$

where

$$
\begin{aligned}
A &= \begin{bmatrix} -1/\tau_k & 0 & -k_e / k_c \tau_k \\ -k'_v / \tau_s & -1/\tau_s & 0 \\ 0 & -1/\tau_e & 0 \end{bmatrix}; \\
B &= \begin{bmatrix} 1/k_c \tau_k & 0 \\ 0 & -1/\tau_s \\ 0 & 0 \end{bmatrix}.
\end{aligned}
\tag{38}
$$

6. NUMERICAL SIMULATIONS

Having the values of the parameters of ARS

$$
\begin{aligned}
&Q_{t_{\max}} = 0.153 \text{ kg/s}, V_c = 0.5 \text{ m}^3, R = 287 \text{ J/kgK}, \\
&t_c^0 = 22^\circ\text{C} \left(T_c^0 = 295 \text{ K}\right), p_N = 10^5 \text{ N/m}^2, \tau_s = 0.1 \text{ s}, \\
&\tau_e = 10 \text{ s}, S_s = 5 \cdot 10^{-4} \text{ m}^2, k_s = k_a = 10^3 \text{ N/m}, \\
&S_a = 5 \cdot 10^{-5} \text{m}^2, \Delta p_c / p_N = 10^{-2}, \\
&\Delta Q_r / Q_{t_{\max}} = 8 \cdot 10^{-3}, \Delta Q_f / Q_{t_{\max}} = 2 \cdot 10^{-3}, \\
&\left(\Delta Q_k / Q_{t_{\max}}\right)_{p_v = const.} = 5 \cdot 10^{-2}, \\
&\left(\Delta Q_e / Q_{t_{\max}}\right)_{S_e = const.} = 2 \cdot 10^{-2}, \\
&\left(\Delta Q_r / Q_{t_{\max}}\right)_{p_e = const.} = 2 \cdot 10^{-2}, \hat{p} = 10^{-1},
\end{aligned}
\tag{39}
$$

one computes the coefficients of the equations in the mathematical model. Using (8) one results $\tau_c \cong 4$ s and using (9) one gets

$$k = 1, k_e = 5, k_v = 1. \tag{40}$$

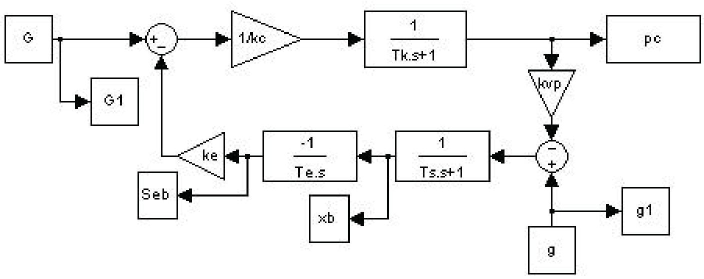

Figure 5: The Matlab/Simulink model of the system from figure 4

The value of disturbance $F(t)$ is obtained using (10); $F(t) \cong 10^{-2}$. With equations (21) one results $k_v' = 5$ and $f(t) \cong 0.11$, while using (28) one obtains $k_c = 2, \tau_k = 2\,\mathrm{s}$, $G \cong -0.99$, $g \cong -0.39$.
Using the Matlab/Simulink model from figure 5 and a Matlab program one has obtained the graphic characteristics from figures 6, 7, 8 and 9.
The poles of the system prove the system's stability

$$\begin{aligned} s_1 &= -10.064; \\ s_2 &= -0.217 + 0.757\,\mathrm{i}; \\ s_3 &= -0.2175 - 0.757\,\mathrm{i}. \end{aligned} \tag{41}$$

The two transfer functions in closed loop are

$$\begin{aligned} H_1(\mathrm{s}) &= \frac{\overline{p}_c(\mathrm{s})}{G(\mathrm{s})} = \frac{\mathrm{s}^2 + 10\,\mathrm{s}}{4\,\mathrm{s}^3 + 42\,\mathrm{s}^2 + 20\,\mathrm{s} + 25}, \\ H_2(\mathrm{s}) &= \frac{\overline{p}_c(\mathrm{s})}{g(\mathrm{s})} = \frac{5}{4\,\mathrm{s}^3 + 42\,\mathrm{s}^2 + 20\,\mathrm{s} + 25}. \end{aligned} \tag{42}$$

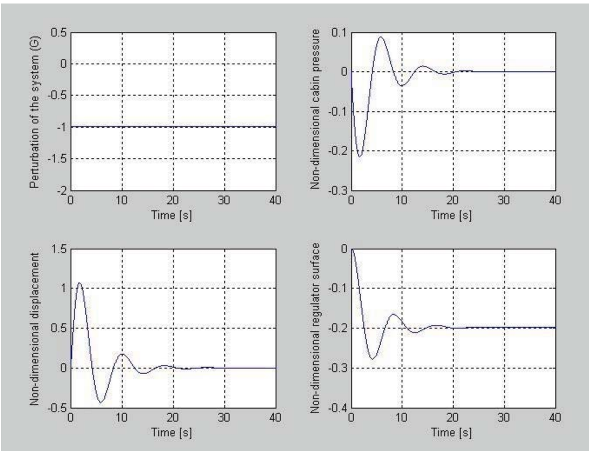

Figure 6: Time characteristics for the system with the input G

The graphic characteristics from figure 6 correspond to the first input G. These are the time variation of the non-dimensional cabin pressure $\overline{p}_c$, time variations of the non-dimensional displacement of the sensitive element $\overline{x}$ and time variation of the non-dimensional regulator surface $\overline{S}_e$. Same graphics are presented in figure 7 but for the system with only one input (the second input g). In figure 8 one presents the same graphics, but, this time, for the system with 2 inputs. The three variables are the sum of the same three variables corresponding to the system with input one or to the system with input two. In figure 9 the step responses for the system are presented (each step response corresponds to a transfer function and to one input).

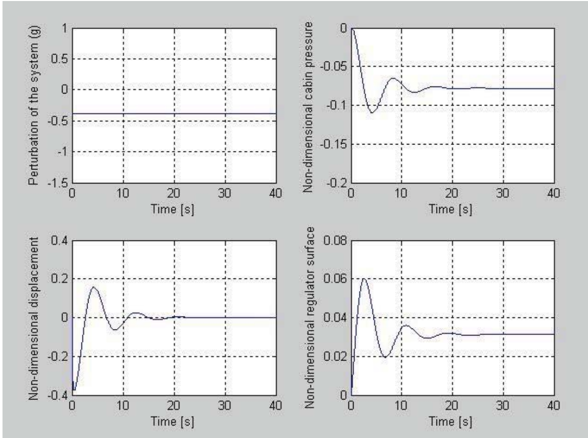

Figure 7: Time characteristics for the system with the input g

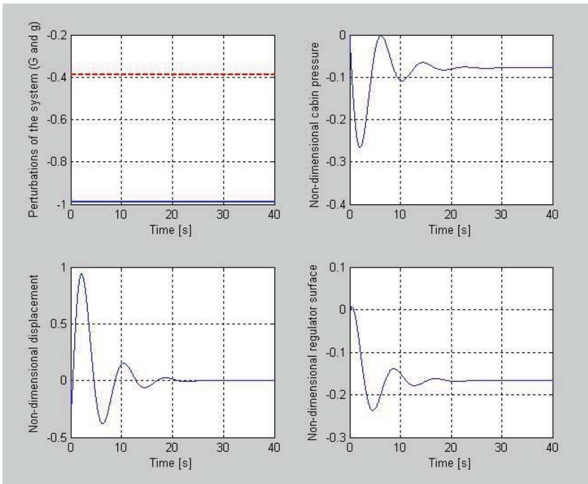

Figure 8: Time characteristics for the system with the inputs G and g

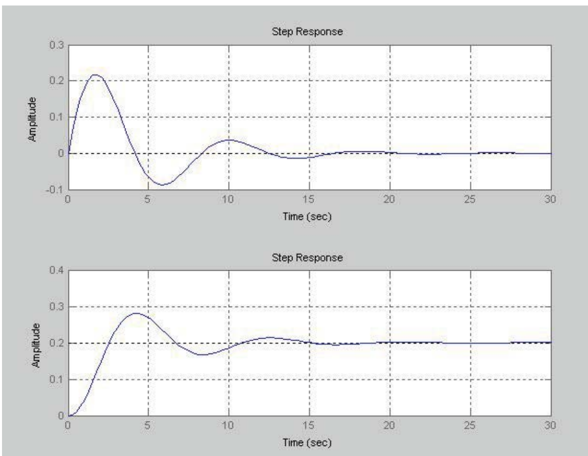

Figure 9: Step responses of the ARS system

As one can see from the previous figures the system is a stable one. The duration of the transient regime is about 18 seconds. In general this transient regime isn't a good one, but, in this case, this transient regime's duration is good because the system of pressure's regulation is slow system (the pressure's regulation must not be done very quickly).

7. CONCLUSIONS

One studies an automatic regulation system of the airflow into the aircrafts' cabins; this is a subsystem of a pressure system. One has presented non-linear and linearised mathematical models in two variants: dimensional and non-dimensional.

One highlights the disturbances that act on the system and the transfer functions which express the response of this system to these disturbances and one makes assessments regarding these disturbances' influence upon stabilization of the cabin pressure.

One may conclude that the influence of disturbance $g(s)$ on the non-dimensional cabin pressure $\overline{p}_c$ is higher than the influence of disturbances $G(s)$.

For a set of system parameters' values one obtained the Matlab/Simulink model which permits the obtaining of graphic time evolution of the system state variables.

The system is a stable one. The duration of the transient regime is about 18 seconds (this transient regime's duration is good because the system of pressure's regulation is slow system - the pressure's regulation must not be done very quickly).

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