A MATRIX APPROACH TO MODEL SHIPPING TYPE COLORED PETRI NETS WITH PRIORITIES ON COLOR MARKS

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Abstract **– This paper presents a transport model which allows coverage in a single model of various components, actions, attributes, priorities of a transport system (military or civilian). A description of the complexity of shipping systems in the introduction of this paper justifies the approach of Petri networks in order to model, simulate and optimize these shipping systems, military or civilian. Furthermore, our research has revealed that the colored Petri networks are offering multiple possibilities for modeling systems of shipping. Extensions of these networks allow modeling changes in position or status of ships in the system, separate representation of vehicles with different functions, simulation of execution order of transitions based on the some priorities, etc. The colored Petri network of transport with priorities on colors of marks is used in this paper to obtain a model of small sizes but with high power of representation and simulation. Matrix representation of the network and then decomposition of the matrix incidence in incidence matrix on color priority differentiate the case of color marks from the transition priorities changes. Matrix treatment of colored Petri transport network allows a proper expression of simulated situations in compliance with related transport systems scenarios. A case study – taking off mission of the aircraft from an aircraft carrier in a certain order and the return in a different order - demonstrate the superiority of this modeling tool which is the colored Petri transport network.**

Keywords: *modeling, colored Petri nets, cellular matrix*

1. INTRODUCTION

The complexity of the current transportation systems (military and civilian) involves the use of a wide range of methods for modeling of some structural and functional aspects having in view the optimization analysis. It is therefore worthwhile to uniform modeling techniques for the wide variety of systems and structural and functional aspects of maritime transport systems. Modeling of shipping systems with colored graphs allows separate representation of different types of vessels operating in a military transport (aircraft carriers, attack and spy submarines, fast accompanying ships troop and weapons carries etc.). These models based on colored transportation networks allow that in the same model both ships and the related resources (fuel, weapons on board, human resources, etc.) and the cargo transported on those ships to be represented separately. In this way one can simulate some processes of interaction between various types of ships (the supply of arms and fuel underway, taking off and landing of the aircrafts on aircraft carrier, etc.). In case of military navigation systems modeling through colored transportation networks different types of ships and participating friends, the positions and their transitions in modeled system can be distinctively represented. These important aspects were considered when establishing the tools used for navigation systems and maritime transport simulation. They determined the carrying out of researches aimed at adaptation some existing tools in the category of Petri nets and bipartite graph theory.

This paper aims to demonstrate that a model can encompass the diversity of components, actions, attributes, priorities of a transport system (military or civilian). The system evolution is described by the equation of state in which the incidence matrix that relates to color and priority.

The first part is an overview of colored Petri nets and works are cited where they were approached for modeling of some studied systems. In the following part a shipping system in whose evolution some priorities exist is presented. The cellular matrix of incidence that describes the system is decomposed on color priority.

2. COLORED TRANSPORTATION PETRI NETS - A SURVEY

Petri Network structure is described by the network graph (or incidence matrix) and the marking system. Therefore for a colored Petri net two problems are considered: firstly, colorful network arcs and secondly colorful markings network. In PRE and POST matrices, color arcs are expressed by m-dimensional vectors, except that the incidence matrix element for the color effect is +1 where the arc is of type POST and respectively -1 if the arc is of type PRE [1],[2],[3]. Automatically this results from the calculation of the incidence matrix (matrix difference Post and Pre colored transport network) positions and their transitions in modeled system.

In the graphical representation of a colored Petri network, the colors of arcs (and marks) can be represented by different graphical symbols such as white squares, black squares, circles, triangles, etc). In

Fig. 1 one can see a colored Petri transport network in which only two colors are used: the first color is represented by the white square and the first item is reserved in the color vector; the second color (black square) is reserved in the last element of vector.
With this information, the color vector attached to the six arcs of this colored network will be:

k_2 = [0 | 1] for the arcs (P_1,T_1), (T_2,P_3),(P_3,T_4) and (T_4,P_4)

k_1 = [1 | 0] for the arcs (P_2,T_2) and (T_1,P_2)

The PRE matrix of a colored network is a cell matrix 4x3 with the number of lines equal to the number of positions (lines related to that of P_1, P_2, P_3, P_4) and the number of columns equal to the number of transitions (columns related to that of T_1 T_2, T_3). Figure 1 represents an example of Wpost and Wpre matrices for a network that was not previously colored. The figure also depicts the uncoloured bigraph W = Wpost-Wpre. One can also note the matrices corresponding to the same network but colored by: Apost, Apre and respectively, A. Apre and Apost matrices are derived from the Wpost and Wpre by substituting the matrix elements equal to 1 through the color vectors (k_1 and k_2 defined above), corresponding to those arcs. The A incidence matrix, in case of colored network, results from the other two matrices as follows:

A= Apost – **A**pre

where A, Apost and Apre are cellular matrix. These matrices, for the considered network, are also shown in Figure 1. Cellular elements of these matrices are separated by vertical bars. One can see a perfect similarity of these cellular matrices with those of the colorless bigraph, W, Wpre and Wpost which are shown in the same figure.
The advantages of structure's representation through cellular matrix are presented in [4].
To a colored transportation Petri net one can assign a colored cellular marking vector whose size equals the number of network's positions.
Each cell element m_1, m_2, ... of this vector is a vector - line of colors whose size equals the number of colors corresponding to arcs. In the line- vectors, the

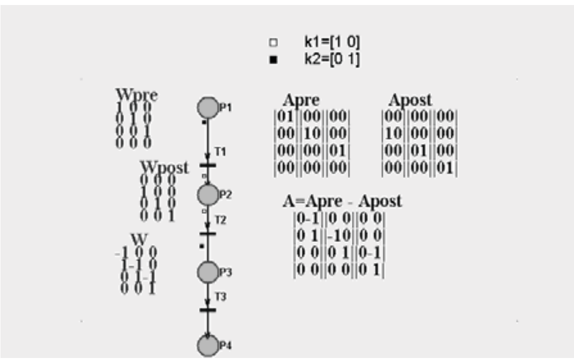

Figure 1: Colored transportation Petri net: white color = white square; black color = black square.

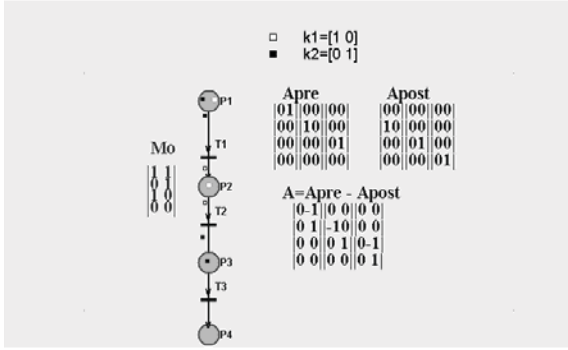

Figure 2: Vector of initial marking M_0 of colorful transportation network.

colors are ordered in ascending order, from left to right, as in the case of color vector of arcs. The difference here is that the related element for a particular color of a vector-line m_k contains the number of marks of the related color, present in position P_k of the network. If in the position P_k there is no mark of the appropriate color, in the reserved element of color the digit zero is registered. In the graphical representation of the related graph of Petri net, color marks can be represented by different graphic symbols as shown in Figure 2.

Color line - vectors attached to the four positions of this transportation network will be:

m_1 = [1 1], vector of colors to mark the position P_1 ;

m_2 = [0 1], vector of colors to mark the position P_2 ;

m_3 = [1 0], vector of colors to mark the position P_3 ;

m_4 = [0 0] (the color vector expresses the absence of marks in position P_4).

Fig. 3 depicts a graphical image of changes that occur in the marking vectors after the execution of various transitions that have completed the execution preconditions. One can consider the conditions met for execution of a transition Tk, only if the positions PRE of transitions contain at least one mark of the same color with PRE arcs between transition and these positions upstream.
If one considers M_1 the initial marking of the network in Fig. 3, this condition is accomplished only for the transition T_1. The implementation of this transition causes the withdrawal of a mark (color of PRE arcs) from PRE positions and the addition of one mark of color of POST arc in all POST positions of this transition. Thus, the marking network becomes the one expressed by the vector M_2 shown in Fig. 3. In the same figure one can see the other changes caused by implementation of other transitions which in turn become executable.
The basic equation allows the calculation by recurrent marking vectors based on the initial marking M1 and the incidence matrix:

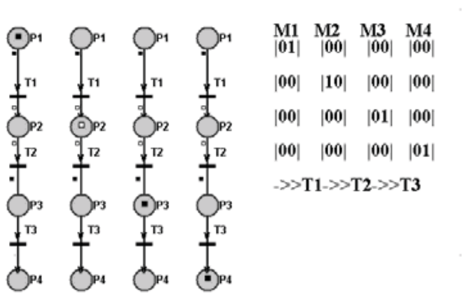

Figure 3: Vector of initial marking M_0 of colorful transportation network.

$$M_k = M_{k-1} + (T_q \text{ column of the A}) \qquad (2.1)$$

where T_q is the transition which in M_{k-1} has satisfied the conditions for execution.
An incidence matrix of the network from Fig. 3 is shown in Figure 2. In this example $T_q = T_1$ and the column related to T_1 is the first column of A. With these observations one can calculate the other marking vectors from the initial given M_1 vector:

$$M_2 = M_1 + \begin{matrix} 0\,\text{-}1 \\ 1\;0 \\ 0\;0 \\ 0\;0 \end{matrix} = \begin{matrix} 0\;0 \\ 1\;0 \\ 0\;0 \\ 0\;0 \end{matrix}\ ;\ M_3 = M_2 + \text{column } T_2$$

All results of Petri transportation networks theory can be applied to colored transport networks if one uses the cellular matrices and cellular vectors defined above.

3. COLORED PETRI TRANSPORT NET WITH DISTINCT PRIORITIES FOR COLOR OF MARKS

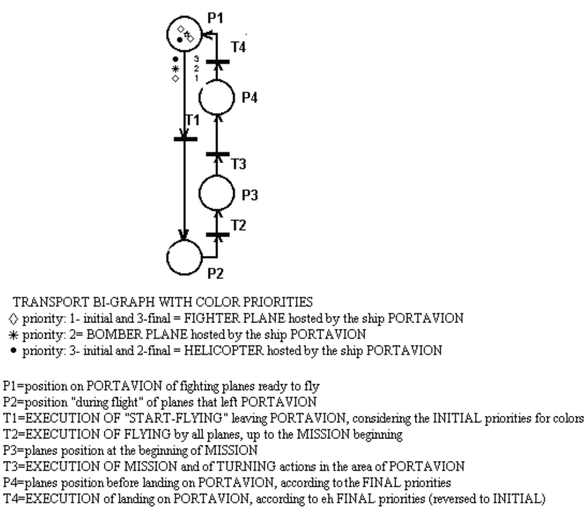

Figure 4: Colored Petri transport net with distinct priorities colors of marks.

The coloring process of Petri networks for shipping systems' modeling enables the representation of distinct treatment when modeling various elements in real system such as: military or civilian ships of different types and capacities, ships of the same type but with different functions and tasks in the transportation system, priorities and other specific attributes of the vessels in the system, type, the use mode and different routes of different loads of ships and of carrying ships (aircraft carrier and load consisting of various types of airplanes, helicopters and missiles or other attack or defense weapons).
In Fig. 4, initial priorities (1, 2, 3, when taking-off) are assigned for arc colors (P_1 T_1). The final priorities (3, 2, 1) are assigned to arc colors (P_4 T_4). Because the priorities of colors associated to the arc (P1 T1) are different, the implementation of the transition T1 is firstly accompanied by the withdrawal of rhombs (of priority 1) from P1, moving them to P2. Afterward the withdrawal of star occurs (of priority 2) and finally the circle with priority 3 is withdrawn (it means that T1 is executed three times in the row, but each execution is related to other color in compliance with the priorities associated to colors). Here, the events associated to marks movement are conducted sequentially (in series according to the order imposed by priorities).
The initial marking cellular vector Mo of the bi-graph from Fig. 4 has 4 colors cells (one for each position P_1, P_2, P_3, P_4, of the bi-graph). Each marking cell is a line-vector of size equal to the number of colors of the bi-graph. One should separate the cells of matrices vectors by vertical bars as in expression (3.1). Cell elements are associated to colors in compliance with the priorities and each element has a value equal to the number of marks of the related color (present in that position). For instance for the initial P_1 position the initial cell is |2 1 1| and the Mo cellular vector of initial marking of the bi-graph and the incidence cellular matrix involved are:

$$M_0 = \begin{bmatrix} |2\;1\;1| \\ |0\;0\;0| \\ |0\;0\;0| \\ |0\;0\;0| \end{bmatrix} \quad A = \begin{array}{cccc} T_1 & T_2 & T_3 & T_4 \\ \left[\begin{matrix} |\text{-}1\text{-}1\text{-}1| \\ |1\;1\;1| \\ |0\;0\;0| \\ |0\;0\;0| \end{matrix}\right. & \begin{matrix} |0\;0\;0| \\ |\text{-}1\text{-}1\text{-}1| \\ |1\;1\;1| \\ |0\;0\;0| \end{matrix} & \begin{matrix} |0\;0\;0| \\ |0\;0\;0| \\ |\text{-}1\text{-}1\text{-}1| \\ |1\;1\;1| \end{matrix} & \left.\begin{matrix} |1\;1\;1| \\ |0\;0\;0| \\ |0\;0\;0| \\ |\text{-}1\text{-}1\text{-}1| \end{matrix}\right] \end{array} \qquad (3.1)$$

One gets the following vector marking M_1 :

$$M_1 = \text{Mo} + \text{column } T_1$$

$$\text{of A} = \begin{bmatrix} |2\;1\;1| \\ |0\;0\;0| \\ |0\;0\;0| \\ |0\;0\;0| \end{bmatrix} + \begin{bmatrix} |\text{-}1\text{-}1\text{-}1| \\ |1\;1\;1| \\ |0\;0\;0| \\ |0\;0\;0| \end{bmatrix} = \begin{bmatrix} |1\;0\;0| \\ |1\;1\;1| \\ |0\;0\;0| \\ |0\;0\;0| \end{bmatrix} \qquad (3.2)$$

4. DECOMPOSITION OF THE MATRIX OF INCIDENCE IN MATRICES OF INCIDENCE ON PRIORITIES OF COLOR

The marking cell vector from (3.2) expresses the fact that three of the four aircrafts present on the aircraft

carrier took off simultaneously, while a fighter plane remained in place.
To ensure the withdrawal of marks in the order of color prioritization the use of different incidence matrices for each color is needed: A_1 for *rhomb*, A_2 for *star*, A_3 for *circle*. Thus, separate arcs for the three colors are taking into consideration and they will be able to be withdrawn in the desired order, using the equation of state several times, replacing every time the incidence matrix related to color and to priority respectively. In the example considered, the three matrices of incidence related to priorities for the arc (P_1, T_1) are Pr (helicopter) = 3, Pr (bomber) = 2, Pr (fighter plane) = 1, are:

$$\begin{array}{rcccc} & T_1 & T_2 & T_3 & T_4 \\ A1(1,1)= & \left[\begin{array}{c|c|c|c} -100 & 000 & 000 & 100 \\ 100 & -100 & 000 & 000 \\ 000 & 100 & -100 & 000 \\ 000 & 000 & 100 & -100 \end{array}\right] \end{array}$$

$$\begin{array}{rcccc} & T_1 & T_2 & T_3 & T_4 \\ A2(1,1)= & \left[\begin{array}{c|c|c|c} 0-10 & 000 & 000 & 010 \\ 010 & 0-10 & 000 & 000 \\ 000 & 010 & 0-10 & 000 \\ 000 & 000 & 010 & 0-10 \end{array}\right] \end{array} \tag{3.3}$$

$$\begin{array}{rcccc} & T_1 & T_2 & T_3 & T_4 \\ A3(1,1)= & \left[\begin{array}{c|c|c|c} 00-1 & 000 & 000 & 001 \\ 001 & 00-1 & 000 & 000 \\ 000 & 001 & 00-1 & 000 \\ 000 & 000 & 001 & 00-1 \end{array}\right] \end{array}$$

In parentheses, A_k (i, j) are indications of the position P_i and of transition T_j, from the ends of the arc to which priorities k = 1,2,3 are assigned.
If there is a number m(k)> 1 of marks with the same priority (and the same color respectively) to be withdrawn all at once in P_i, then in the equation of state the following change must be made:

$$M_t = M_{t-1} + mp(k) * (\text{ column } Tq \text{ from } A_k) \tag{3.4}$$

We must note that when two or more objects of the same type (eg two fighter planes) must behave differently, they must be represented in bigraph by different colors or, if the case may be, different priorities must be associated to them. In the case expressed in (3.4), it is considered that the two fighter planes have to take off simultaneously. If in fact the two planes of the same type must take off in order then (2.1) is applied, where, A = A_1 until all marks of the same priority are withdrawn:

$$M_1=M_0+(\text{column } T_1 \text{ of } A_1) = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$M_2=M_1+(\text{column } T_1 \text{ of } A_1) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 \\ 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

After withdrawal of priority marks k = 1 we move to the next k = k_1 = 2 :

$$M_3=M_2+(\text{column } T_2 \text{ of } A_2) = \begin{bmatrix} 0 & 1 & 1 \\ 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 2 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$M_4=M_3+(\text{column } T_3 \text{ of } A_3) = \begin{bmatrix} 0 & 0 & 1 \\ 2 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 2 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

To illustrate the use of equation (3.4) we consider the case in which the two fighter planes have to take off simultaneously from the aircraft carrier and before the bomber and the helicopter. Then from the inspection of matrices A_1, A_2 and A_3, corresponding to the bi-graph of the given system and of initial marking cell vector results k = 1, mp (k) = 2 and Tq=T_1 and we obtain at t = 1:

$$M_1=M_0+2*(\text{column } T_1 \text{ of } A_1) = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + 2* \begin{bmatrix} -1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 \\ 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

This final marking vector M_1 correctly expresses the simulated situation .

5. CONCLUSIONS

Colored transportation Petri networks are instruments with high capacity for modeling military and civilian shipping systems, given their complexity and diversity. Extending the use of colors in the colored Petri transportation networks for arcs and marks, improves the modeling through separate representation of ships' cargo and carrying transportation modes.
The matrix treatment of colored Petri transportation networks allows a correct expression of simulated situations according to scenarios related to systems of transport. The evolution of the system complying with an order imposed by the priorities of colors is determined by the equation of state where the incidence matrix is related to color and to priority respectively. The decomposition of cellular matrix of incidence in matrices of incidence on the priorities of color is what differentiates the case of priorities of color of marks from the case of priorities of transitions.

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