COMPARATIVE ACTIVE CURRENT CALCULATION BY p-q AND CPC THEORIES

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Abstract **– This paper studies the possibilities of decomposition of the current in three-phase, three-wire systems based on the classic p-q theory proposed by Akagi and further developed and, respectively, on the Czarnecki's theory. In the case of distorted current absorbed by a nonlinear load, the resulted components can be used to generate the reference compensating current in three-phase active power filters. As the major compensation goal of both current harmonics and reactive power involves a current drawn from the network which has the same phase and shape as the grid voltage, the attention in current decomposition is especially directed to the active component. Two case studies were analyzed by simulation in Matlab/Simulink environment, under both sinusoidal and nonsinusoidal voltage conditions. Linear and nonlinear loads were taken into consideration. For each case, the current components had been calculated according to Akagi's original decomposition, the second interpretation of the p-q theory, and Czarnecki's expressions based on Current Physical Components theory. It is shown that only the Current Physical Components – based decomposition is able to provide a correct active component of the load current as defined by Fryze for single-phase circuits, irrespective of voltage shape and load nonlinearity. Some additional modifications should be made in p-q based current decomposition in order to obtain an active current which preserves the voltage waveform under nonsinusoidal voltage conditions.**

Keywords: active current, CPC theory, non-sinusoidal voltage, p-q theory.

1. INTRODUCTION

The non-sinusoidal conditions analysis is important because of the disturbing effects produced to the power grid.
In the power grids working with distorted current the power factor diminishes and the existing reactive power cannot be compensated using capacitive compensators, due to the oscillating circuits that this capacitors form with the grid reactance which leads to the amplification of some harmonic currents existing in these circuits.
Active power filtering is an efficient method to improve the shape of the current absorbed from the power grid and to improve the power factor.
The purpose of this paper is to analyze the current decomposition possibilities based on the classical p-q theory [7] and its development [3] using some case studies.

2. THE p-q THEORY

Starting from expression of the instantaneous apparent complex power [1],

$$\begin{aligned} \underline{s} &= p + jq = \frac{3}{2}\underline{u}\cdot\underline{i}^{*} = \\ &= \frac{3}{2}\left[u_d i_d + u_q i_q + j\left(-u_d i_q + u_q i_d\right)\right] \end{aligned} \tag{1}$$

H. Akagi has proposed the compensation for the AC components of the real and imaginary parts of the complex instantaneous apparent power, respectively the calculation of the reference currents of an active filter with the following expression:

$$\underline{i} = i_d + ji_q = \frac{2}{3}\frac{1}{u_d^2 + u_q^2}\underline{u}\cdot\underline{s}^{*} \tag{2}$$

Developing the scalar product, the expression (2) becomes:

$$\begin{aligned} \underline{i} &= i_d + ji_q = \\ &= \frac{2}{3}\frac{1}{|\underline{u}|^2}\left[pu_d + qu_q + j\left(-qu_d + pu_q\right)\right] \end{aligned} \tag{3}$$

On this basis, Akagi, Nabae and their co-authors defined [1]:
- the instantaneous active current with the following components:

$$i_{ad} = \frac{2}{3}\frac{u_d}{|\underline{u}|^2}p \qquad i_{aq} = \frac{2}{3}\frac{u_q}{|\underline{u}|^2}p \tag{4}$$

- the instantaneous reactive current with the following components:
-

$$i_{rd} = \frac{u_q}{|\underline{u}|^2}q; \quad i_{rq} = -\frac{2}{3}\frac{u_d}{|\underline{u}|^2}q \tag{5}$$

These definitions had been criticized by Czarnecki [5], who found some examples in which the current defined with (4) is not corresponding to active power, and the current defined with (5) is not corresponding to reactive power. This observation is justified because, as it was shown, p and q contain both active/reactive power and the components corresponding to the non-sinusoidal and asymmetrical regime. This vagueness can be eliminated by the second interpretation of the p-q theory, defining four components for the distorted current [2], [8], which emphasize the mean values (P and Q) and the alternative components ($p_{\sim}$ and $q_{\sim}$).

Now, expression (3) becomes:

$$\underline{i} = \frac{2}{3}\frac{1}{|\underline{u}|^2}\Big[(P+p_{\sim})u_d + (Q+q_{\sim})u_q + + j\big(-(Q+q_{\sim})u_d + (P+p_{\sim})u_q\big)\Big] \quad (6)$$

Based on the previous expression, the current components can be defined:

- the active current vector, $\underline{i}_a$, with the following components:

$$i_{ad} = \frac{2}{3}\frac{u_d}{|\underline{u}|^2}P \qquad i_{aq} = \frac{2}{3}\frac{u_q}{|\underline{u}|^2}P \quad (7)$$

- the reactive current vector, $\underline{i}_r$, with the following components:

$$i_{rd} = \frac{2}{3}\frac{u_q}{|\underline{u}|^2}Q \qquad i_{rq} = -\frac{2}{3}\frac{u_d}{|\underline{u}|^2}Q \quad (8)$$

- the supplementary useless current vector for the AC component of p, $\underline{i}_{sp}$, with the following components:

$$i_{spd} = \frac{2}{3}\frac{u_d}{|\underline{u}|^2}p_{\sim} \qquad i_{sqp} = \frac{2}{3}\frac{u_q}{|\underline{u}|^2}p_{\sim} \quad (9)$$

- the supplementary useless current vector for the AC component of q, $\underline{i}_{sq}$, with the following components:

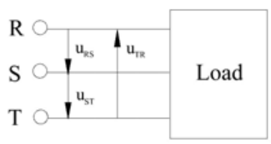
$$i_{sdq} = \frac{2}{3}\frac{u_q}{|\underline{u}|^2}q_{\sim} \qquad i_{sqq} = -\frac{2}{3}\frac{u_d}{|\underline{u}|^2}q_{\sim} \quad (10)$$

3. CURRENT PHYSICAL COMPONENTS

Considerations in this Section are confined to three-phase, three-wire circuits, shown in Fig. 1(a), with linear, time-invariant loads supplied with a sinusoidal symmetrical voltage of positive sequence.

Because a three-phase, three wire load was taken into consideration, the phase voltages were calculated based on the line measured voltages.

For any such load there exists an equivalent resistive and balanced load, shown in Fig. 1(b), that at the same voltage has the same active power, P, as the original load [4].



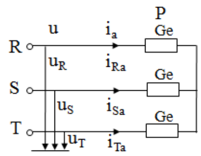


Figure 1: (a) Three-phase load and (b) its equivalent load with respect to active power, P

The active power of the load in Fig. 1(b) is equal to:

$$U_R^2 G_e + U_S^2 G_e + U_T^2 G_e = P \quad (11)$$

Thus, this load is equivalent to the original load with respect to the active power, if its phase conductance has the value:

$$G_e = \frac{P}{U_R^2 + U_S^2 + U_T^2} \quad (12)$$

The line current vector of the equivalent resistive load is equal to:

$$\underline{i}_a = \sqrt{2}Re\{G_e \underline{U} e^{j\omega_1 t}\} = G_e \underline{u} \quad (13)$$

and is referred to as the *active current*.

It is the smallest current needed for energy permanent conversion in the load with power P.

Under non-sinusoidal voltage conditions, the grid voltage can be also decomposed, using Fourier analysis, in a sum of harmonic components.

In order to obtain the maximum efficiency, the power must be transferred by harmonics as well. Thus, the current must contain the same harmonics as the voltage. In other words, the current must have the same shape as the voltage.

The conductance G_e specified by (12) is valid irrespective of the supply voltage shape.

Thus, the active current vector can be expressed as:

$$\underline{i}_a = \sqrt{2}Re\sum_{n\in N} G_e \underline{U}_n e^{jn\omega_1 t} \quad (14)$$

4. CASE STUDIES

Two case studies were created for a three-phase sinusoidal and then nonsinusoidal voltage system, with different symmetrical loads (linear and non-linear), in which the current had been calculated with Akagi's expressions, the second interpretation of the p-q theory and Czarnecki's definitions.

These case studies will emphasize that Akagi's interpretation of the p-q theory, related to the active current, leads to correct results only if the load is symmetrical and linear.

The active component defined by the second interpretation of the p-q theory represents the active component of load current only if the voltage is sinusoidal.

The only definition for the active current which keeps its validity, in the both cases related to voltage conditions and load type, is the definition given by Czarnecki in the CPC theory.

All the waveforms that will be presented for these case studies have been obtained in Matlab/Simulink environment.

4.1. Sinusoidal voltage, symmetrical nonlinear load

The symmetrical nonlinear load consists of a three phase thyristor bridge rectifier with a passive RL load. The sinusoidal grid voltage and distorted load current are shown in Fig. 2.

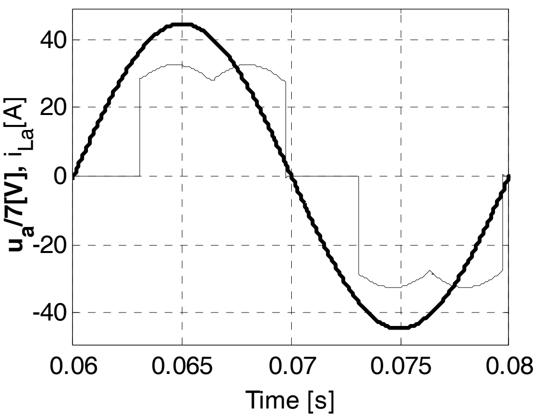


Figure 2: The grid voltage and current waveforms for the three-phase bridge rectifier

As it can be seen, the active current defined by Akagi is nonsinusoidal and its fundamental component has the same phase as the voltage (Fig.3).

On the other hand, the active current defined by the second interpretation of the p-q theory (Fig. 4) is sinusoidal with the same phase as the voltage [6]. Therefore, besides the actual active component, the current proposed by Akagi contains a distorted current component.

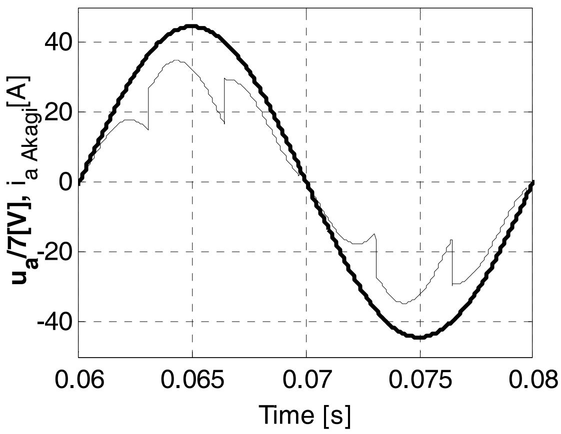


Figure 3: The grid voltage and the Akagi's active current waveforms

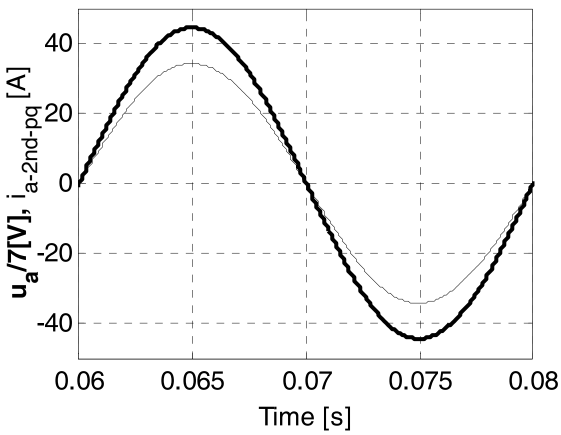


Figure 4: The grid voltage and the active current waveforms for the 2nd interpretation of p-q

As expected, the CPC's active current is a sinusoidal one, with the same phase as the grid voltage (Fig. 5).

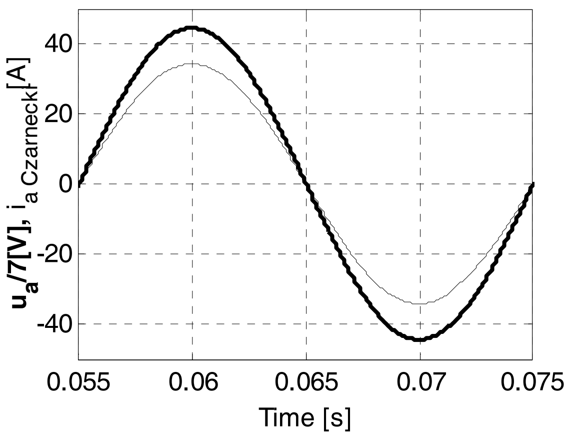


Figure 5: The grid voltage and the Czarnecki's active current waveforms

It can be seen in Fig. 6 and 7 that the locus of the active current space vector follows the locus of the voltage space vector only in case of the second interpretation of the p-q theory.

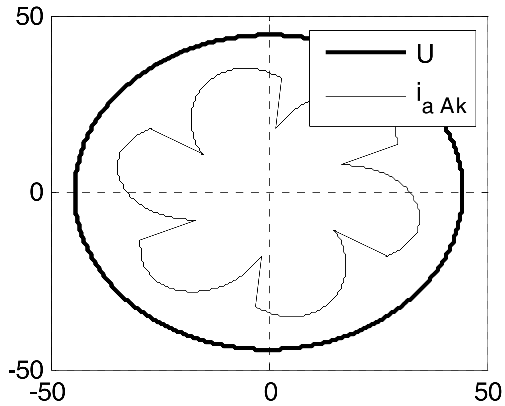


Figure 6: The locus of the voltage space vector and of Akagi's active current space vector

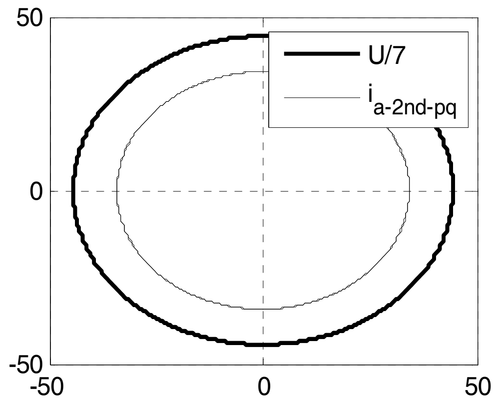


Figure 7: The locus of voltage space vector and of the active current space vector for the second interpretation of the p-q theory

This case study shows that the "active" name is not suitable for the currents defined by Akagi, because, under sinusoidal voltage conditions, the active power is transmitted only on the fundamental component of the distorted current. This does not happen in the case of active current defined by the second interpretation of the p-q theory nor in the case of CPC's active current.

4.2. Nonsinusoidal voltage, symmetrical resistive load

Let us consider a symmetrical resistive load supplied by a nonsinusoidal voltage system.
This voltage system was obtained by reconstructing the shape of an actual measured voltage, based on its harmonics spectrum.

Obviously, the load current and grid voltage have identical shapes and phase (Fig. 8).

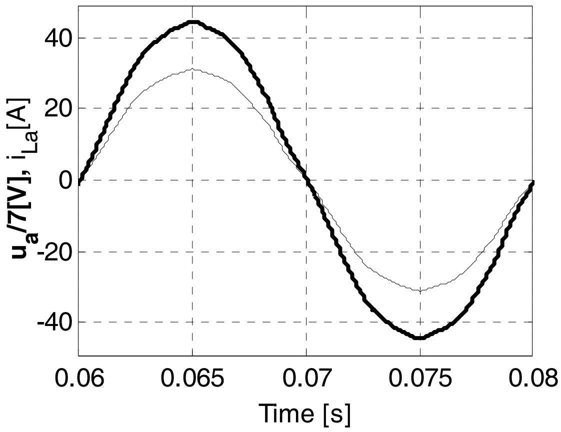
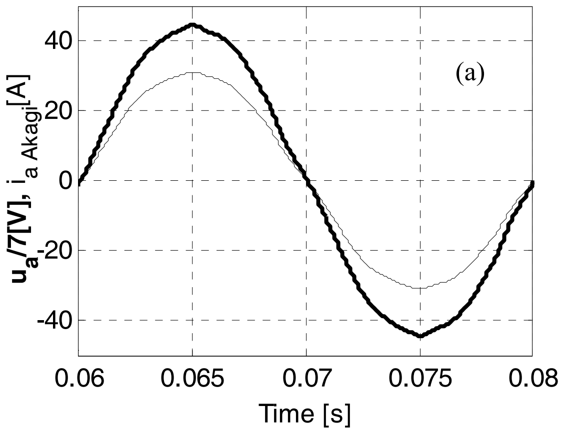


Figure 8: The grid voltage and current waveforms for the resistive load

In this case, the Akagi's p-q theory leads to correct results because the active current has the same phase and shape as the grid voltage (Fig. 9).



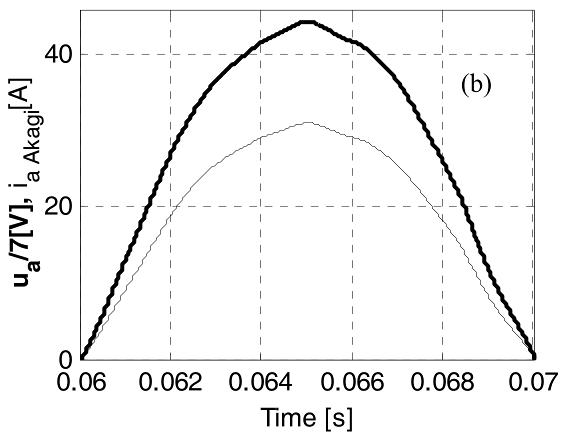


Figure 9: (a)The waveforms of grid voltage and Akagi's active current; (b) Detail view which demonstrates the similarity between the voltage waveform and the active current waveform

According to the second interpretation of the p-q theory, the resulted active current system has a different shape referring to the voltage (Fig. 10).

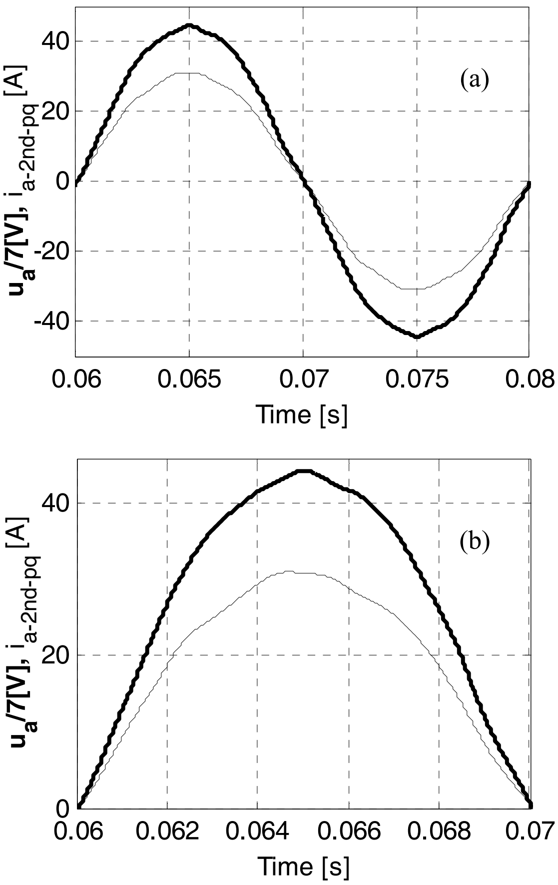


Figure 10: (a) The waveforms of grid voltage and the 2nd interpretation of p-q active current; (b) Detail view which shows the difference between the voltage waveform and the active current waveform

It can be noticed from Fig. 11, that the active current based on the CPC theory, also, preserves the supply voltage shape.

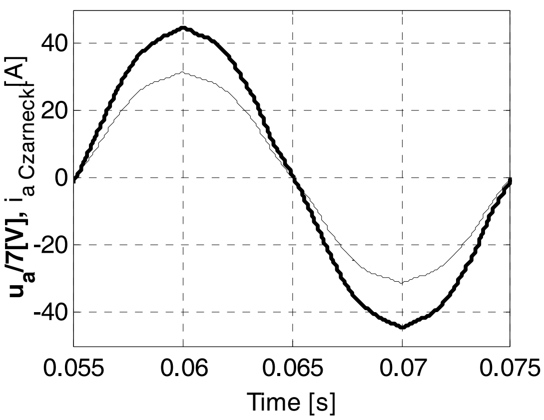


Figure 11: The grid voltage and the Czarnecki's active current waveforms

As shown in Fig. 12 through the locus of active current and voltage space vectors, Akagi's decomposition leads to correct results only if the load is symmetrical and linear.

Unfortunately, under non-sinusoidal voltage conditions, the active current defined by the second interpretation of the p-q theory does not represent the active component of the load current (Fig. 10 and Fig. 13).

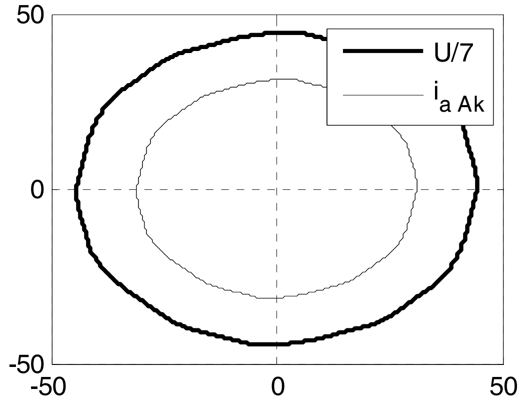


Figure 12: The locus of voltage space vector and of Akagi's active current space vector

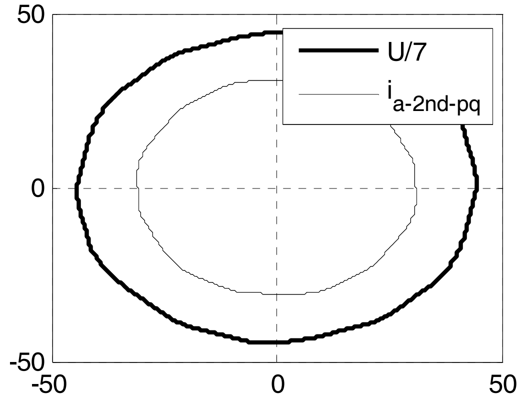


Figure 13: The locus of the voltage space vector and of the 2nd interpretation of p-q active current space vector

5. CONCLUSIONS

Considering the obtained results after analyzing the two case studies, some conclusions regarding the current components in a three phase system can be outlined.

The active current defined by Akagi represents the active component of the load current only for symmetrical linear loads. The active current defined by the second interpretation of the p-q theory represents the active component only if the grid voltage is sinusoidal, no matter the character of the

load. If the grid voltage becomes nonsinusoidal, this interpretation, also, loses its validity.
The active current defined by the CPC theory remains valid in all the imposed study cases. This happens because, even if the grid voltage is nonsinusoidal, the active current keeps the voltage shape for both linear and nonlinear loads.

Acknowledgments

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