OPTIMAL SOLUTION OF WEAK SPOTS PROBLEM IN A GRID SYSTEM BY MEANS OF FACTS DEVICES

Tudor RADILOV

Moscow Power Engineering Institute, radilov07@gmail.com

Abstract – **Electrical power system functioning is permanently exposed to different disturbances which cause the regime parameters change. Every grid system contains elements whose regime parameters variations influence system's sensitivity most of all. These elements are called weak spots and may represent nodes or branches of the electrical network. An original method of weak spots detection and efficient ways of decreasing grid system's sensitivity are presented in this paper. The technique used to estimate the number of weak spots in a grid system is based on singular value decomposition of Jacobi inverse matrix called sensitivity matrix. The theory taken for analysis implies the investigation of right and left singular vectors components related to the minimal singular value of sensitivity matrix in order to detect the weak nodes of the power network. The optimal way to strengthen the weak spots adds up to the use of FACTS devices that represent a powerful technology capable to solve this problem. In this paper steady-state power flow models of Static VAR Compensator (SVC) and Static Synchronous Compensator (STATCOM) were used to solve the problem set above owing to their characteristics. Consequently their inclusion in Newton-Raphson method has shown the opportunity to control the nodal voltage magnitudes. Case studies are carried out for a test power grid and the results show that either of the FACTS devices is eminently suitable to strengthen the weak nodes that were found.**

Keywords: *weak spot, SVC, STATCOM, SVD, sensitivity.*

1. INTRODUCTION

The reliable functioning of electrical power system ensures the basis of development for all branches of industry and household use that fit to all human needs. The permanent exposure of grid system to small and big disturbances in grid system's operating mode is a common trait. This exposure to different factors determines electrical power system's sensitivity or its reaction to appeared perturbations and is identified with variations in regime parameters. The changes in regime parameters are influenced by external factors – the type and magnitude of disturbance, as well as by internal factors – the structure of grid system scheme and the parameters of its component elements that could be considered invariable to regime factors. Big perturbations represent such regime violations as scheme topology changes caused by emergency disconnections of the electrical equipment as well as short-circuits, power cuts and other causes; small perturbations are identified with the changes of loads, generation of active and reactive power etc. The elements of power grid's scheme, whose parameters are more sensitive to sudden disturbances were called sensors [1]. According to [1, 3] every complex electrical power system is inhomogeneous. This statement leads to the idea that grid systems contains elements, whose regime parameters mostly react to different disturbances. To a certain extent the existence of sensors is conditioned by the parameters of grid system's structural elements (impedances, admittances). Thereby it's reasonable to assess the connection between sensors and electrical power system parameters, as well as to find those elements of the scheme, whose regime parameters variations determine sensors existence and offer the possibility to change the electrical power system reaction to different disturbances. Weak spots are the grid system elements, whose sensitivity depends mostly on the change of regime parameters. It becomes possible to improve grid system properties, thereby to decrease its reaction to disturbances by changing the parameters of such spots (sections, nodes, branches). The applied perturbations to different points of power network most of all arouse the regime parameters reaction of the same elements. Thus sensors reaction depends on weak spots existence. Consequently there should be examined the factors that determine weak branches and weak nodes existence, so as to propose ways of branch strengthening and node admittance change with the purpose to change grid system's behavioral properties. Original analytical methods for grid system sensitivity investigation are presented in [3]. The methods imply the study of sensitivity matrix and are based on singular value analysis or spectrum analysis. Their subject of investigation is the Jacobi inverse matrix, being called sensitivity matrix. The applied method in this paper is based on the technique of singular value decomposition (SVD) of Jacobi inverse matrix, in consecutive order analyzing the left and the right singular vectors components that are related to the minimal singular value of sensitivity matrix. In some sources of information [1] ,[3] it is stated that in most of the cases more sensitive are the nodes to whom the maximal components of the right singular vector correspond. These maximal values must be related with the minimal singular value of sensitivity matrix.

According to [3] Jacobi matrix may be presented as follows, in the case of current balance:

$$J = \begin{pmatrix} -B & G \\ G & B \end{pmatrix} + \begin{pmatrix} A_{11} & A_{12} \\ A_{12}^T & A_{22} \end{pmatrix} \quad (1)$$

where A_{11}, A_{12}, A_{22} – submatrix, whose elements depend on the powers drawn by the load and nodal voltage magnitudes.

The analysis of correlation (1) leads to the conclusion that Jacobi matrix values, thereby the sensitivity of nodal voltage magnitudes depends on regime factors as well as network topology and its parameters that are invariable unto the regime during electrical power system functioning. The maximal decreasing of grid system sensitivity, realizable due to the change of scheme's nodal admittances and due to active and reactive power additional generations, can be reached by the strengthening of the branches or the nodes electrical power system.

Large-scale use of a new class of reactive power compensation installations, classified by the work group of IEEE as flexible alternating current transmission systems, represent the optimal and complex solution of weak spots strengthening and creation of a full-strength grid system. Flexible transmission systems include controllers, based on power electronic devices or other principles of static control, which consist of power electronic circuits using conventional thyristors or voltage source converters and one of the storage elements – inductive or capacitive.

One of the operational policies that must be observed by the electrical energy suppliers is to maintain nodal voltage magnitudes within narrow boundaries. FACTS devices provide the possibility to control the nodal voltages in a power system and therefore to improve the security of the system.

The goal set in this paper to achieve boils down to detect weak spots and to find the optimal solution to their strengthening on the basis of the results obtained from load flow analysis. The results obtained by these techniques will be useful to assess the steady-state operation of electrical power systems and could represent a benefit for future planning of the grid system as well as improving the conditions of its functioning in transient regimes.

2. THE MATHEMATICAL ASPECTS OF WEAK SPOTS DETECTION PROBLEM

Singular value decomposition is one of the most powerful techniques in linear algebra. It represents an easy-to-use method to perform different operations with matrix. SVD of any matrix shows its structural aspects and has a lot of valences to be applied.

Singular nonnegative values σ_i, $i = 1{:}r$, of a matrix $A \epsilon R^{k \times n}$ are the pozitive square roots from nonzero eigenvalues λ of the symetric positive semidefinite matrix [4]

$$B = A^T A \quad (2)$$

Then for $k < n$

$$\sigma_i(A) = \sqrt{\lambda_i(A^T A)} = \sqrt{\lambda_i(AA^T)}, \text{ if } i = 1, \dots, k,$$

$$\text{and } \sigma_i(A) = 0, \text{ if } i = k+1, \dots, n.$$

Sensitivity analysis of the electrical power system using SVD technique mathematically is founded on the hypothesis that every matrix can be expressed as the product of three matrix [1].

$$J = W \Sigma V^T = \sum_{i=1}^{k} w_i \sigma_i v_i^T \quad (3)$$

where J is Jacobi matrix, but W and V are $(k \times k)$ ortogonal matrix; Σ is a diagonal matrix with nonnegative real numbers on the diagonal.

The k columns $(w_i \in R^k)$ of W and $(v_i \in R^k)$ of V are called the left singular vector and right singular vector of J, respectively. For these vectors can be written the correlation:

$$w_i^T v_i = 1, \qquad w_i^T v_j = 0 \text{ for } i \neq j \quad (4)$$

$$\Sigma = diag(\sigma_1, \sigma_2, \dots, \sigma_k) \quad (5)$$

so as $$\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_r > 0 \quad (6)$$

The diagonal entries of Σ are known as the singular values of J.

By this way it may be analyzed the influence of each singular value on the grid system's sensitivity according to the equation written below.

$$J_{\sigma_1} = \begin{bmatrix} w_{11} & \dots & \dots & w_{1k} \\ w_{21} & \dots & \dots & w_{2k} \\ \dots & \dots & \dots & \dots \\ w_{k1} & \dots & \dots & \dots \end{bmatrix} \cdot \begin{bmatrix} \sigma_i \\ 0 \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} v_{i1} & \dots & \dots & v_{ik} \end{bmatrix} \quad (7)$$

According to these correlations, it follows that for the existence of Jacobi inverse matrix, that is all of the σ_i ($i = 1, \dots, n$, $n = k$) are nonzero values, can be written the equation:

$$J^{-1} = (W \Sigma V^T)^{-1} = \sum_{i=1}^{k} v_i w_i^T / \sigma_i \quad (8)$$

or $$J_{\sigma_1}^{-1} = \begin{bmatrix} v_{11} & \cdots & \cdots & v_{1k} \\ v_{21} & \cdots & \cdots & v_{2k} \\ \cdots & \cdots & \cdots & \cdots \\ v_{k1} & \cdots & \cdots & \cdots \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{\sigma_i} \\ 0 \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} w_{i1} & \cdots & \cdots & w_{ik} \end{bmatrix} \quad (9)$$

If among all the singular values, ordered on the decrease, the last one is much less than the others, then for equal conditions the last component of the sum from (8) will maximally influence the grid system sensitivity and the existence of weak spots.

The analysis of grid system's reaction disturbances shows that sensitivity indicator is reflected by the minimal singular value decreasing for Jacobi inverse matrix. That's why the influence of power network scheme's parameters and regime parameters on its sensitivity can be assessed in accord with the correlation:

$$\frac{\partial \sigma_i}{\partial f} = w_i^T \left(\frac{\partial J}{\partial f} \right) v_i \quad (10)$$

where σ_i is the minimal singular value of sensitivity matrix.

If the controlled nodal voltages are taken as parameter *f*, then it is possible to detect that weak node that will mostly influence grid system's sensitivity. For this reason the second term which is placed in the right part of (10) can be presented as follows:

$$\frac{\partial J}{\partial U_j} = \begin{pmatrix} \frac{\partial^2 P}{\partial \delta \partial U_j} & \frac{\partial^2 P}{\partial U \partial U_j} \\ \frac{\partial^2 Q}{\partial \delta \partial U_j} & \frac{\partial^2 P}{\partial U \partial U_j} \end{pmatrix} \quad (11)$$

The technique of weak spots detection was applied to a three-bus test network [2]. Its schematic circuit is depicted in figure 1.

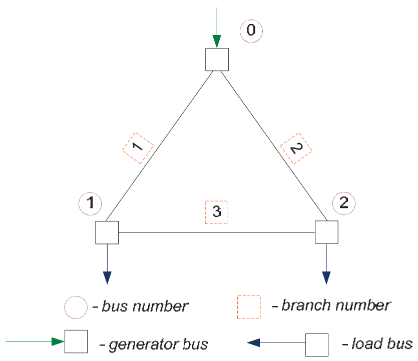

Figure 1: Three – bus test network.

The powers drawn by the load in the electric network illustrated in figure 1 are included in table 1.

Bus number	P	Q
1	35 [MW]	15 [MVAr]
2	45 [MW]	20 [MVar]

Table 1: The values of the powers drawn by the load.

The „0" bus is considered to be the slack bus in this power grid, so its voltage magnitude is maintained at $U_0 = 115\ kV$.

Power grid parameters are presented in table 2.

Branch number	R	X
1	3,54 [Ω]	12,16 Ω]
2	4,72 [Ω]	16,20 [Ω]
3	4,08 [Ω]	8,40 [Ω]

Table 2: Power grid scheme's parameters.

The power flow results are included in table 3.

Parameter	Bus number	
	1	2
U	111,522 [kV]	110,945 [kV]
δ	-2,062 [°]	-2,320 [°]

Table 3: Power flow results.

Singular value decomposition of sensitivity matrix has shown that the last singular value is much less than the others. The structure of marix Σ is depicted bellow.

$$\Sigma = \begin{pmatrix} 3535 & 0 & 0 & 0 \\ 0 & 878 & 0 & 0 \\ 0 & 0 & 31{,}20 & 0 \\ 0 & 0 & 0 & 7{,}37 \end{pmatrix}$$

This state offers the possibility to carry out weak spots detection in conformity with (10). In this case it is necessary to present the components of the left and the right singular vectors of Jacobi inverse matrix which relates to the minimal singular value.

$$w_4 = \begin{pmatrix} -0{,}206824 \\ -0{,}241226 \\ -0{,}63799 \\ -0{,}701429 \end{pmatrix}, \qquad v_4 = \begin{pmatrix} -4{,}587 \cdot 10^{-4} \\ -4{,}887 \cdot 10^{-4} \\ -0{,}671 \\ -0{,}7409 \end{pmatrix}$$

If to apply the equation (10) it could be obtained the derivatives of minimal singular value with respect to nodal voltages.

$$\frac{\partial\sigma_4}{\partial U_1} = w_4^T\left(\frac{\partial J}{\partial U_1}\right)v_4 = 0{,}0555;$$

$$\frac{\partial\sigma_4}{\partial U_2} = w_2^T\left(\frac{\partial J}{\partial U_2}\right)v_4 = 0{,}0812.$$

The obtained values show that the node "2" is weaker than the node "1". Thereby appears the necessity to strengthen this node by its admittance change or reactive power injection. For this reason the author considered the installation of FACTS devices in this node as the most efficient way to attain the set goal. STATCOM and SVC are the members of FACTS family that have been chosen to strengthen the node "2" owing to their characteristics.

3. POWER FLOW MODELS OF FACTS DEVICES

It was necessary to analyze the models of FACTS devices with reference to steady-state operation because the choice and location of these devices is made up in compliance with steady-state operating mode requirements.

The theory covered in this section boils down to the steady-state models of FACTS devices inclusion in Newton – Raphson method.

3.1 Power grid with SVC

The modeling approach used to represent SVC in power flow studies is based on firing angle utilization as state variable that describes the device in Newton – Raphson method.

In the advanced power flow models the SVC is described as a variable admittance which combines the admittances of the compensator and the step-down transformer coupled in series. This model allows for the voltage magnitude control at the bus where the power transformer is coupled. The equivalent scheme [7] that illustrates the series combination of the admittances mentioned above is presented in figure 2.

The total admittance of this model [7], which takes into consideration thyristor's firing angle, can be calculated using the equation below:

$$Y_{T-SVC(\alpha_{SVC})} = \frac{Y_T \cdot Y_{SVC}}{Y_T + Y_{SVC}} \qquad (12)$$

$$Y_{T-SVC(\alpha_{SVC})} = G_{T-SVC} - jB_{T-SVC} \qquad (13)$$

$$G_{T-SVC} = \frac{R_T}{R_T^2 + X_{Ec}^2} \qquad (14)$$

$$B_{T-SVC} = \frac{X_{Ec}}{R_T^2 + X_{Ec}^2} \qquad (15)$$

$$X_{Ec} = X_T + X_{SVC} \qquad (16)$$

$$X_{SVC} = \frac{X_C \cdot X_{TCR}}{X_C - X_{TCR}} \qquad (17)$$

where X_C - the reactance of capacitor bank; X_{TCR} - the reactance of thyristor controlled reactor.

$$X_{TCR} = \frac{\pi \cdot X_L}{2(\pi - \alpha_{SVC}) + \sin(2\alpha_{SVC})} \qquad (18)$$

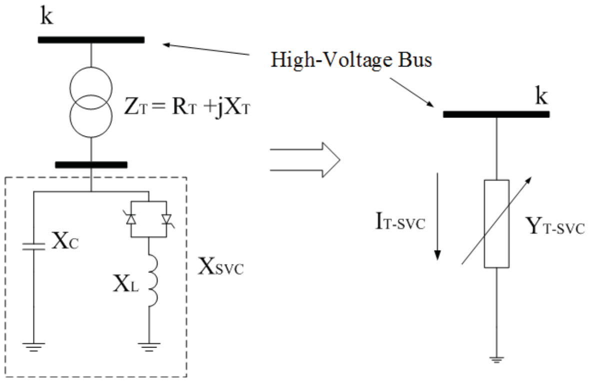

Figure 2: Advanced SVC model.

The parameters of the SVC designed for the power grid depicted in figure 1 are:

$X_C = 58{,}82\ \Omega$; $X_L = 26{,}71\ \Omega$; $X_T = 6{,}225\ \Omega$.

$R_T = 0{,}284\ \Omega$; $Q_C = 22{,}5\ MVAr$; $Q_{TCR} = 40\ MVAr$;

The mismatch reactive power equation at the bus where SVC is installed is presented bellow.

$$\omega Q_2 = -Q_2 - U_2^2 \cdot B_{22} + U_0 \cdot U_2 \cdot G_{02} \cdot \sin(\delta_2 - \delta_0) - \\ - U_0 \cdot U_2 \cdot B_{02} \cdot \cos(\delta_2 - \delta_0) + U_1 \cdot U_2 \cdot G_{12} \cdot \sin(\delta_2 - \delta_1) - \\ - U_1 \cdot U_2 \cdot B_{12} \cdot \cos(\delta_2 - \delta_1) + k_T^2 \cdot U_2^2 \cdot B_{TSVC}$$

The linearised power flow equations for the SVC connecting bus are given as:

$$\begin{bmatrix} \Delta P_k \\ \Delta Q_k \end{bmatrix}^{(i)} = \begin{bmatrix} 0 & U_k^2 \dfrac{\partial G_{T-SVC}}{\partial \alpha_{SVC}} \\ 0 & -U_k^2 \dfrac{\partial B_{T-SVC}}{\partial \alpha_{SVC}} \end{bmatrix}^{(i)} \cdot \begin{bmatrix} \Delta\theta_k \\ \Delta\alpha_{SVC} \end{bmatrix} \qquad (19)$$

The necessity to reduce the nodal voltage magnitude at bus “2” to 110 kV corresponds to inductive operating region of SVC. The obtained results from this operation are presented in table 4. These results rely on the firing angle value that lies in inductive operating region, $\alpha_{SVC} = 116{,}249°$.

Parameter	Bus number	
	1	2
U	110,962 [kV]	110 [kV]
δ	-2,008 [°]	-2,186 [°]

Table 4: Power flow results for SVC inductive operating mode.

The SVC absorbed *11,14 MVAR* at the value of total susceptance $B_{T-SVC} = 1{,}037 \cdot 10^{-4}\ S$ in order to maintain the nodal voltage magnitude at bus 2 at 110 kV. Singular value decomposition of sensitivity matrix which refers to the power grid with SVC shows that its sensitivity decreased with the minimal singular value increase.

$$\Sigma = \begin{pmatrix} 4405 & 0 & 0 & 0 \\ 0 & 3158 & 0 & 0 \\ 0 & 0 & 841 & 0 \\ 0 & 0 & 0 & 20{,}649 \end{pmatrix}$$

The necessity to increase the nodal voltage magnitude at bus “2” to 112 kV involves SVC to operate in capacitive regime. The obtained results are presented in the table 5.

Parameter	Bus number	
	1	2
U	112,147 [kV]	112 [kV]
δ	-2,122 [°]	-2,469 [°]

Table 5: Power flow results for SVC capacitive operating mode.

These results rely on the firing angle value that lies in capacitive operating region, $\alpha_{SVC} = 116{,}575°$.

The SVC injected *12,30 MVAR* at the value of total susceptance $B_{T-SVC} = 1{,}139 \cdot 10^{-4}\ S$ in order to maintain the nodal voltage magnitude at bus 2 at 112 kV.

Power grid sensitivity decreases for capacitive operating mode of SVC. This leads to the idea that reactive power injection may be considered as a mean of grid system strengthening. The matrix of singular values that shows this statement is given bellow.

$$\Sigma = \begin{pmatrix} 4549 & 0 & 0 & 0 \\ 0 & 3238 & 0 & 0 \\ 0 & 0 & 853 & 0 \\ 0 & 0 & 0 & 20{,}868 \end{pmatrix}$$

Consequently it was reasonable to use another device connected in derivation which belongs to the third generation of FACTS family based on voltage source converter – STATCOM.

3.2 Power grid with STATCOM

Generally STATCOM can be modeled as a regulated voltage source connected to the node through a step-down transformer. The equivalent scheme that illustrates the series combination of STATCOM and step-down transformer is depicted in figure 4.

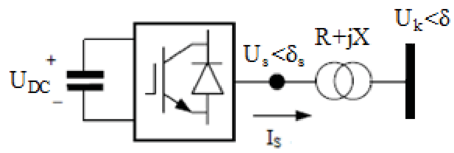

Figure 3: STATCOM equivalent scheme.

The real part of the impedance (figure 3) represents the losses of the coupling transformer, while the imaginary part of the impedance is identified with leakage reactance of the step-down transformer. The losses of the power electronic devices were neglected.

The series impedance calculated for the power network depicted in figure 1 is given as:

$$Z_T = 2{,}54 + j55{,}55\ \Omega$$

The nominal voltage at low-voltage bus of coupling transformer is 35 kV. The mismatch reactive power equation at the low-voltage bus of step-down transformer where STATCOM is installed is presented bellow.

$$\omega Q_S = -Q_S - k_T^2 \cdot U_S^2 \cdot B_S + k_T \cdot U_S \cdot U_2 \cdot G_{2S} \cdot$$
$$\cdot \sin(\delta_S - \delta_2) - k_T \cdot U_S \cdot U_2 \cdot \cos(\delta_S - \delta_2)$$

where k_T is the turn ratio of the coupling transformer, Q_s is the injected or absorbed reactive power by the STATCOM.

The linearised STATCOM equations are given as:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} \frac{\partial P}{\partial \delta_S} & \frac{\partial P}{\partial U_S} & \frac{\partial P}{\partial \delta_k} \\ \frac{\partial Q}{\partial \delta_S} & \frac{\partial Q}{\partial U_S} & \frac{\partial Q}{\partial \delta_k} \end{bmatrix} \quad (20)$$

Similarly to SVC the STATCOM was set to operate in both the inductive and capacitive modes. The necessity to reduce the nodal voltage magnitude at bus "2" to 110 kV gives the results included in the table 6.

Parameter	Bus number		
	1	2	3
U	110,960 [kV]	110 [kV]	32,2 [kV]
δ	-2,009 [°]	-2,186 [°]	-2,188 [°]

Table 6: Power flow results for STATCOM inductive operating mode.

The amount of reactive power absorbed by STATCOM in this operating mode is equal to SVC's and constitutes 11, 135 MVAR.

The Σ matrix that refers to the power grid with STATCOM is given bellow.

$$\Sigma = \begin{pmatrix} 16355 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3066 & 0 & 0 & 0 & 0 \\ 0 & 0 & 697 & 0 & 0 & 0 \\ 0 & 0 & 0 & 32,10 & 0 & 0 \\ 0 & 0 & 0 & 0 & 10,14 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3,83 \end{pmatrix}$$

The necessity to increase the nodal voltage magnitude at bus "2" to 110 kV gives the results included in the table 7. The amount of reactive power injected by STATCOM in the capacitive operating mode is equal to SVC's and constitutes 12, 31 MVAR.

Parameter	Bus number		
	1	2	3
U	112,144 [kV]	112 [kV]	37,6 [kV]
δ	-2,121 [°]	-2,469 [°]	-2,471 [°]

Table 7: Power flow results for STATCOM capacitive operating mode.

The minimal entry of Σ matrix is $\sigma_6 = 4{,}705$ for capacitive operating mode of STATCOM.

4. CONCLUSIONS

An original technique of weak spots detection was analyzed and efficient ways of their strengthening were proposed. The sensitivity matrix dimension that refers to the power grid with STATCOM has increased due to grid topology change. That's why it is difficult to compare the effect of these FACTS devices with reference to minimal singular value. One can be said their effect is similar according to the same amount of reactive power absorption/injection to maintain the desired nodal voltage magnitude at the bus. The minimal singular value increases in capacitive regime for both devices. This leads to the idea that capacitive regime of FACTS devices connected in derivation is eminently suitable for nodes strengthening, thereby grid systems sensitivity decrease.

References

[1] Gamm, A., Golub, I. *Sensorî i slabâe mesta v electroenergheticeschih sistemah.* SĂI SO RAN, Ircutsc, 1996;

[2] Licev, P., Fedin, V, Electriceskie sistemi i seti. Reshenie practiceskih zadach. Design Pro, Minsk, 1997;

[3] Voitov, O., Voropai, N., i drugie. Analiz neodnorodnostei electroenergeticeskih sistem. Novosibirsk, Nauka, 1999;

[4] Golub, J., Van Loun, C. Matricinâe vâcislenia. Izdatelistvo „Mir", Moscva, 1999;

[5] Cocichin, V. Upravleaemâe staticeschie ustroistva compensații reactivnoi moşinosti dlea linii electroperedaci. Electricestvo, №9, 2000;

[6] Eremia, M., Trecat, J. Reseaux electriques, aspects actuels. Editura Tehnică, Bucureşti, 2000;

[7] Acha, E., Claudio r. Fuerte-Escquivel, Hugo ambrez Perez, Angeles-Camacho, C. *FACTS Modelling and Simulation in Power Networks.* John Wiley & Sons Ltd. The Atrium, Southern Gate, Chichester, West Sussex, PO19 8SQ, England, 2004;

[8] Râjov, I., Moti Birjandi, A. Rejimnâe haracteristichi linii peremennogo toka s vectornâm regulirovaniem. Electricestvo, №9, 2005;

[9] Xiao-Ping, Zhang, Rehtanz, C. Flexible AC Transmission Systems: Modelling and Control. Springer – Verlag Berlin Heidelberg, 2006;

[10] Mohamed, Osman Hassan, Cheng, S. J., Zakaria Anwar, Z. *Steady – State modelling of SVC and TCSC for power Flow Analysis.* Proceeding of the International Multiconference of Engineers and Computer Scientists 2009 Vol. II IMECS, Hong Kong, 2009.