ESTABLISHMENT OF THE OPTIMAL CONSTRUCTIVE DIMENSIONS FOR THE ASYNCHRONOUS MOTORS WITH ROTOR IN CAGE

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Abstract **– In our days, the designers want to find answers to the current questions, by adopting solutions in concordance with the global tendencies, using different instruments. So, we can define the optimal design for the asynchronous motors as the process of selection of the machines which respond correctly to the imposed criteria. The optimal design solves the actual problems of the producers of electrical machines like: reducing the consumption of energy, obtaining of superior operating characteristics and also, perspective demands regarding the diminishing of the operating cost. It analyzes the optimal design of induction motor having regard as the main criterion minimum total cost, considering all the 10 variables: D –machine diameter; ; N_{c1}, N_{c2} – number of stators slots, respective of rotors; $\beta_{c1}=b_{c1}/t_1$ – form factor of the stator slot; b_{01}, h_{01} – the width and height of the isthmus of stator slots; $\beta_{c2}=b_{c2}/t2$ – form factor of the rotor slot; b_{02}, h_{02} – the width and height of the isthmus of rotor slots; $\beta_i=b_i/h_i$ – form factor of the shorting ring. The advantages of the optimization are seen by presenting simultaneous of the three variants of analyzed motors: V_m –real variant of motor; V_o – optimal variant (with the minim total cost), V_{def} – unfavorable variant (with maximal total costs). Thus: to the optimized motor reported to the real one, the total cost decreased by 9.04%, and if compared with unfavorable variant (likely), it follows a decrease of 26.5%. Because in use there are many motors of this type, the savings which are made could be significant.**

Keywords: *asynchronous motors, optimal design.*

1. INTRODUCTION

Researchers and designers are concerned with the decrease of polluting substances that alter the quality of air, water and soil by: adequately using resources, waste reduction; rational use of the energy; results dissemination. In the design of an electric machine, the first stage has to be analysis of the impact on the environment. In order to solve the problem of power consumption, actual actions are necessary for stimulating optimal design, technological development and implementation studies for the new technologies [2], [3], [6], [10].

The paper shows that the machine savings are mainly influenced by the cost of energy losses during operation depending on the choice of electromagnetic loads, the quality of electro-technical materials used and the values of constructive sized adopted.

2. ASPECTS TAKEN ACCOUNT TO THE OPTIMAL DESIGN OF THE ASYNCHRONOUS MOTORS

2.1. Mathematic pattern and optimization criterion

The mathematic pattern includes calculation and dimensioning relations, provided by the expert literature and accepted as precision level, tables with conductors standardized values, magnetization curves, etc, under the form:

$$\begin{aligned} h_i(x_1, x_2, \ldots, x_n) &= 0, \quad i = 1,2,\ldots,l \\ g_j(x_1, x_2, \ldots, x_n) &\le 0 \quad j = 1,2,\ldots,m \end{aligned} \tag{1}$$

where x_i , i=1, 2, 3,.. n – variables described next.

The restrictions required for certain sizes during the design stage shall be verified after their calculation and in case of failure to comply with their mathematic pattern, it is performed again for other values assigned to variables.

In the paper, the term of constructive solution means that machine that meets the following requirements: it complies with the technical and economic requirements established by the beneficiary through the theme; materials mainly from the country are used in the construction of the machines; technologies are those existing in the current fabrication.

Based on the studies performed, it was established that in case of required electromagnetic loads, constructive sizes play an important role in designing asynchronous machine with major implications on operation characteristics and optimization criteria.

2.2. Objective function and its restrictions

As *an objective function* for the optimal design of asynchronous machine, the criterion C_t – total minimum cost is established,

$$f(\bar{x}) = C_t = C_f + C_e \tag{2}$$

where: C_f –manufacturing cost, C_e –power losses cost in operation.

The exact value of C_f is difficult to establish because it includes the cost of active materials used C_{ma}, manpower, power consumption, various divisions specific to the production process. This is why we propose an estimation for the fabrication cost in relation to the cost of active materials through a factor, k_f, determined in similar asynchronous machine,

$$C_f = k_f C_{ma} \tag{3}$$

C_e cost shall be calculated through the losses and their related cost price.

$$C_e = N_{ou} C_{el} T_{ri} \Sigma p \tag{4}$$

where: N_{ou} –the number of operation hours of the engine during a year; C_{el} –the cost of a kWh of power; T_{ri} – investment recovery time; Σp -total losses in the machine during the operation in rated load.

At the optimal design of asynchronous machine, *rated load* shall mean the minimum of the objective function, $f(\overline{x}) = \min$. The achievement of an optimum solution requires the use of an adequate searching method related to the available calculation facilities.

This is why the calculation volume is very high and efforts are still made for reducing the working time.

The objective function restrictions are determined by the terms required from weight sizes, those specific to the design of asynchronous machine.

The main variables appear in the expression of the objective function, optimization is made in relation to them [3], [9], and their number is established from the condition of achieving an accurate mathematic pattern and from reducing working time. For an accurate optimization 12 variables were established, being constructive sizes: D – machine diameter; N_{c1}, N_{c2} – the number of slots in the, respectively rotor; $\beta_{c1}=b_{c1}/t_1$ – reported width of the stator slot; b_{01}, h_{01} –width and height of the stator slot isthmus; $\beta_{c2}=h_b/b_b$ –relative height of the rotor bar; b_{02}, h_{02} –width and height of the rotor slot isthmus; $\beta_i=b_i/h_i$ –ratio between short-circuit ring sizes. Based on the indications from the expert literature intervals are established in which these independent variables take values. The other variables used in the mathematic pattern shall be determined according to the literature and represent the category of *auxiliary variables*.

The paper proposes to perform a broader study in order to establish the optimal values of constructive sizes, considering the aforementioned criterion: minimum total cost of the engine.

2.3. Calculation of the objective function minimum

This paper considered the *method of going through spatial knots (exhaustive exploration method).*

The minimization of the objective function is required $f(\overline{x}) = C_t$,

$$\begin{aligned} C_t = k_f(c_{Fe1}m_{Fe1} + c_{Fe2}m_{Fe2} + c_{Cu1}m_{Cu1} + c_{Cu2}m_{Cu2}) + \\ + N_{ou}C_{el}T_{ri}(p_{Fe} + p_{sFe} + p_{Cu1} + + p_{Cu2} + p_{sCu} + p_{mec+v}) \end{aligned} \tag{5}$$

Depending on the following variables:

$$C_t = f(D, N_{c1}, N_{c21}, \beta_{c1}, b_{01}, h_{01}, \beta_{c2}, b_{02}, h_{02}, \beta_i) \tag{6}$$

The relation (4) c_{Fe1}, c_{Fe2}, c_{Cu1}, c_{Cu2}, includes costs for stator and rotor iron, respectively for copper, m_{Fe1}, m_{Fe2}, m_{Cu1}, m_{Cu2}, and related masses. The objective function goes through the restrictions system on variables, and the restrictions system specific to electric machines.

In relations (5), (6) notations used have well-known meanings in the expert literature.

We next briefly describe the stages of this algorithm adapted to the optimal design of electric machines in order to solve the minimization of the function $f(\overline{x}) = C_t$.

The method supposes the calculation of the objective function in the network slots and successive elimination of the slots where the function has maximum value. The following working stages are performed:

a) the search step is established for every direction (ΔD, ΔN_{c1}, ΔN_{c2}, $\Delta\beta_{c1}$, Δb_{01}, Δh_{01}, $\Delta\beta_{c2}$, Δb_{02}, Δh_{02}, $\Delta\beta_i$) with relations of the form:

$$\Delta D = \frac{D_{\max} - D_{\min}}{n_D} \qquad \Delta N_{c1} = \frac{N_{c1\max} - N_{c1\min}}{n_{Nc1}} \; ..... \tag{7}$$

where, n_D, n_{Nc1}, n_{Nc2}, $n_{\beta c1}$, n_{b01}, n_{h01}, $n_{\beta c2}$, n_{b02}, n_{h02}, $n_{\beta i}$, n_{Nhj1}, n_{Nhj2}, are the number of intermediary points for every search direction, conveniently chosen in relation to the desired precision;

b) "spatial slots" are established as points: $P_1, P_2, P_3, .., P_N$ where the value of the objective function will be calculated;

c) search will start from the minimum point:

d) passing from one searching point k, to another, is made by altering only one component of the searching point. This provides an ordered and easy to follow character to the search.

e) After the evaluation of the objective function in all the slots of the network we get a *minimum global calculation*, which is *the real minimum of the function:*

$$f(\overline{x}_m) = \min[f(\overline{x}_1), f(\overline{x}_2), ..., f(\overline{x}_N)] \tag{8}$$

The total number of searching points, therefore evaluations of the objective function shall be determined with the relation:

$$N = (n_D + 1)(n_{Nc1} + 1)(n_{Nc2} + 1)...(n_{h01} + 1) \tag{9}$$

3. RESULTS AND CONCLUSIONS

For exemplification we used a three phase high voltage asynchronous engine with normal construction short-circuit rotor having: rated data: P_N= 250 kW – rated power; U_N=6 kV –rated voltage; n_1=1000 rot/min – synchronism speed; starting features: m_p=1,1 –specific starting torque; i_p= 6,1 –specific starting current; the following weight restrictions: l_{max} <924 mm –maximum length; D_{max}<845 mm –maximum exterior diameter, and for costs determination, based on the documentation, we considered: N_{ore}=365*14=5110 hours/year – the number of operation hours per year; T_{ri} =15 years –investment recovery time; c_{Cu} =95 E/kg –the cost of a kg of copper; c_{Fe}= 25,1 E/kg –the cost of a kg of iron (cherty sheet); c_{enel} =0,095 E/kWh –the cost of a kWh of power.

All the engine related sizes resulted from a traditional design (known from the specialized literature) are further considered as values of reference (relation).

For instance: $\cos\varphi_m$=0,84; η_m=0,925; $M_{max.m}$=2,424*M_N; $M_{p.m}$=1,061*M_N; $I_{p.m}$=5,264*I_N; A_m=494 A/cm ; B_m=0,82 T; $J_{1.m}$=6,2 A/mm^2; $J_{2.m}$=4,5A/mm^2; $C_{f.m}$=106400E; $C_{e.m}$=117900E; $C_{t.m}$=224300E.

The study performed and described next considered for every analyzed variation (D, N_{c1}, N_{c2}, β_{c1}, b_{01}, h_{01}, β_{c2}, b_{02}, h_{02}, β_i, h_{J1}, h_{J2}), a variation within limitations -15%, respectively +15% compared to the value of reference known, calculated according to the design method in the expert literature.

For instance for the diameter of the machine, we have:

$D=[D_{min} \div D_{max}]=[0{,}85 \div 1{,}15]D_m=[382 \div 517]$ mm

At the optimal design of the asynchronous engine we consider the optimization (minimization) of the total cost function, $C_t = f(\overline{x})$, simultaneously following the evolution of other important markers (lower weight criteria): costs (C_f, C_e –fabrication and operation cost); starting characteristics (m_p, i_p, m_m –starting torque and current, maximum torque).

In order to easily follow the influence of every variable on the aforementioned criteria, graphic representations are given in relative units.

3.1. Analysis of the cost criteria

In order to determine the influence of every variable on the established optimization criteria, at first optimizations by only one variable were made, being able to determine the percentage of the variation limitations for every cost.

Fig. 1.a, …, fig.1.1 describe the variation curves for all the costs considered: c_f –reported fabrication cost (red color), c_e-reported operation cost (blue color), c_t-total reported cost (grey color).

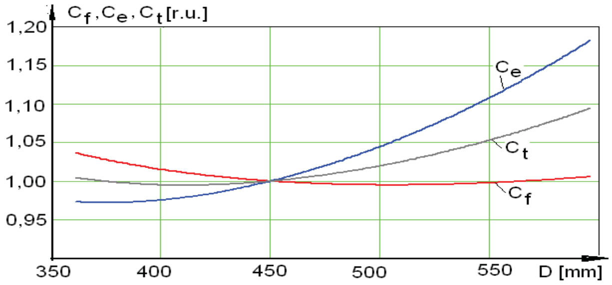


Figure 1.a. Costs variation curves in relation to D – machine diameter.

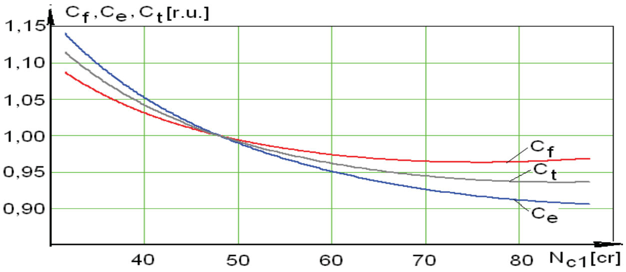


Figure 1.b. Costs variation costs in relation to N_{c1} – the number of slots in the stator.

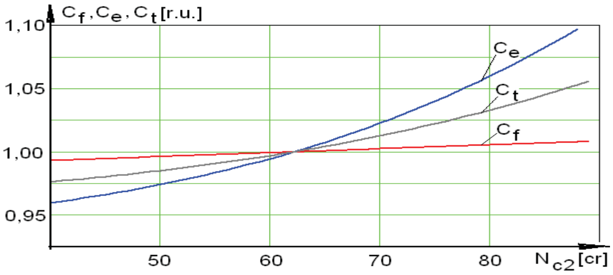


Figure 1.c. Costs variation costs in relation to N_{c2} – the number of slots in the rotor.

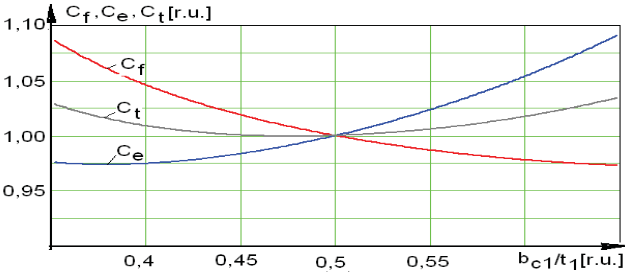


Figure 1.d. Costs variation costs in relation to β_{c1}=b_{c1}/t_1 – the reported width of the slot in the stator.

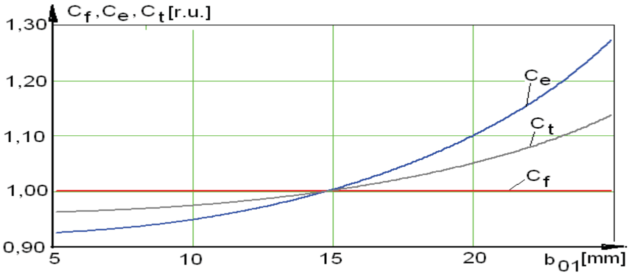


Figure 1.e. Costs variation costs in relation to b_{01} –the width of the stator slot isthmus.

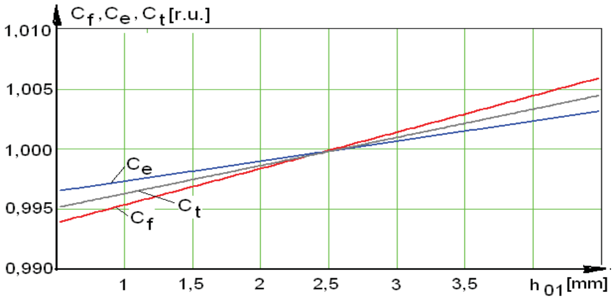


Figure 1.f. Costs variation costs in relation to h_{01} –the height of the stator slot isthmus.

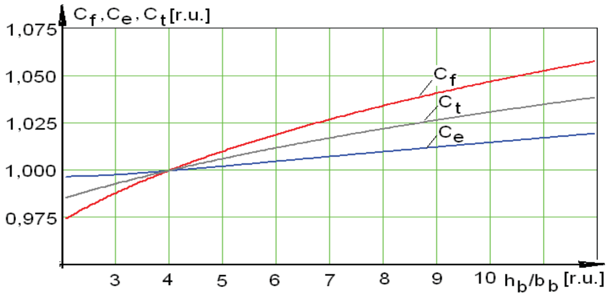


Figure 1.g. Costs variation costs in relation to $\beta_{c2}=h_b/b_b$ –the relative height of the rotor bar.

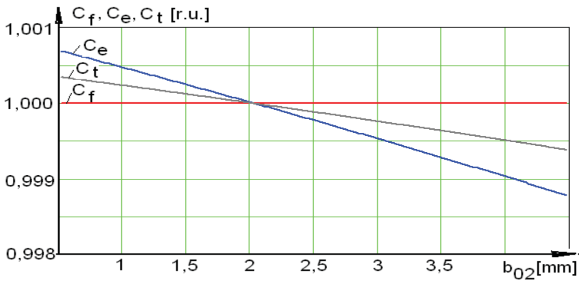


Figure 1.h. Costs variation costs in relation to b_{02} –the width of the rotor slot isthmus.

Analyzing fig.1.a,..fig.1.h and considering the main criterion of total minimum cost, table no. 1 was drawn-up where we notice the influence of every variable.

Table 1

	D	N_{c1}	N_{c2}	β_{c1}	b_{01}	h_{01}	β_{c2}	b_{02}
C_t (%)	11	20	9,5	7	21	1	5,3	0,1

Next, for the most important variables (N_{c1}, b_{01}), resulting from the analysis of table no. 1, all the results of the study are presented, where the other criteria were also considered: C_f , C_e , C_t –fabrication, operation and total cost, D_e –exterior diameter, l_e –total length, m_p, i_p –starting torque and current, k_s –magnetic saturation factor, η –output, cosφ –power factor, m_m –critical torque, m –weights, m_{Cu} , m_{Fe} –copper and iron consumption, p_{Fe}, p_{Fes}, p_{Cu} –iron losses, additional in iron and windings.

3.2. Optimization study considering variable N_{c1} –the number of slots in the stator

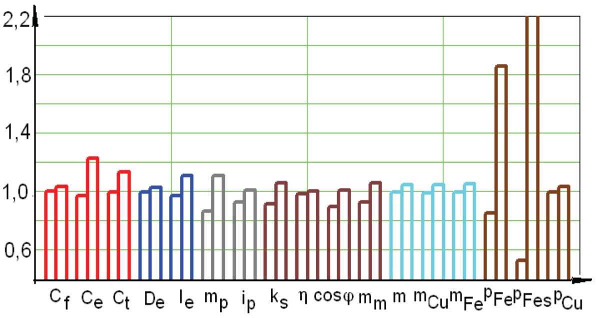


Figure 2. Extreme values for analyzed criteria in relation to variable N_{c1} –the number of slots in the stator.

Table 2

Criterion Achieved values	c_t (u.r.)	c_f (u.r.)	c_e (u.r.)	m_p (u.r.)	i_p (u.r.)	m_m (u.r.)	η_r (u.r.)
V_o optimal value	0,94	0,975	0,91	1,020	0,914	1,015	1,01
V_i initial value	1,12	1,085	1,14	0,855	0,914	0,930	0,99
V_f final value	0,94	0,975	0,91	0,845	0,924	0,945	1,01

When variable N_{c1} takes values in the established interval, $N_{c1}=[N_{c1.min}\div N_{c1.max}]=[0{,}85\div 1{,}15]N_{c1.m}$ slots for the important criteria considered, table no. 2 reveals: V_o –optimal values, V_i –initial values (for $N_{c1.min}$) and V_f –final values (for $N_{c1.max}$).

The increase of the number of slots in the stator, according to fig.1.b has the following effects:

-advantages: it increases the output and power factor, there are optimal values for the maximum and starting torque, it decreases the total length of the machine, decreases weight, iron and copper consumption, all costs decrease, power, main and additional iron losses decrease;

-disadvantages: magnetic saturation and starting current grow.

3.3. Optimization study considering variable b_{01} –the size of the stator slot isthmus

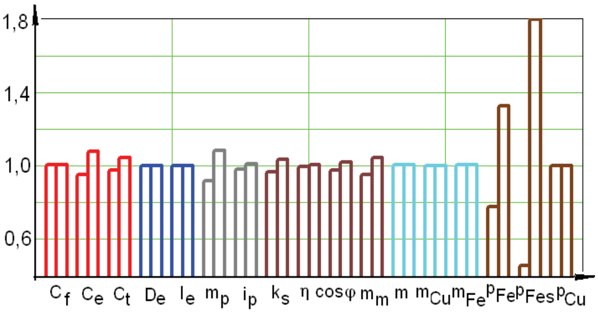


Figure 3. Extreme values for analyzed criteria in relation to variable b_{01} –the size of the stator slot isthmus.

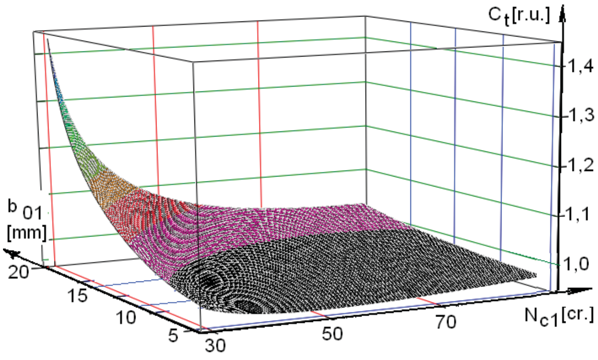
The increase of the isthmus opening at the stator slot, according to fig.1.e has the following effects:
-advantages: increase of the starting and maximum torque, decrease of the magnetic saturation factor, exterior diameter, machine length, fabrication costs remain constant;
-disadvantages: increase of the starting current, decrease of the power output and factor, increase of the operation and total cost, increase of main and additional iron losses.

3.4. Optimization of asynchronous engine design with pairs of variables

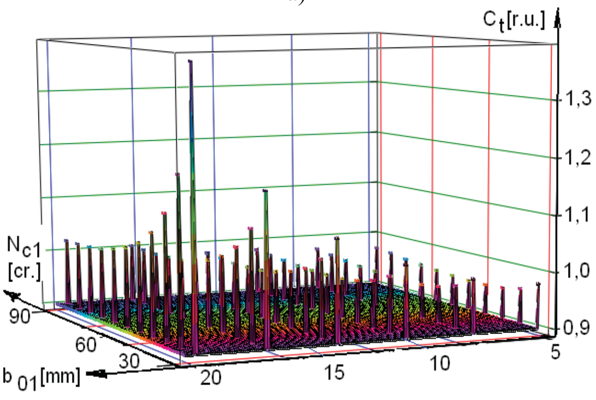
This time de study considered two main variables, with maximum effect on costs, within the same ranges of values. These are: N_{c1} –the number of slots in the stator, which determine a variation of the total cost (final cost without the initial one), $\Delta c_t = (c_{t\,f} - c_{t\,i}) = 20\%$ and b_{01} –the width of the stator slot isthmus for which we get $\Delta c_t = (c_{t\,f} - c_{t\,i}) = 17\%$.

3.4.1. Optimization study in relation to variables N_{c1} and b_{01}

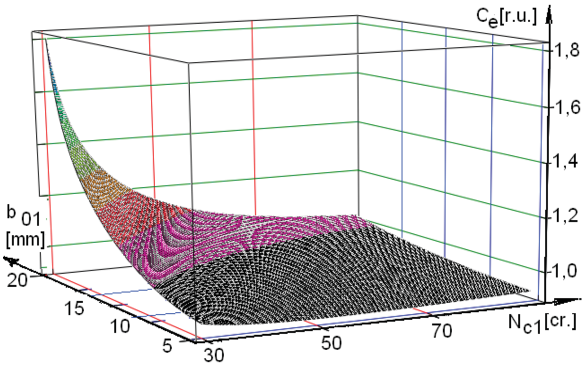
The answer sizes are surfaces in the three-dimensional space, and we can notice that the final result differs from the superposition of individual optimization presented. Next, costs criteria are described: the answer surface in space of the total cost criterion (4.a), the same criterion but for a small number of values assigned to variables (fig.4.b).



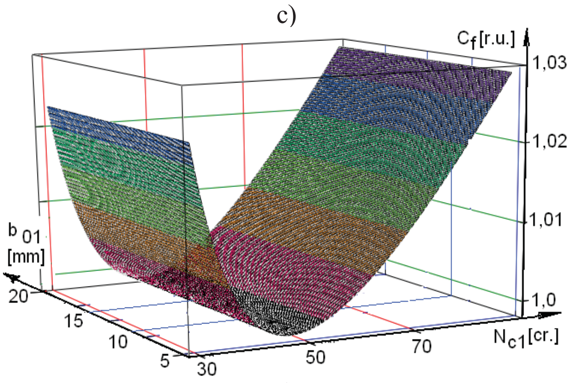
a)



b)



c)



d)

Figure 4. Answer surfaces in space for all analyzed criteria: a) and b) total cost; c) operation cost; d) fabrication cost, considering as variables: the number of slots in the stator and the width of the stator slot isthmus.

3.4.2. Optimization study in relation to all variables

This time, optimal design of the asynchronous engine was made considering the main criterion of **minimal total cost,** considering all the 10 variables: D –machine diameter; ; N_{c1}, N_{c2} – the number of slots in the stator, respectively rotor; $\beta_{c1}=b_{c1}/t_1$ – the reported width of the stator slot; b_{01}, h_{01} –stator slot isthmus width and height; $\beta_{c2}=b_{c2}/t2$ – reported width of the rotor slot; b_{02}, h_{02} – rotor slot isthmus width and height; $\beta_i=b_i/h_i$ – ratio between the sizes of the short-circuit ring. Optimization was made by using the method of exhaustive exploring, where 8 intermediary points were taken for every searching direction, resulting $N_t=8^{12}=6,871*10^{10}$ – variants of calculated machines. For the optimized asynchronous engine (with minimum total cost), related sizes are noted with factor “o”, as follows: m_o=1007 kg – engine weight; $m_{Cu.o}$=92,1 kg –copper consumption; $m_{Fe.o}$=627,5 kg –iron consumption; $M_{max.o}=2,321*M_N$ – maximum torque; $\cos\varphi_o$=0,83 –power factor; η_o=0,932 -output; $C_{f.o}$=97720 E –fabrication cost; $C_{e.o}$=105500 E –operation cost; $C_{t.o}$=203200 E –total cost.
The optimal variant was achieved for the following main variables: D_o=409,5 mm; $N_{c1.o}$=48 slots; $N_{c2.o}$=40 slots; $\beta_{c1.o}=b_{c1}/t_1$=0,592; $b_{01.o}$=5 mm; $h_{01.o}$=0,5 mm; $\beta_{c2.o}=h_b/b_b$=2; $b_{02.o}$=0,5 mm; $h_{02.o}$

3.4.3. Optimization conclusions

In order to reveal the advantages of general use asynchronous engine optimal design, the same calculation programs were used to determine which is the most unfavorable type of engine (maximum total cost), that can be achieved using the method used in literature.

The results of this study may be seen in table no. 4 which simultaneously describes the three types of analyzed machine: V_m –real engine type; V_o –optimal type, V_{def} – unfavorable type. The real values of the analyzed sizes result from the multiplication of the results from the table with the reference sizes of the real engine: $M_{max.m}$=2,424*M_N; $M_{p.m}$=1,061*M_N; $I_{p.m}$=5,264*I_N; $\cos\varphi_m$=0,84; η_m=0,925; $C_{f.m}$=106400E; $C_{e.m}$=117900E; $C_{t.m}$=224300 E. For instance, the maximum torque for the optimal engine type will be:

$M_{max.o}=m_{m.o}*M_{max.m}=0,577*2,424*M_N=1,40*M_N$.

The analysis considered the main criterion of total cost. We notice that from this point of view, in the case of the optimized engine, as compared to the real one, the total cost decreased by:

$\Delta C_{to}=(c_{to}-c_{tm})C_{tm}=(0,906-1,00)*224300=-19100$ E.

and if we compare to the unfavorable type, which is possible:

$\Delta C_{tdef}=(c_{to}-c_{tdef})C_{tm}=(0,906-1,444)*224300=-120700$E

Considering that there are many machine of this type in operation, savings would be important.

Table 3

Criterion Achieved values	c_t (u.r.)	c_f (u.r.)	c_e (u.r.)	m_p (u.r.)	i_p (u.r.)	m_m (u.r.)	η_r (u.r.)	$\cos\varphi$ (u.r.)
V_m -real engine type	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00
V_o -optimal type	0,906	0,918	0,895	0,88	1,07	0,98	1,01	0,89
V_{def} -type unfavorable	1,444	1,462	1,428	0,91	0,61	0,58	0,97	0,78

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