A MODIFIED PETRI NET FRAMEWORK FOR HYBRID SYSTEM MODELING

Mircea Adrian DRIGHICIU, Anca PETRIŞOR

University of Craiova, Department of Engineering in Electromechanics, Environment and Industrial Informatics, adrighiciu@em.ucv.ro, apetrisor@em.ucv.ro

Abstract – **In order to achieve the framework for analysis of hybrid dynamic systems, different approaches of modeling are used and at present is already an abundance of such paradigms, including Petri Nets. Often, the continuous dynamic is represented by differential – algebraic equations; on the other hand, the discrete dynamic is modeled by automata or input – output transitions systems, with a finite and countable number of states. This paper focuses on the modeling of hybrid systems with autonomous commutation of the model generated by a hysteresis phenomenon through a particular Petri Nets structures, called Modified Petri Nets (MPN). The main goal of this approach is to get a formal description language for such hybrid systems, which combines the advantages of a graphical description with the possibility of a transparent visualization, simulation and analysis. The proposed concepts are illustrated with a case study, which refers to a classical temperature control process in a room, using a thermostat with anticipative resistance. Starting from a generic mathematical model and using the Visual Object Net ++ software tool, several simulation scenarios was proposed, in order to verifying the Petri Nets structures achieved. The simulation results can be used for building a modular hierarchical structure of the models.**

Keywords: *dynamics, Petri nets, control process, , simulation, switched systems.*

1. INTRODUCTION

A dynamical system is generally considered a hybrid structure if it is difficult to deal with it either as a purely continuous-variable system or as a purely discrete-event system without ignoring important phenomena that result from the combination of continuous and discrete movements of this system. This situation is due to the fact that the theories of continuous and discrete systems have been elaborated completely separately until recently. Hybrid systems pose the problem of bridging the gap between both theories. This has been done until now not only by considering a combination of continuous and discrete subsystems but also by investigating different extensions of either continuous or discrete systems. Hybrid dynamical systems generate variables or signals, that are mixed signals consisting of combinations of continuous or discrete value or time signals, and through them interaction with other systems and the environment occurs. More specifically, some of these signals take values from a continuous set (e.g. the set of real numbers) and others take values from a discrete, typically finite set (e.g. the set of symbols {α, β, γ}). Furthermore, these continuous or discrete-valued signals depend on independent variables such as time, which may also be continuous or discrete. Another distinction that could be made is that some of the signals could be time-driven while others could be event-driven in an asynchronous manner [1], [2], [3], [16].

A hybrid system is a dynamical system that cannot be represented and analyzed with sufficient precision either by the methods of the continuous systems theory or by the methods of the discrete systems theory. It is known that continuous systems theory assumes that the system under consideration can be described by some differential equation:

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{u}(t), t),\ \boldsymbol{x}(0) = \boldsymbol{x}_0 \tag{1}$$

$$\boldsymbol{y}(t) = \boldsymbol{g}(\boldsymbol{x}(t), \boldsymbol{u}(t), t) \tag{2}$$

where $\boldsymbol{x} \in \mathrm{R}^n$ is the state vector, $\boldsymbol{u} \in \mathrm{R}^m$ the input vector and $\boldsymbol{y} \in \mathrm{R}^r$ the output vector. $\boldsymbol{x}_0$ denotes the initial state. More generally, (1) can be replaced by a set of difference and algebraic equations, which then is called a differential-algebraic system (DAE system) [16], [17], [20], [21].

The key assumption of continuous systems theory concerns the fact that the functions $\boldsymbol{f}$ and $\boldsymbol{g}$ satisfy a Lipschitz condition. With respect to the state **x** this smoothness assumption means for the function $\boldsymbol{f}$ that a constant L has to exist for which the inequality:

$$\left\| \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}, \mathrm{t}) - \boldsymbol{f}(\hat{\boldsymbol{x}}, \boldsymbol{u}, \mathrm{t}) \right\| \leq L \cdot \left\| \boldsymbol{x} - \hat{\boldsymbol{x}} \right\| \tag{3}$$

holds for all $\boldsymbol{x}, \hat{\boldsymbol{x}}$, $\boldsymbol{u}$ and t, where $\| \cdot \|$ symbolises a vector norm. A similar condition should be satisfied with respect to $\boldsymbol{u}$. Under this assumption, uniqueness and existence results can be derived for the solution of the differential equation (1). Furthermore, many analysis methods assume the property (3).

In the paper, we are interested on a class of hybrid dynamical systems with commutation. An abrupt change of the vector field $\boldsymbol{f}$ if the state $\boldsymbol{x}$ reaches a given bound is called switching. Formally, the system can be represented by two or more different vector fields $\boldsymbol{f}_q$ together with conditions that describe the validity of these vector fields, for example by:

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) \tag{4}$$

with:

$$\boldsymbol{f} = \begin{cases} \boldsymbol{f}_1(\boldsymbol{x}) & \text{for} \quad \boldsymbol{h}(\boldsymbol{x}) \le \boldsymbol{0} \\ \boldsymbol{f}_2(\boldsymbol{x}) & \text{for} \quad \boldsymbol{h}(\boldsymbol{x}) \ge \boldsymbol{0} \end{cases} \tag{5}$$

If the system is currently described by the vector field $\boldsymbol{f}_1$ and the state reaches the border $\boldsymbol{h}(\boldsymbol{x}) = \boldsymbol{0}$ of the region of validity of this vector field, the vector field switches to $\boldsymbol{f}_2$ which is valid until the border described by $\boldsymbol{h}(\boldsymbol{x}) = \boldsymbol{0}$ is reached from the other side.
In order to do a unitary conception of the hybrid systems representation, different approaches of modeling are used and at present there is already an abundance of such models.
In fact, the modeling of hybrid systems needs a combination of description methods for discrete systems and for continuous systems. The classical timed Petri Nets approach with its discrete state space is well suited for the field of discrete systems, but not for continuous systems. For the field of continuous systems, the continuous Petri Nets approach [5], [6], [7], [10], [12], [13], [14], is useful because it offers a continuous sate space. The combination of a discrete and a continuous state space is a main condition for the hybrid structures modeling. Frequently in hybrid systems, the event-driven dynamics were studied separately from the time-driven dynamics via automata or Petri nets models or via differential or difference equations.
Hence, if it is possible to describe the behavior of continuous systems with continuous Petri Nets and then to combine these models with the discrete world of common timed Petri Nets, we would be able to model the complex behavior of hybrid dynamical systems using a single graphical or analytical formalism [6], [8], [9], [10], [16], [21].

2. REVIEWS ON HYBRID PETRI NETS

In a discrete Petri Net (PN), the marking of a place may correspond either to the Boolean state of a device, or to an integer. A general analysis method is to compute the set of reachable states of the model and deduce the different structural and behavioral properties of the system. But when the Petri net places contain a large number of tokens, the number of reachable states explodes, limiting the use of model. This observation led us to define continuous and hybrid Petri nets.
An autonomous HPN is a sextuple Q = {P, T, **Pre**, **Post**, $\mathbf{m}_0$, h} such that: P = {P_1, P_2, …P_n} is a finite, not empty, set of places; T = {T_1, T_2, …, T_m} is a finite, not empty, set of transitions; $P \cap T = \varnothing$ (P and T are disjointed); h - called "hybrid function" indicates for every node whether it is a discrete or a continuous node; **Pre** : $P \times T \rightarrow R^+$ or N^+, is the input incidence mapping; **Post** : $P \times T \rightarrow R^+$ or N^+, is the output incidence mapping and $\mathbf{m}_0 : P \rightarrow R^+$ or N^+ - the initial marking of the net [5], [6].
The basic model of a non-autonomous HPN consists in a combination between non – autonomous discrete and continuous sub-models. So, generally speaking, the discrete transition in a non-autonomous HPN may be fired as the transition in a discrete PN (i.e. they may be synchronized, or timed with constant or stochastic timings). Similarly, the continuous transitions in a non-autonomous HPN may be fired with a flow rate, as transitions in a continuous PN (they can be synchronized, or maximal speeds may be constant, or function of time, or function of the marking) [12], [15].
Informally, there are two parts in a hybrid Petri net, a discrete part and a continuous part, and these parts are interconnected thanks to arc linking a discrete node (place or transition) to a continuous node (transition or place), (Fig.1). In some cases, one part can influence the behavior of the other part without changing its own marking. In other cases, the firing of a D – transition can modify both the discrete and the continuous marking.

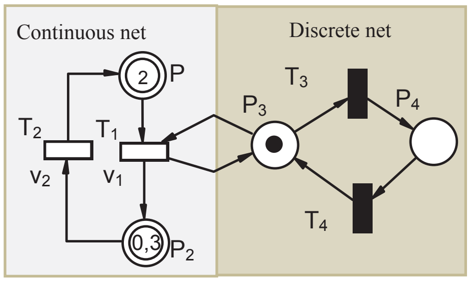

Figure1: A Hybrid Petri net model.

Continuous places of the model are P_1 and P_2; the continuous transitions are T_1 and T_2, the discrete places – P_3 and P_4, and the discrete transitions – T_3 and T_4. Starting from the initial marking (initial conditions) of the model and firing T_1 with a constant flow (firing speed – v_1) 0.1 another marking (state of the system) in reached - (2.4, 0.4, 1, 0). A marking quantity 0.1 has been removed from P_1 and P_3, which are input transitions, and the same quantity has been added to P_2 and P_3 - output transitions.
The HPN, as were defined are not sufficient to model hybrid systems because they do not cover the possibility of modeling the dynamic behavior of continuous systems. Since continuous Petri Nets allow the modeling of real values, they can be used to model continuous state variables only. Therefore, a

powerful extension of basic formalism, called MPN allows introducing several enhancements:
- the firing speed of continuous transitions can be given as a function of token quantities, opening the possibility of modeling the behavior of continuous dynamics, due to fact that the values of this function can become positive as well as negative;
- the token quantity of continuous places can take as well positive as negative values for modeling positive as well as negative continuous system variables, whereas HPN only allow positive values.

Usually, a continuous system is described by its input, output and by its system behavior. Using a MPN model (Fig.2), the input and output variables are each described with a continuous place. The transition T_1 is always active and the system behavior is described with the firing speed function (flow rate) $v = f(u, y)$ depending on P_1 and P_2 marking. Moreover, due to P_1 – T_1 test arc (dotted line represented) which not allows the modification of the P_1 marking quantity, during the dynamic behavior of the model the token quantity of P_1 is not influenced [8], [14], [16].

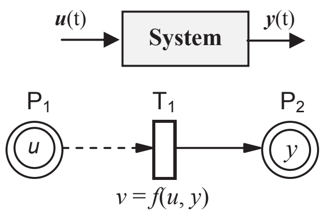

Figure 2: Continuous basic element of MPN model.

If a single input arc is directed to P_2, we get:

$$v = \frac{dy(t)}{\mathrm{d}t} = \dot{y}(t) \qquad (7)$$

For a general place P_j with the marking m_j and i input arcs we get:

$$\dot{m}_j(t) = \frac{dm_j(t)}{dt} = \sum_i v_i \qquad (8)$$

In the model behavior, continuous input and output transitions supply their part to increase or to decrease m_j. We can model different basic elements in this way. Even non-linear coupled subsystems can be described since v may be a non-linear function.

The main property of hybrid systems is the interaction between discrete and continuous subsystems. This interaction is also the main reason that classical methods of description are not sufficient in order to model such systems. So, the use of MPN allows to representing various interactions between continuous and discrete subnets. In most cases, the supervisory control of continuous processes is represented by discrete sub – nets which can influence the flow of continuous transitions in accordance with the firing rules [4], [11], [19], [20].

3. CASE STUDY

In order to illustrate the resources of HPN formalism for hybrid systems representation and analysis, a model for the control process of temperature during the heating of a room, using a bi–positional thermostat with anticipative resistance was proposed.

3.1 Physical System Structure

Fig. 3 illustrates the structure and the behavior of the system [14]. During the temperature control process, for different fluctuations against the reference value of the room temperature, the environmental thermostat switches between “on” and “off” positions and the anticipative resistance is connected into the circuit, bringing the heating of the bi – metal. In the mean time, the relay -7 turns on the supply valve - 4 and the gas – generating station is started. When the temperature rises and become greater than reference value, the anticipative resistance is disconnected.

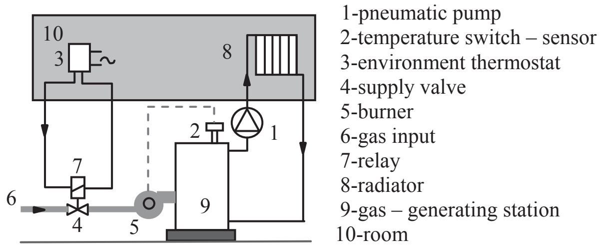

Figure 3: Physical system structure.

In a qualitative representation of the control process (Fig.4), discrete variable „q” reaches two values (i.e. 1 and 0) according with one or other of steps θ_{t1} and θ_{t2}, considered as lower, respectively upon limits of the room temperature.

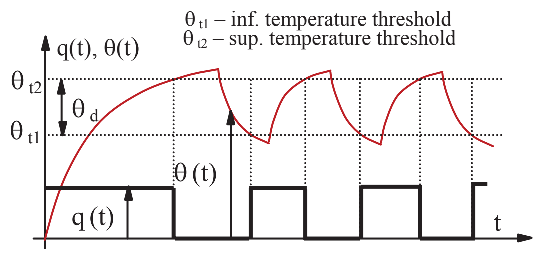

Figure 4: Illustrative for temperature control process.

Temperature fluctuations induced by thermal inertia of the device can be reduced using one anticipative resistance R. Hence, the exceeding of θ_d value (Fig.4) is restricted and the thermostat switches “off” before the reference value of temperature being reached. In this way, the temperature oscillations can be diminished and, in the meantime, releasing frequency of the device can be increased.

3.2 Mathematical Model

The MPN model of the process was synthesized with a hybrid technique starting from a simplified mathematical model, established according to the Fourier's low of the heating process, supposing a proportional dependence between the heat flows and gradients of the temperature. Thus, room temperature variation – θ can be expressed from a linear dependence between temperature values of the external environment - θ_e and the radiator - θ_r [14], [18]:

$$\frac{d\theta}{dt} = -c_1(\theta - \theta_e) + c_2(\theta_r - \theta) \tag{9}$$

Similarly, temperature of the thermostat - θ_{th} depends on heat changed between that device and the room and on the thermal energy – Q_1 produced by the anticipative resistance – R, during it connection to the power supply:

$$\frac{d\theta_{th}}{dt} = -c_3(\theta_{th} - \theta) + q(\theta_{th}) \cdot Q_1 \tag{10}$$

On the other hand, the temperature variation of the radiator – θr depends on the thermal changing between the radiator and the room and, also, on the thermal energy – Q2 produced by radiator itself:

$$\frac{d\theta_r}{dt} = -c_4(\theta_r - \theta) + q(\theta_{th}) \cdot Q_2 \tag{11}$$

In (9), (10) and (11), c_i (i = 1, …4) denotes the global coefficients of heat transfer, and „q" is a discrete variable who can reaches only two different values (0 or 1), according to the hysteresis thermostat cycle (Fig.5) and controls the starting and the stopping process of the heating system [4], [14], [18].

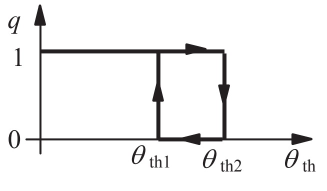

Figure 5: Hysteresis thermostat cycle.

Considering as state vector of the system $\boldsymbol{\theta}^t = (\theta \;\; \theta_{th} \;\; \theta_r)^t$ and as input vector $\boldsymbol{u}^t = (\theta_e \;\; Q_1 \;\; Q_2)^t$, we can represent the simplified mathematical model of the process through a linear system of equations, in accordance to (1).

The switching between the two values of the variable q causes two distinct operated services for the whole process: ON (for q = 1) and OFF (q = 0). The commutation of the hybrid system between its states is released when the state space vector $\boldsymbol{\theta}$ (more precisely it θ_{th} component) reaches for the first time the threshold value θ_{th1} (q = 0 and $d\theta_{th}/dt < 0$), then the other threshold value θ_{th2} (q = 1 and $d\theta_{th}/dt > 0$).

$$\frac{d}{dt}\begin{bmatrix}\theta \\ \theta_{th} \\ \theta_r\end{bmatrix} = \begin{bmatrix}-(c_1+c_2) & 0 & c_2 \\ c_3 & -c_3 & 0 \\ c_4 & 0 & -c_4\end{bmatrix} \cdot \begin{bmatrix}\theta \\ \theta_{th} \\ \theta_r\end{bmatrix} + \\ + \begin{bmatrix}c_1 & 0 & 0 \\ 0 & q & 0 \\ 0 & 0 & q\end{bmatrix} \cdot \begin{bmatrix}\theta_e \\ Q_1 \\ Q_2\end{bmatrix} \tag{12}$$

3.3 Hybrid Petri Net Model of the Process

The Petri Net model achieved starting from the control process of temperature analysis was obtained based on previous observation on the autonomous commutation of the system, due to hysteresis cycle threshold values θ_{th1} and θ_{th2} respectively. Hence, starting from the initial state, until the thermostat temperature becomes θ_{th2}, the behavior of the system is described by (12) with q = 1. Then, during [θ_{th1}, θ_{th2}] interval of temperature, when the anticipative resistance R and the gas – generating station are turned off, the process dynamic can be described by the same mathematical model, for q = 0. Hence, the model switches periodically between its two structures, according to the achievement of θ_{th1} or θ_{th2} temperature values.

Particularities of the mathematical model of the process, considered as a hybrid system representation, and the behavioral particularities due to a permanent switching between its states leads to a Hybrid Petri Net topology for the looking model. The commutation between the continuous space sates values of the system is induced by the occurrence of some external events (commands or disturbances). In order to synthesize easier a basic frame of the model in accordance with the behavior of the process, the main idea was to use all the facilities dues to MPN formalism.

Thus, the topology of the whole framework (Fig.6) contains continuous sub-models (continuous Petri Nets) activated or no by a permanent interaction with a discrete control sub-model (a discrete Petri Net). For the continuous part of the model were used elements of continuous Petri Nets with variable firing speed of continuous transitions, dependent on continuous places marking.

The discrete supervisory control sub–model ensures that only the behaviors consistent with the specification may occur in the system; it can be

synthesized - for example - using T-timed discrete Petri Nets elements [5], [6], [15].
The structure of the model was synthesized starting from the state space model equations (12) according to the modified Petri nets evolution rules generated by particular expressions of variable speeds of continuous transitions.

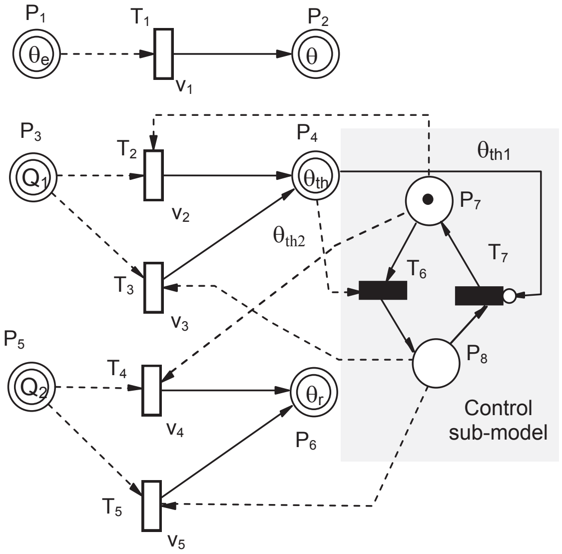

Figure 6: Modified Petri Net model of the process.

The coupled in–and outputs of the system were represented by arcs. The firing speed functions, assigned to the continuous transitions, can be gained line by line from the equation system, from both values (0 and 1) of q variable:

$$v_1 = -(c_1 + c_2)\cdot\theta + c_2\cdot\theta_r + c_1\cdot\theta_e \tag{13}$$

$$v_2 = c_3\cdot\theta - c_3\cdot\theta_{th} + Q_1 \tag{14}$$

$$v_3 = c_3\cdot\theta - c_3\cdot\theta_{th} \tag{15}$$

$$v_4 = c_4\cdot\theta - c_4\cdot\theta_r + Q_2 \tag{16}$$

$$v_5 = c_4\cdot\theta - c_4\cdot\theta_r \tag{17}$$

The initial state of the system was indicate by the initial set of variables values - θ, θ_{th}, θ_r and θ_e, Q_1, Q_2. Thus, the linear equations of the initial mathematical model were represented by continuous sub – nets, composed of $P_1 \div P_6$ places and $T_1 \div T_5$ transitions which allows the system behavior dynamic between commutation processes. The changing of Petri net structure according a commutation process is obtained due to discrete sub – net (control sub – model : P_7, P_8 places and T_6, T_7 transitions), which activates or disables – through test arcs – some of continuous transitions. In this way, the Petri net switches permanently between two similar structures, this behavior been induced thanks to the weight of arcs P_4 – T_6 (test arc with θ_{th2} weight) and P_4 – T_7 (inhibitor arc with θ_{th1} weight) respectively. Specification of all Petri net elements was made according to the hybrid model and, also, according to the evolution rules, due to the specific net formalism: P_1 is assigned with the value of external environment temperature, P_2 – controlled room temperature, P_3 – thermal energy produced by R, P_4 – the current thermostat temperature, P_5 – thermal energy due to the gas – generating station, P_6 – radiator temperature. The marking non – null of P_7 activates the firing of T_2 and T_4 transitions (q = 1) and, similarly, the marking non – null of P_8 activates the continuous firing of T_3 and T_5, for $q = 0$.
The behavior of the process allows – according to switching operations – to enrich two mathematical models, M_1 and M_2, obtained from (12) for the two values of q:

If $\theta_{th} < \theta_{th\,2}$ and $d\theta_{th}/dt \geq 0$

Then M_1 (for $q = 1$)

else

M_2 (for $q = 0$)

Until $\theta_{th} = \theta_{th\,1}$ and $d\theta_{th}/dt < 0$

Even for a less complex topology of Petri Nets, it is difficult to find always the most simply solution to synthesize a compact model, with a minimum number of elements and, also, with goods behavioral and structural properties. Moreover, the model achieved it is not unique. On the other hand, to verify the correctness of the solutions using analytical methods may be a very laborious process, especially for a large number of system states of or when the places marking of the model becomes very great. Therefore, both in the primary stage, when the model is developed and in the other stage, when the analysis of goods properties is started, various dedicated software tools with a friendly graphical user interfaces are used.
Hence, the Petri net model was synthesized and then its behavioral properties were verified by on-line simulation using Visual Object Net++ tool [9], [10]. For a quantitative analysis, the values for global coefficients of heat transfer c_i ($i = 1 \div 4$) and, also, for Q_1 and Q_2 were the same used in [18]. The model allows a very easily modification of all initial variable values. Thus, the reference value of room temperature can be initially adopted and then eventually changed only by setting the threshold values of the thermostat.
Fig.7 shows a set of on-line simulation scenario obtained with Visual Object Net++ tool, using the modified Petri net model of the process, for: $c_1 = 10^{-4}$ s^{-1}, $c_2 = 5\bullet10^{-5}$ s^{-1}, $c_3 = 5\bullet10^{-3}$ s^{-1}, $c_4 = 10^{-3}$ s^{-1}, Q_1 = 0,0132 °C/s, Q_2 = 0,06 °C/s, $\theta_{th\,1}$ = 20 °C; $\theta_{th\,2}$ = 22 °C, θ_e = 15 °C. The threshold values $\theta_{th\,1}$ and $\theta_{th\,2}$ of the thermostat were set so as to ensure a reference room temperature θ = 20 °C.

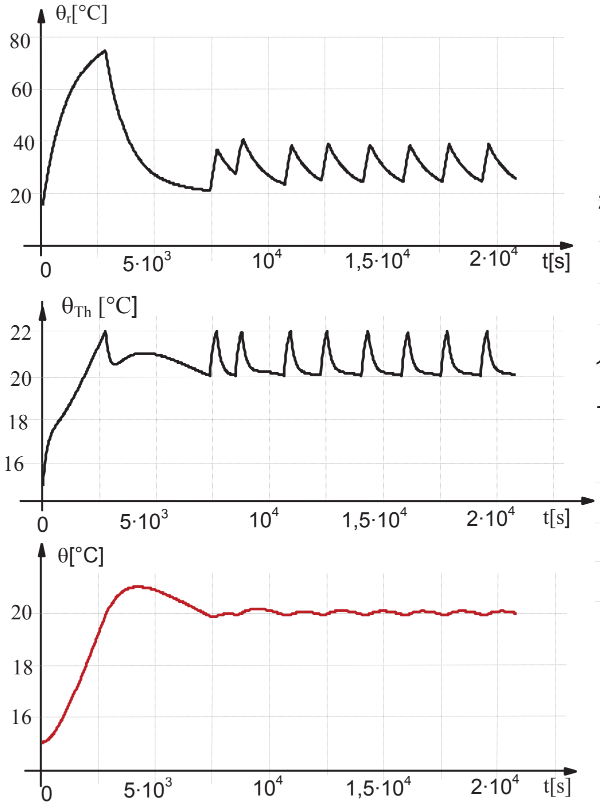

Figure 7: On-line simulation results of the MPN model.

It is important to specify that, in the model structure the effect of external perturbation on whole dynamic process was neglected, the process dynamic being roughly the same whit that of an isolated system. The system response confirms all the observations stated since the initial stage of model development about the great time constant value of the process and about the advantages due to use a thermostat with anticipative resistance for the temperature control.

In order to prove the good properties of the model, various simulation scenarios were considered, using the same MPN topology and setting different environmental initial conditions for many control output values of room temperature (Fig.8, Fig.9).

As noted, the MPN representation of the temperature control process analysis is a modular and flexible structure, providing the opportunity to be refined or modified according with various simulation scenario modifications.

These behavioral changes can be easily caught by adding new elements (places, transitions or arcs) at initial model. Sometimes, the modification of arcs weight only is sufficient for adapt the model to new conditions.

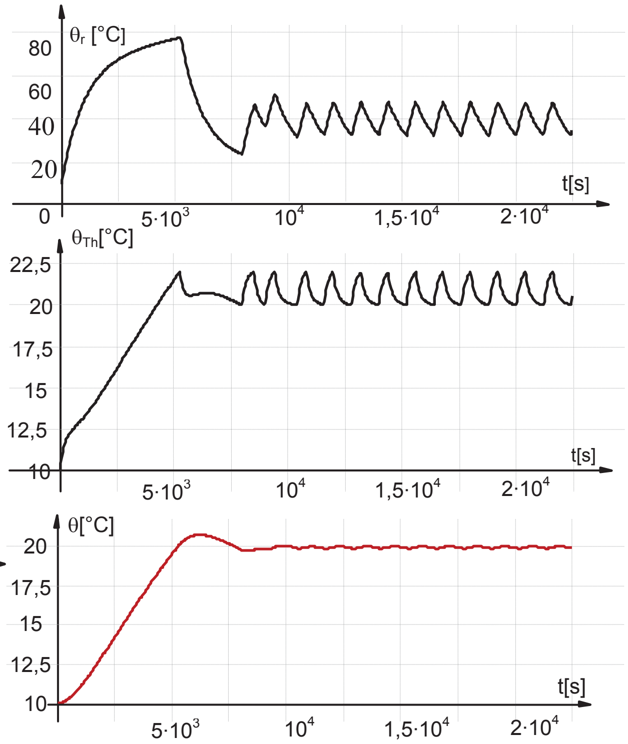

Figure 8: On-line simulation results for c_1=10^{-4}s^{-1},c_2 =5•10^{-5}s^{-1},c_3=5•10^{-3}s^{-1},c_4=10^{-3}s^{-1},Q_1=0.012°C/s,Q_2 = 0,06 °C/s,θ_{th1}=20°C,θ_{th2}=22°C,θ_e=10 °C.

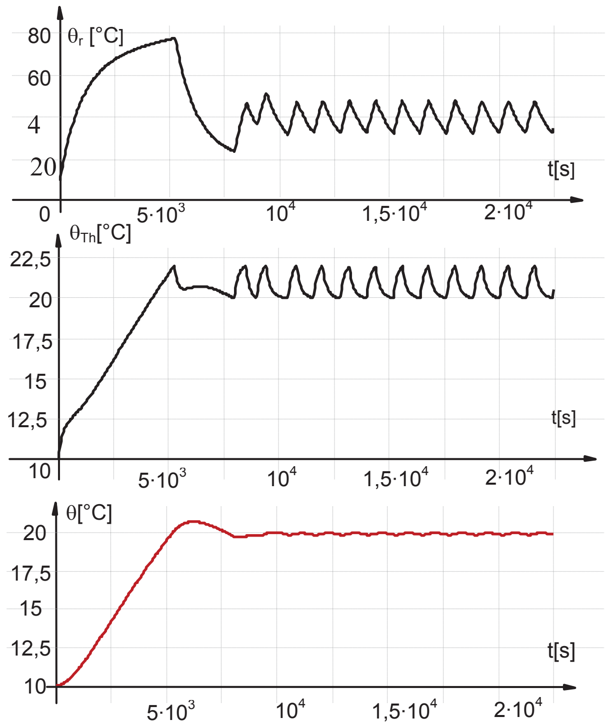

Figure 9: On-line simulation results for c_1=10^{-4}s^{-1},c_2 = 5•10^{-5}s^{-1},c_3=5•10^{-3}s^{-1},c_4=10^{-3}s^{-1},Q_1 = 0.0168°C/s,Q_2 = 0,06°C/s, θ_{th1}=20°C,θ_{th2}=22°C,θ_e = 16 °C.

4. CONCLUSIONS

The main idea of this paper was to consider an temperature control process such a particular hybrid system, with autonomous state commutation and, starting of this point, to find a specific frame for describing his behavior. A mathematical model of such a process has thus to be a hybrid model involving discrete variables (integers or with a domain in a finite set) and continuous variables (real numbers). Both dynamics (discrete and continuous) have to be modeled: a discrete event based dynamics for discrete variables (sequence of operations) and a continuous time dynamics for continuous variables (differential algebraic equations). The general approaches have strong similarities with hybrid automata, but the discrete dynamics is represented by Hybrid Petri Nets in place of automata in order to address in an explicit way resource allocation policies. True concurrency is indeed required and the interleaving semantics of automata based approaches is not sufficient.

Adding new elements, with a grown power of analysis, may enrich the general model proposed. Hence, in the hybrid Petri Net model achieved, a first step for increasing the details was made due to variable delays associated to the discrete transitions. The concept "variable delay" is more often utilized in a determinist way. The delays are in any time defined in occurrence with an external event, a priori estimated, and generated by the decisional structure of the entire system. A major step to refine the initial model can be made using several powerful extinctions of basic Petri Nets models, i.e. stochastic, colored or fuzzy Petri Nets.

In this context, another problem, which leads to interesting results can be formulated is the controller synthesis of hybrid dynamical systems. Briefly, such approach is generally based on three steps: the behavioral description of the system (called open loop system) by a HPN model, the definition of specifications required on this behavior and, finally, the synthesis of the controller, which restricts the model behavior to the required one, using a controller synthesis algorithm. These algorithms use traditionally automata (finite state, timed and hybrid automata) because of their ease of formal manipulation; however, a model like HPN or MPN is preferred in the first step, of behavioral description.

Concluding, hybrid systems represent a highly challenging area of research that encompasses a variety of challenging problems that may be approached at varied levels of detail and sophistication. Also, it is very important to have good software tools for the simulation, analysis and design of hybrid systems, which by their nature are complex dynamical structures.

Acknowledgments

This report is part of the ROMANIAN NATIONAL UNIVERSITY RESEARCH COUNCIL (CNCSIS) contribution to the project PNCDI – II - 289/2008.

References

[1] P.J. Antsalkis, W. Khone, A. Nerode, S. Sastry (Coordinators), *Hybrid Systems II*, Lecture Notes in Computer Science, 999, Springer-Verlag, 1995.

[2] P. J. Antsalkis, J. A. Stiver, M. D. Lemmon, *"Hybrid System Modeling and Autonomous Control Systems"*, Workshop on Theory of hybrid Systems, Lecture Notes in Computer Science 736, Springer-Verlag, pp.336-392, 1993.

[3] M. S. Branicky , *"A Unified Framework for Hybrid Control"*, Proc. 33rd Conference on Decision and Control, Lake Buena Vista, Fl., USA, pp. 4228 – 4234, 1994.

[4] B. Cébron, *"Commande de systèmes dynamiques hybrides"* , Thèse, Université d'Angers, France, 2000.

[5] R. David, H. Alla, "On Hybrid Petri Nets", Discrete Event Dynamic Systems: Theory and Applications, 11, 2001, pp. 9 – 40.

[6] R. David, H. Alla, Discrete, *Continuous and Hybrid Petri Nets*, Springer-Verlag 2005.

[7] Demongodin, N. T. Koussoulas, *"Modelling of hybrid control systems using Petri nets"*, (Zaytoon, 1998), pp.330 – 337, 1998.

[8] R. Drath, *"Objektorientierte Modellierung hybrider Prozesse – Vorstellung eines neuen Werkzeuges"*, IWK , 42, pp. 533 – 540, 1997.

[9] R. Drath, *Tool Visual Object Net ++*, Available: http://www.daimi.au.dk/PetriNets/tools

[10] R. Drath, *"Hybrid Object Nets: An Object Oriented Concept for Modelling Complex Hybrid Systems"*, Dynamical Hybrid Systems, ADPM98, Reims, 1998.

[11] M. A. Drighiciu, *"Petri Net formalism for a possible hierarchical structure of complex hybrid systems"*, Proceedings of 6th International Carpathian Control Conference, Miskolc – Lillafüred, Hungary, pp. 371- 376, 2005.

[12] M. A. Drighiciu, Anca Petrişor, *"Hybrid Systems Modeling Using Petri Nets; A Case Study"*, Buletinul Institutului Politehnic Iaşi, Tomul LII (LVI), Fasc.5A, ISSN:1223-8139, pp.89-94, 2006.

[13] M. A. Drighiciu, Gh. Manolea, D. C. Cismaru, Anca Petrişor, *"Hybrid Petri Nets as a New Formalism for Modeling Electrical Drives"*, Proceedings of International Symposium on Power Electronics, Electrical Drives, Automation

and Motion – SPEEDAM 2008, ISBN: 978-1-4244-1664-6, IEEE Catalog Number: CFP0848A-CDR, Ischia, Italy, 11th – 13th June, pp.626-631, 2008.

[14] M. A. Drighiciu, Gh. Manolea, Anca Petrisor, M. C. Popescu, *"On Hybrid Systems Modeling with Petri Nets"*, Recent Advances in System Science and Simulation in Engineering, Proceedings of the 7th WSEAS International Conference on System Science and Simulation in Engineering (ICOSSSE'08), Venice , 21- 23 November, pp.73-78, 2008.

[15] E. Dubois, *"Continuous Petri net with maximal speeds depending on time*", Proc.Of 15th International Conference on Application and Theory of Petri Nets, Zaragoza, Spain, 1994.

[16] S. Engell, G. Frehse, E. Schneider (Coordinators), *Modelling, Analysis, and Design of Hybrid Systems*, Lecture Notes in Control and Information Sciences 279, Springer-Verlag, 2002, ISBN: 3-540-43812-2, pp.3-36.

[17] R. L. Grosmman, *Hybrid Systems*, Lecture notes in Computer Science, Springer, New York, Vol.736, 1993.

[18] J.-C. Joly, *"Recherche de cycles pour une classe de systèmes dynamiques hybrides à commutation autonomes. Application à un thermostat à résistance d'anticipation"*, Rapport LAAS, CNRS, LISA, Université d'Angers, France, 2004.

[19] C. Valentin-Roubinet, *"Hybrid Dynamic System verification with Mixed Petri Nets*", The 4th International Conference on Automation of Mixed Processes: Hybrid Dynamic System, Dortmung, Shaker Verlag, Aachen, pp.231-236.

[20] Emilia Villani, P. E. Miyagi, R. Valette, *Modelling and Analysis of Hybrid Supervisory Systems; a Petri Net Approach*, Springer-Verlag, 2007, ISBN 978-1-84628-650-6, pp. 1-13.

[21] J. Zaytoon (Coordinator), *Systèmes dynamiques hybrides*, Hermes-Science,2001.