MODELS AND MODELLING THE SUPERCAPACITORS FOR A DEFINED APPLICATION

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Abstract **– Supercapacitors, located in the middle of the energy storage devices' hierarchy, present unique advantages that make them indispensable for applications which require high power delivered in a short time. Establishing the appropriateness of strategies for using these devices is through knowing the supercapacitor's behavior in different loads and its control.**
Starting from the establishment of a mathematical-econometric model that allows the supercapacitor characteristics' optimization and of a basic electrical equivalent model, the paper presents the realization of a new complex, adaptive model of a supercapacitor, used as a storage device and power supply, taking into account temperature influences on key parameters, the effect of high frequencies on the capacitance and the effects of aging based on application-specific usage.
Based on thorough analysis of the classic measurements that are made in pre-choosing a type of supercapacitor to perform the function of an energy source in a system and taking into account all the above factors that may influence the behavior of a supercapacitor, the new electrical equivalent, conceptually model is tested and validated by successive simulations using dedicated software. The model is stable during the simulation and the size of the error to the measured value does not exceed 2.1% of nominal voltage. The solution is even more important as the power supply system must ensure continuity all operating conditions.

Keywords: *supercapacitor's behavior, modeling accuracy, storage efficiency.*

1. INTRODUCTION

Energy storage is a pervasive requirement and is essential for all space endeavors. Supercapacitors are under development because of the large advantage they have over batteries for storage, in static and dynamic applications. There is demonstrated that replacing the classical storage system with a hybrid one based on supercapacitors reduce the mass of the system by 60% or more.
Several unique features of supercapacitors: high power, high energy density and reliability, allow use of the storage device in a wide variety of applications in the backup power sources, auxiliary sources, for instantaneous power compensation, for peak power or simply as energy storage element. There are now developed several models of supercapacitors, sufficiently accurate, but these models are difficult to adapt and used in different applications, and also very few of these models take into account the temperature dependence, the effect of high frequencies on the capacitance or the aging effects depending on specific application usage. Designing an efficient application requires implementation of key elements in mathematical and econometric systems, then identifying the most appropriate electric equivalent model for the specific storage application and electricity supply.

2. MATHEMATICAL AND ECONOMETRIC MODEL OF A SUPERCAPACITOR

Storing energy in a power system can be defined as any other state of a system, with which it is possible to store the energy generated, to conserve it and use it in times of need. According to this definition, energy storage in power systems can be employed in the following arrangements: charge, storage, discharge.
In each of these three regimes, one must maintain a balance between power and energy of the system so that the stored energy has a rated power and a capacity of appropriate values. The duration of each regime, the switching time and the storage efficiency is subject to the requirements of power system. We define the system requirements for energy storage as a limit to the rated power P_s, to the density of energy E_s, to the yield, to t_{res} switching time, to the duration t_ω of the regime. It is obvious that these limits - requirements of the system - are subject to functions performed by the storage devices in power systems, which can be used for the following:
- Improving the operational efficiency of a system;
- Reduction of primary fuel used for energy conservation;
- As a primary source if there is no other alternative source of energy available;
- Ensuring security in electricity supply.

There are two different types of energy systems - hybrid systems and combined systems. We define a hybrid system as an energy system with two inputs and one output. Combined system, in return, has a primary source as input and two or more different types of output energy. The hybrid system was used in the transport sector. Combined system was widely used in cogeneration (electricity and heat), when a power plant uses heat recovered from the loss of heat production for electricity generation. Storage devices can be used in both cases.

2.1. Mathematical model

The mathematical model of energy storage systems require a model formulation for each of its three elements – central storage (CS), the transformation system (TS) and control system charging / discharging (CSCD) [1].

Since the central storage requires energy storage, its sole function is to accumulate energy in needed quantities and to discharge it with a predetermined speed. E_s stored energy is a function that depends on the following parameters:

- CP_i, construction parameters are constant for a given energy storage device;
- VP_i variables that depend on the current regime of storage;
- Time t (time constant of the storage process is determined for each method used in part).

The equations that describe the general mathematical model are:

$$E_s = f_{CS}(CP_i, VP_i, t) \quad (1)$$

$$P_{CS} = dE_s / dt = df_{SC}(CP_i, VP_i, t) / dt \quad (2)$$

$$P_{TS} = f_{TS}(CPP_i, RP_i, PRSEi, VP_i) \quad (3)$$

where PRSE are regime parameters of the energy system energy in the reference node and P_d is the power needed to be discharged;

$$P_d = f_{CSCD}(PRSE_i) \quad (4)$$

$$RP_i = f_{CSCD}(Pdes, VP_i) \quad (5)$$

where RP are special regulation parameters;

$$P_{CS} = P_{TS} = P_d \quad (6)$$

As the storage doesn't represent an energy source, is mandatory to establish the energy balance equation:

$$E_c - E_d - E_s(1-\eta_s)/\eta_c = 0 \quad (7)$$

The stored energy transfer characteristics are:

$$\delta P_d = \frac{\delta P_s}{1+T_s P} \text{ and } \delta Q_d = \frac{\delta Q_s}{1+T_s Q} \quad (8)$$

where T_s is the time constant of the storage process and δP_s and δQ_s are the operating characteristics of the active and reactive power control.

It should be noted that the particular functions as f_{CS}, f_{TS} and f_{CSCS} are given by the type of storage device, system transformation and the number of functions that the storage system must accomplish within the energy system.

Customizing for a supercapacitor and considering its constant and variable parameters, the mathematical model is supplemented by the following equations:

$$CP_i = f(U, C, R_{es}, I_v, P_d, E_{max}) \quad (9)$$

Where U is the rated voltage, C the nominal capacitance, R_{es} the equivalent series resistance, I_v is the maximum peak current for 1 second, P_d is the discharged power and E_{max} is the maximum available energy, all mentioned in the product's datasheet.

$$VP_i = f(I_d, R_{ep}, R_{sd}, f, \theta, n) \quad (10)$$

Where I_d is the output current, R_{ep} is the parallel equivalent resistance, R_{ds} is the self discharge resistance, f the frequency, θ the operating temperature, and n is the supecapacitor's endurance. Every of these parameters can be determined with known formulas.

The use of complete mathematical model can be justified only if we have a certain storage system and there is needed a parameters' optimization or an optimal control of the system.

2.2. Econometric model

Econometric model has the advantage of operationalization, has a practical purpose and becomes an instrument of control, guidance, simulation and forecasting of investment involved in the chosen application. C_s, energy storage cost, consists of two components. One refers to storable energy, the other one depends on the peak power the device must provide for the system and depends on the CSCD, based on energy demand. In other words, the total cost depends on the CS, TS and CSCD, proportionally with installed power and energy storage capacity.

It is preferable to calculate the cost per unit of energy stored (C_e), respectively per unit of installed power (C_p), so the econometric model will be:

$$C_{CS} = C_e * E_s \quad (11)$$

$$C_{TS} + C_{CSCD} = C_p * P_s \quad (12)$$

$$C_s = C_e * E_s + C_p * P_s \quad (13)$$

If referring to specific cost per unit of generation, which is very common in power systems analysis and design, the model can be completed with the relation:

$$\frac{C_S}{P_d} = \frac{C_e^* E_S}{P_d} + \frac{C_p^* P_S}{P_d} \quad (14)$$

The annual cost of one unit of storage, K_s, when operating costs also include capital repairs, forced discharging, maintenance, cost of energy losses, can be given by the relation:

$$K_s = RC_s + n_C C_u \delta E_s \quad (15)$$

Where R represents all the parameters on which the total cost depends: repairs, accidental or forced discharge, operating or maintenance costs, n_c is the number of charge / discharge cycles in a year, C_u is the specific cost of energy used for charging the device.

Econometric model of a storage system is usually given according to the composition of the total annual cost:

$$K_c = R(C_e E_s + C_p P_s) + n_c C_u E_s (1-\eta_s) / \eta_c \quad (16)$$

Where η_s and η_c are the storage and charging efficiency.

All these allow us to optimize the system depending on key parameters - the installed capacity, the stored energy and costs.

3. ELECTRICAL MODEL OF SUPERCAPACITOR

Energy storage is an integral part of the system and P_s and E_s must meet special requirements of the system. To properly define these applications, it is necessary to formulate a general model of the system, including a custom model of a supercondensator used as power storage device and its electric equivalent model. Depending on the specific application (static or dynamic) there are used supercapacitors with aqueous or organic electrolyte.

Modeling on micro-scale (the electrochemical process, the electrode material, geometric patterns of the surface, electrolyte composition) based on performance and dynamic modeling, on the macro-scale (where the supercapacitor is seen as a block studied in frequency and time) are two major categories of techniques mentioned above. Researchers have proposed two dynamic approaches in recent years, based on equivalent electrical circuit analysis and artificial neural networks (ANN).

In 2004, Spyker and Nelms [2] have proposed an equivalent circuit comprising a capacitor, an equivalent series resistance and equivalent parallel resistance (Figure 1.a.). Parameters were determined by studying the performance on charge / discharge cycles. Subsequently, Gualous introduced impedance spectroscopy method for measuring equivalent series resistance, which served to build a second equivalent electrical circuit for supercapacitors and could quantify, on the basis of polynomial equations, the effects of temperature on supercapacitor's performance (fig.1.b.) [3]. Based on impedance spectroscopy, Buller started researches on a dynamic model of supercapacitor in frequency [4] and suggested introducing a transfer function of order 3 and its correlated parameters. In 2006, Michel, using the same method of impedance spectroscopy, has developed a new model that describes factors that influence the open circuit voltage and operating temperature of supercapacitor [5]. In 2007, Lajnef proposed an equivalent circuit for a supercapacitor [6] that operates only to provide a peak power. He also studied the relationship between temperature, voltage and frequency to the load. Also in 2007, Rafik has developed 14 equivalent RLC models to describe the influences of frequency, voltage and temperature on supercapacitor's operation [3]. In 2008, Shi and Crow have proposed two equivalent circuits based on the previous model: circuit with RC series- parallel branches and RC circuit type transmission line (Figure 1.c and 1.e).

In 2009, Brouji studied the behavior of a supercapacitor on its entire life cycle, using electrical impedance spectroscopy [5]. He built an electric equivalent model and calculated a number of specific parameters. In the same year, Sakka elaborated the first thermal model based on the electrical one [7].

For the second approach to modeling, neural networks technique, there were developed multiple linear models, inspired by indistriale needs.

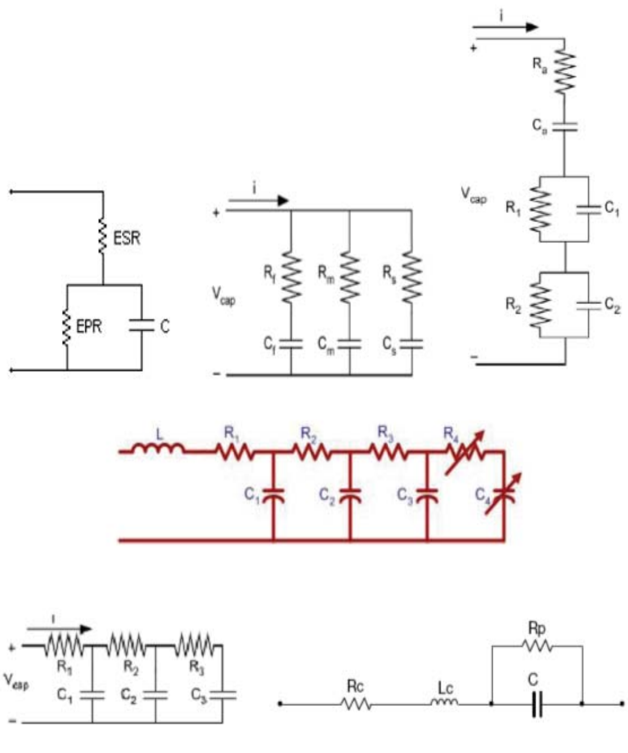

Figure 1: Electrical equivalent models of a supercapacitor: a) RC series equivalent circuit, b) RC circuit branches parallel, c) parallel series RC circuit, d) nonlinear model, e) RC circuit type transmission line, f) equivalent circuit model-order I.

Complementing the database continuously with measured values to inputs and outputs of ANN layers, system dynamics can be simulated by training the network neurons in several iterations. This approach is modeled as a black box system, so that the mutual effects of all factors of influence and simulation can be predicted. In 2000, Bhatikar applied neural networks technique in an energy storage system, molding it to an application for hybrid vehicles [8]. ANN approach plays with precision complex correlations, nonlinear, in a system with batteries. Later, Shen and Monfared shaped, also with ANN technique, the effects of temperature on performance, of load current on the state of charging (SOC) of battery [5]. Although many studies can determine the nonlinear dynamic of an energy storage system based on neural networks, very few of these studies have adopted this approach to model a supercapacitor. Only Farsi, in 2007, began calculating a linear model of supercapacitor [9] using a feedforward neural network type (which permits only to connect the neurons of different layers in a single direction, from input to output). ANN modeling techniques advantage is that they don't use complex differential equations to describe the system, but a very precise model can result by a combination of methods.

3.1. Considerations in choosing the proper model

Before testing any simulation program, there must be first build a model. Analyzing a series of measures (for two supercapacitors with organic electrolyte, ESSP8 and ESSP18) whereby was tested the capacitance and series resistance dependence on frequency and temperature (see figure 2), in different states of discharge, there are presented some important features to be taken into consideration when establishing a conceptual model for a supercapacitor:

- Series resistance increases with decreasing temperature due to reduced mobility of ions in the electrolyte at low temperatures.
- At a supercapacitor with 16 cells, the variation (resistance with temperature) is 2 times higher than in the case of a supercapacitor with one cell.
- Resistance range is 3.7 times higher at -30°C to the value measured at 40°C.
- Capacitance depends on physical parameters of supercapacitor, so it is not influenced by temperature.
- At higher currents, the voltage fluctuates greatly and can not accurately determine the effects on capacitance (ions in the electrolyte migrate to the harder surfaces of carbon at low temperatures).
- Capacitance variation with voltage introduces a nonlinear component in the system (to increase voltage, the internal electric field attract more ions and the concentration near the electrode increases).
- At the same voltage, at different temperatures, the capacitance varies more in the charging state. A possible explanation is that during the charging cycle, the diffusion and electrical forces are of rejection and during the discharge cycle, they are of attraction.

3.2. The basic optimal model

The equivalent model shown in figure 3 is chosen based on measurements analyzed in Section 3.1., developed to fit the curves obtained by processing precision measurements - RC series and parallel linear model in Simulink.

A supercapacitor can be modeled using standard circuit components, as shown in figure 3 (for some applications or projects, the model is provided in the manufacturer's data sheet). The circuit includes all components that are used in the base model. Because this is a conceptual circuit more than one effective functional, there is not shown the control of the switch or of the capacitors. The two variable capacities are nonlinear, dependent on the voltage applied to the entire circuit, as deduced from measurements' analysis presented above.

Capacitance C is the one on which depends the behavior of the entire circuit, determining the maximum state of charge of the supercapacitor.

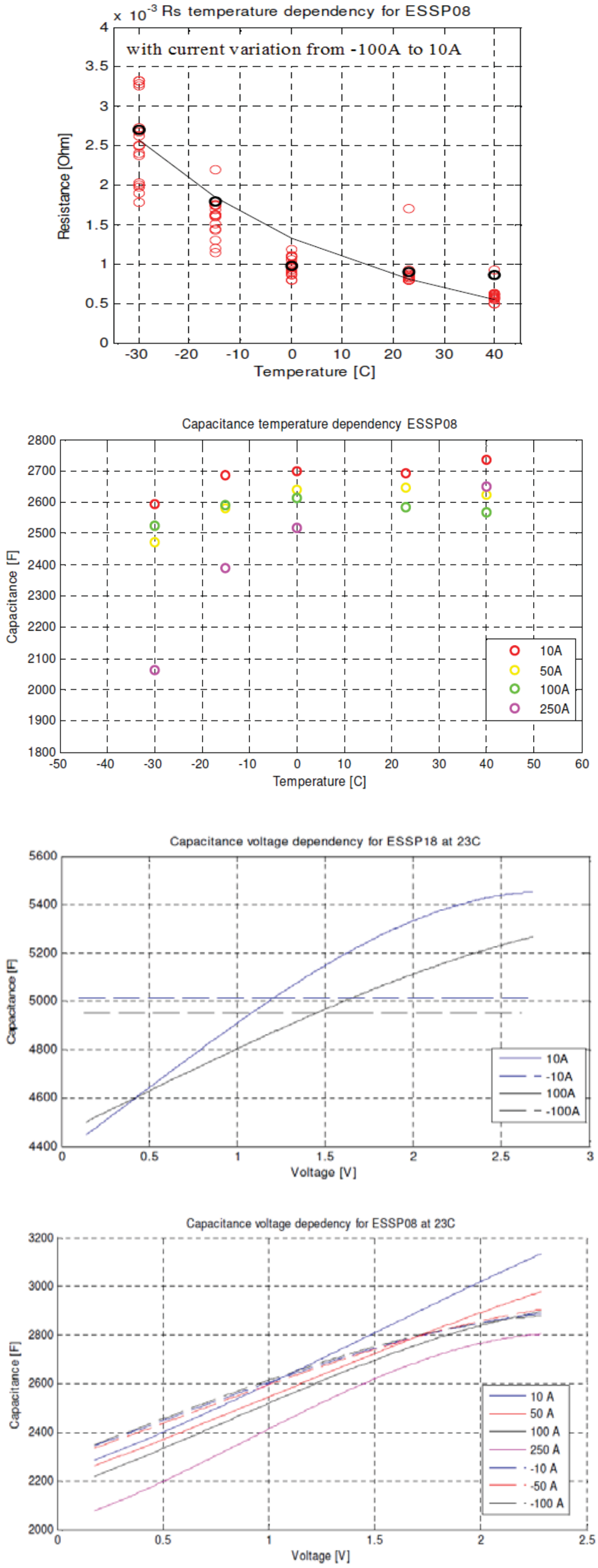

Figure 2 – Resistance and capacitance variation with temperature and voltage

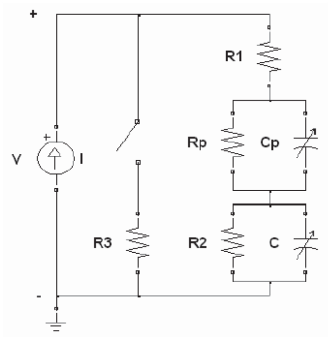

Figure 3 – The basic conceptual model of a supercapacitor

The amount of stored energy and the value of energy levels variations are determined mainly by the capacitance. Resistance R_2, connected in parallel with the capacitor represents the quantification of auto discharge effect. Resistance R_1 represents losses in charge/discharge cycle, arising due to resistance of the conductor element and the process is not an ideal one.

Over voltage protection, provided by R_3 is necessary to prevent supercapacitor elements' damage, by balancing the voltage of the cells (otherwise, the voltage in an individual cell can increase than the other values, leading to emission of gas or explosion). This difference in voltage can occur if a cell has a smaller capacitance than the other, which is reflected in the amount of stored energy [10]. Resistance R_p and C_p are included in the circuit to shape the fastest processes in the supercapacitor's behavior.

3.3. Testing the adopted model of the supercapacitor

Depending on application type, stationary or dynamic, the supercapacitor is chosen by the type of electrolyte. Then, depending on power and voltage requirements, it's picked a specific number of cells. Subsequent, knowing the specific conditions of application, the performance we want to obtain from the storage device and the environmental characteristics where the supercapacitor will operate, is determined the values of essential variable parameters and the equivalent electrical model to simulate the application which is going to be designed.

Exemplifying on Simulink Matlab software, the initial testing of the model is a simple circuit comprising a resistor in series with a capacitor and another resistor in parallel. This basic circuit led to the identification of the basic functions of a supercapacitor. By adding more components to this simplified circuit until is obtained the adopted model, is grown the precision of the model.

When all blocks of the model are properly connected, there are assigned similar values with those analyzed in section 3.1. Comparing the resulting voltage curve and the measured voltage (until those two confuse), one can determine the accuracy of each parameter for circuit components. By changing the values of all components, the model is adjusted gradually to achieve the values obtained practically. When is obtained the curve which resembles most closely that resulting from measurements (given in Figure 4), component values are recorded and inserted in a table (Table 1). There was not use any mathematical procedure to find the closest values, it is only a qualitative test by changing the circuit parameters and compare.

Component	R1 [mΩ]	R2 [kΩ]	R3 [mΩ]	Rp [mΩ]	C [F]	Cp [F]
Value	6	18	52	3	35	C/13

Table 1: Components values for Simulink model

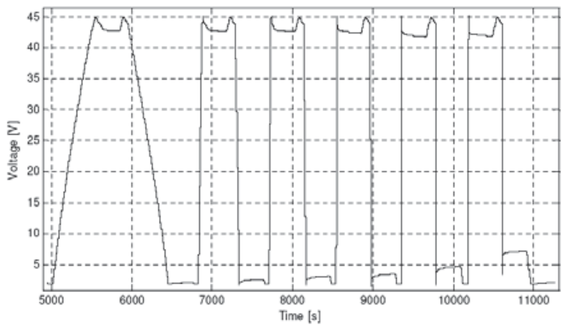

Figure 4 – Voltage variation that must be attained

Capacity can be calculated in two different ways. The first is the voltage derivative during charging, the second method is based on energy stored in supercapacitor, calculated with established formula. The advantage of this method is that it avoids the effects of nonlinearity during charge. Capacitance is calculated at various points of voltage curve (extracted from Figure 2).

In the first part of the test cycle, the supercapacitor is charged from about 0 to 100%, the process shown in Figure 5, which was marked as points. The test is repeated three times. The results are noted in a table similar with Table 1 and introduced into lookup table in Simulink. Since the capacitance varies with voltage, it is better to include dependence in the model, to get one as real. According to figure 5, captured from figure 4, the portion of interest is within the time range of 5018.13 and 5449.23 s. It creates new vectors for time, current and voltage that contains only values recorded during that range of time. With these vectors is done a new graph of voltage, and using the tool "fitting cubic" is identified the curve equation. Differentiating it, we can calculate the capacitance for more voltage levels. Voltage vector and the capacitance vector are connected to the same time vector.

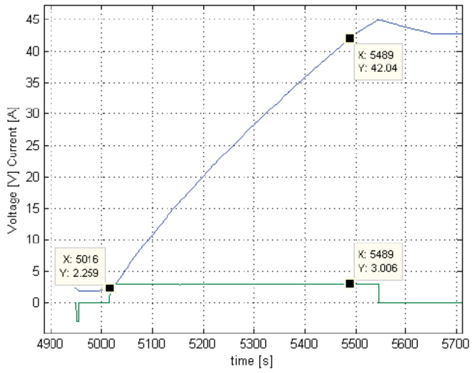

Figure 5: First measurement points on the testing curve

If is desired to model a nonlinear capacitance, one use a current or voltage adjustable source, where the voltage is extracted from the lookup table and calculated using the formula 16:

$$u(t) = \int \frac{i(t)}{C} dt \quad (16); \qquad i(t) = C\frac{d}{dt}u(t) \quad (17);$$

Integral calculation requires historical data (from the lookup table) and its advantage is that errors occurring by high peak current derivation will disappear. Derivative calculation (if formula 17 is used) would require predicted values that were not available through simulation. This issue can be solved by combining electrical equivalent modeling with ANN.

Analog, there are determined the resistance values and all these assigned to each block, to test the accuracy of the model, until we obtain a simulated voltage curve identical to the real, measured one.

4. CONCLUSIONS

Supercapacitors' superior characteristics open up new ways to power microsources development. A dynamic modeling technique for power conversion is important in terms of industrial needs for a fast storage, conversion and supply of a quantity of energy.

Simulink model error seems to be relatively stable during the simulation and the size of the error to the measured value is also low, about 2.1% of nominal voltage.

Simulations were performed for the same conditions, frequency, voltage and temperature remained constant.

It was noted that the same electrical modeling performed in OrCAD, can improve the overall model and reduce the problems of causality.

This paper proposes, as research perspective, to achieve a combined modeling method, which uses electrical equivalent circuits and ANN. This new method will provide historical values and predicted values of supercapacitor´s parameters. This approach can result in a very precise model.

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