GEOMETRY OPTIMIZATION OF SMES COIL

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***Abstract* – Superconducting Magnetic Energy Storage (SMES) device is an attractive solution for energy storage. Because the most important element of SMES system is toroidal superconducting coil, the cryogenic technology involved in the development of SMES is the main disadvantage. More, using the low temperature superconductor materials (LTS), which operate at liquid helium temperatures (4.2 K) is an additional disadvantage in terms of construction and functioning. The maximum stored energy is determined by the size and the geometry of the coil, which determines its inductance, and the characteristics of the superconductor used, which determine the maximum current.**

The Volume or the quantity of superconducting material has a major contribution to the investment cost of such storage systems as the price of the superconducting material and the refrigeration capacity of helium liquefactor.

The purpose of this paper is to establish geometric criteria for pre-sizing of toroidal coil for magnetic energy storage in order to minimize the volume of the superconducting material used. It uses an iterative method for determining relationships between the characteristic dimensions of the toroidal coil. Basically, two parameters are determined. These parameters are ratios of geometrical dimensions and define correlations between three characteristic geometrical dimensions. These correlations are then adjusted for reasons of electromagnetic field for not to exceed certain critical parameters of the superconducting state.

***Keywords*:** *SMES, toroidal coil, superconductor, geometry optimization*

1. INTRODUCTION

Modern Society is characterized by an accelerated increase of electricity consumption. According to [1], the electricity will be 34% of the total energy processed by humans in 2025 to 12% as it was in 1996, in the context of diminishing fossil fuel resources and sustainable development policies and ethics in order to increase the respect to the environment. As a result, electrical energy storage becomes a necessity, presenting a great strategic and economic interest to increase the capacity to meet electricity needs in real time and prevent interruptions in supply. No less important it is the electricity storage to support the introduction of variable and uncertain sources (solar, wind) of renewable sources category. But the electricity, which is a very practical energy vector(carrier), has the inconvenience of being difficult and expensive to be stored in large quantities. For this reason, no remarkable progress has been made in this area. Therefore, recently the U.S. Department of Energy's (DOE) has allocated $4.2 million to develop a superconducting magnet energy storage system [2].

The SMES system (Superconducting Magnetic Energy Storage) is one of the rare techniques for direct storage of electricity through the magnetic energy in superconducting short-circuited coil. The cryogenic technology based SMES seems to be superior to other storage systems regarding more parameters excepting the cost (investment and maintenance) [3]. In this case, the difficulties of using superconducting materials and cryogenic temperatures (building and maintenance) limits the application of SMES systems to very specific applications, some of them not having another solution, that allow acceptance of actual costs assessed to be high [1]. The Volume or the quantity of superconducting material has a major contribution to the investment cost of SMES storage systems as superconducting material prices and by correlation with the refrigeration capacity. For shape of the coil, toroidal version was imposed although it uses more wire than solenoid coil [4].

Regarding the superconducting materials, compared to the new generation of high critical temperature superconducting materials (HTS), at present only NbTi conductors (LTS) meet these requirements. But their operating temperature is low (4.2 K) using the liquid helium with all the implications of this extremely low temperature [5].

The main problem of the SMES device design is to determine the coil system geometry and the current distribution which produces a magnetic field below the critical limit of the superconducting material.

Meanwhile, optimization of storage superconducting coil configuration must reduce the amount of superconducting material as much as possible to limit the cost of storage system [6].

2. CHOOSING OPTIMIZATION PARAMETERS

We consider a classic model of toroidal coil with continuous winding shown in Figure 1. Basic dimensions are *D* (mean diameter of torus), *d* (inner diameter of winding) and *g* (thickness). Coil energy

(storage capacity) and the volume of superconducting material depend on the basic dimensions.
For such a thick torus winding (high g), in the exact analytical expression of inductance, the first additive term is just the expression of thin winding torus inductance, namely [7]:

$$L_{tor} = \frac{\mu_0}{2}\left[D^2 - \sqrt{D^2 - d^2}\right] \quad (1)$$

where μ_0 is vacuum permeability.

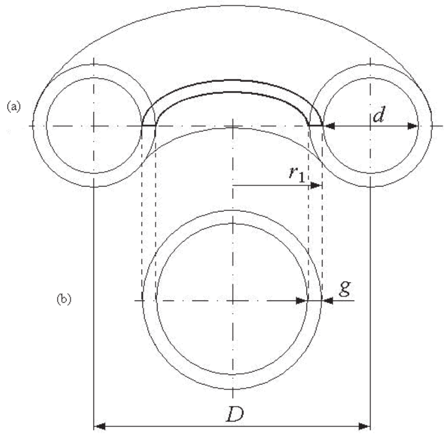

Figure 1. Toroidal coil

Through this simplified expression, we get the expression of torus energy as a function of total current (magnetomotive force) I and inductance L:

$$w = \frac{LI^2}{2} = \frac{\mu_0 I^2}{4}\left[D - \sqrt{D^2 - d^2}\right] \quad (2)$$

To note that expression (2) of energy provides a slightly different value compared to the exact amount of energy due to the use of expression (1) of inductance which is approximately in the case of thick winding torus.
In the hypothesis of using a superconductor NbTi with critical density of current J_c = 530 MA/m^2 at T = 4.2 K and B = 7 T, an admisibile current density (maximum) J = 500 MA/m^2 can be used [8].
Torus energy can be expressed as a function of current leads dimension, respectively minimal section of superconductor, corresponding to inner torus radius, and its length.
According to notations used in Figure 1, the total section of superconductor corresponding to inner radius is:

$$S_{sc} = \pi\, g\,(D - d - g) \quad (3)$$

From relations (2) and (3), the torus energy expression is obtained depending on the geometric dimensions and the current density J which is constant:

$$w = \frac{\mu_0 J^2}{4}\pi^2 g^2 (D - d - g)^2\left(D - \sqrt{D^2 - d^2}\right) \quad (4)$$

Superconducting material volume is:

$$V_{sc} = \pi^2\, g\,(D - d - g)(d + g) \quad (5)$$

To optimize the volume of superconducting material, we introduce two parameters that characterize the geometric torus shape, defined by relations:

$$\alpha = \frac{d}{D} < 1\,;\ \beta = \frac{g}{D} < 1 \quad (6)$$

Torus energy expression (4) according to the mean diameter D and torus parameters α, β becomes:

$$w = \frac{\mu_0 J^2}{4}\pi^2\beta^2 D^5 (1 - \alpha - \beta)^2\left(1 - \sqrt{1 - \alpha^2}\right) \quad (7)$$

Similarly, expression (5) the volume of superconducting material becomes:

$$V = \pi^2 D^3\, \beta(1 - \alpha - \beta)(\alpha + \beta) \quad (8)$$

We define the function:

$$F(\alpha, \beta) = \frac{w}{V} = C\frac{\beta(1 - \alpha - \beta)\left(1 - \sqrt{1 - \alpha^2}\right)}{(\alpha + \beta)} \quad (9)$$

where the constant C with the expression:

$$C = \frac{\mu_0 J^2 D^2}{4} \quad (10)$$

depends on the current density J in superconductor and the mean diameter D of the torus, both sizes set of technical and economic considerations.

3. GEOMETRY OPTIMIZATION

Minimum volume of superconducting material per energy value stored in the toroidal coil is equivalent with maximum amount of stored energy per volume of superconducting material. The maximum of function $F(\alpha,\beta)$ depending on the variables α and β is determined iteratively by identifying maximum of function separately according to each of the two variables.
Maximum of function $F(\alpha,\beta)$ depending on the variable α can be identified quickly from function plot:

$$y(x, \beta) = \frac{\beta(1 - x - \beta)\left(1 - \sqrt{1 - x^2}\right)}{x + \beta} \quad (11)$$

where β can have values between 0.01 and 0.5.
To plot (11) the Equation Grapher program is used. Graphical representation of function $y(x,\beta)$ for β = 0.01, 0.1, 0.3, and 0.4 (Figure 2) indicates that its maximum, respectively of function $F(\alpha,\beta)$ coressponds to values of α in the interval (0.35, 0.55). For example, for β = 0.4, the maximum of function $y(x,\beta)$ is reached for $\alpha = x$ = 0.35. For β = 0.01, in the same figure the maximum of this function is reached approximately for $\alpha = x$ = 0.55. The analysis of

curves in Figure 2, shows that there is a maximum of function $y(x,\beta)$ depending on β. The maximum of function $F(\alpha,\beta)$, depending on the variable β, can be identified from the graphical representation of function:

$$y(x,\alpha)=\frac{x(1-0.45-x)}{0.45+x} \qquad (12)$$

where $\alpha = 0.45$ is considered as an average value.

From the graphical representation of the function $y(x,\alpha)$ for $\alpha = 0.45$ in Figure 3, the maximum of this function, is reached approximately for $\beta = x = 0.22$.
For verification, in Figure 4 the function $y(x,\beta)$, given by (11) is plotted again for three values of β, $\beta = 0.22$ and two neighboring values. We identify maximum of function $y(x,\beta)$ for $\alpha = 0.46$.
Finally, for $\alpha = 0.46$, the maximum of function $y(x,\alpha)$ is identified from the plot of function:

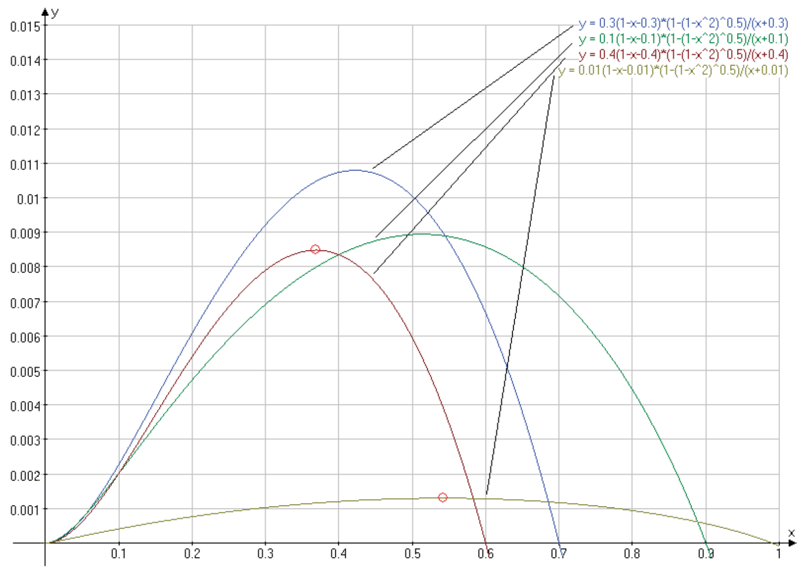

Figure 2. Function $y(x,\beta)$) for $\beta = 0.01, 0.04, 0.1, 0.3$

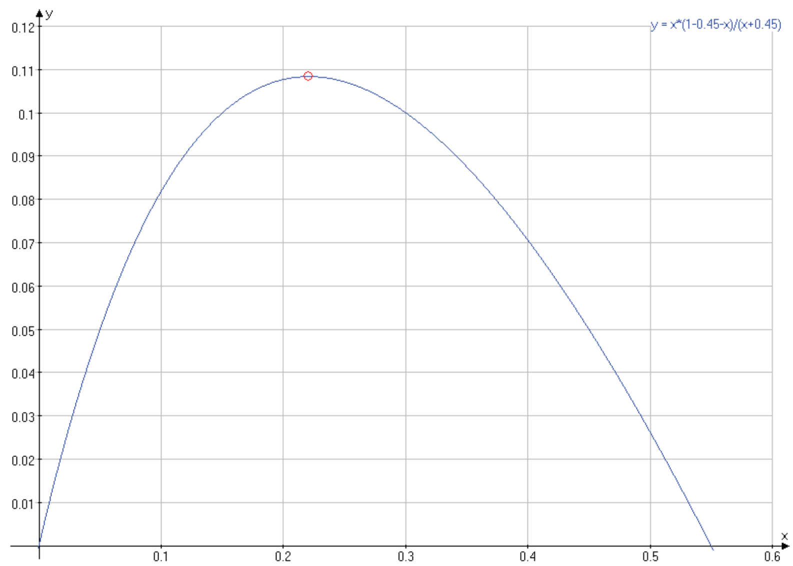

Figure 3. Function $y(x,\alpha)$ for $\alpha = 0.45$

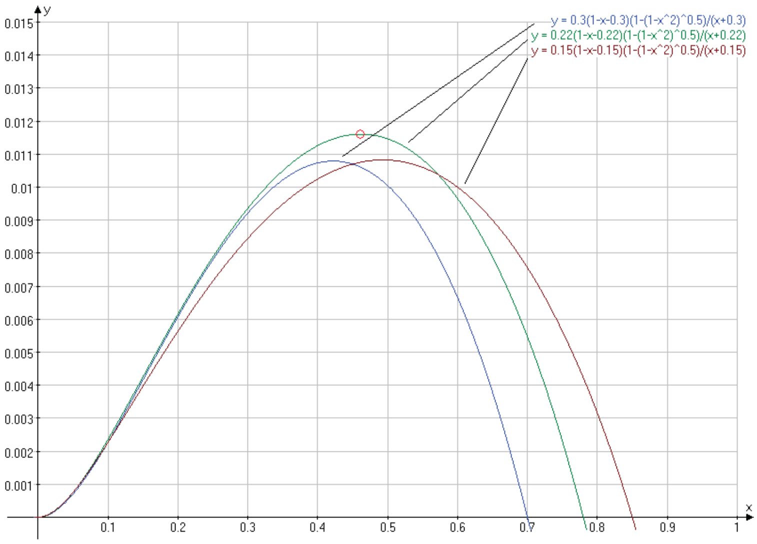

Figura 4. Function $y(x,\beta)$ for $\beta = 0.15$, $\beta = 0.22$ and $\beta = 0.3$

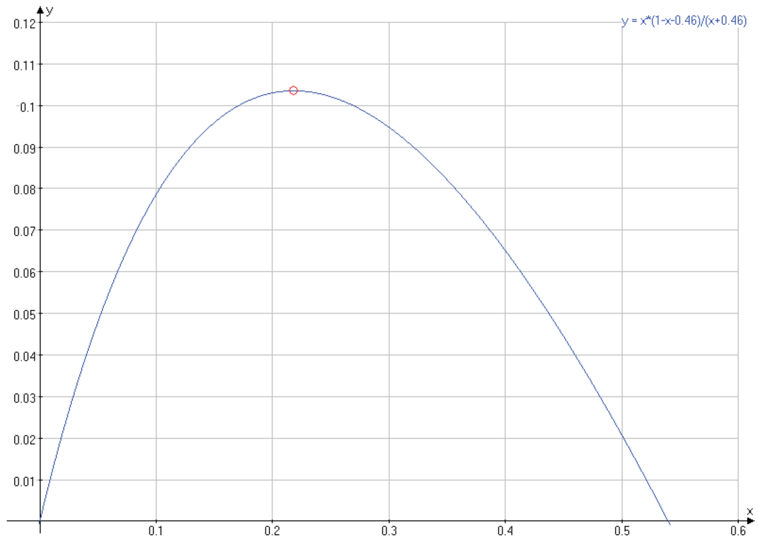

Figure 5. Function $y(x,\alpha)$ for $\alpha = 0.46$

$$y(x,\alpha) = \frac{x(1-0.46-x)}{0.46+x} \tag{13}$$

According to the representation of Figure 5, maximum of $y(x,\alpha)$ given by (13) is obtained for $\beta = 0.22$.

For verification, the maximum of function $y(x,\beta)$ given by (11) is determined for $\beta = 0.22$, having the form:

$$y(x,\beta) = \frac{0.22(1-x-0.22)\left(1-\sqrt{1-x^2}\right)}{x+0.22} \tag{14}$$

From the plot of this function in Figure 6, its maximum is reached again for $\alpha = 0.46$.

Therefore, from considerations to maximize energy per volume of superconducting material, we adopt, according to relations (6), the following expressions of d and g:

$$\begin{aligned} d &= 0.46\,D \\ g &= 0.22\,D \end{aligned} \tag{15}$$

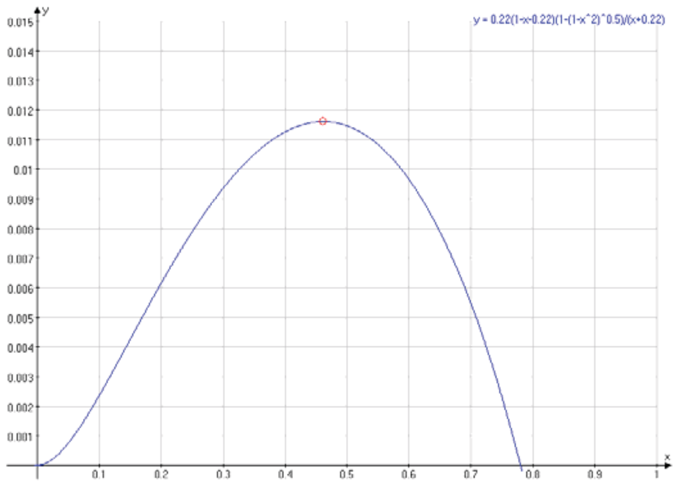

Figure 6. Function $y(x,\alpha)$ for $\beta = 0.22$

4. CONCLUSIONS

Based on analytical calculation, we have established correlations between different dimensions that define the toroidal coil, in order to minimize the volume of superconducting material used. They are approximate relationships based on analytical expression of torus inductance used in calculations.
These correlations are then adjusted for reasons of electromagnetic sizing for not to exceed specific magnetic field of superconducting wire used to manufacture superconducting toroidal coil.
Geometric parameters, α and β, or similar parameters, can be used for other optimization of toroidal coil based on other criteria [9].

Acknowledgments

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