Considerations on Design and Control Modelling of Permanent Magnet Synchronous Motors for Driving Electric Bicycles

Nicolae Digă*, Sergiu Ivanov†, Silvia-Maria Digă† and Virginia Ivanov†
* Doctoral School of Electrical Engineering, University Politehnica of Bucharest, Bucharest, Romania, nicolae.diga@yahoo.ro
† University of Craiova / Faculty of Electrical Engineering, Craiova, Romania, sergiu.ivanov@ie.ucv.ro, sdiga@elth.ucv.ro, vivanov@elth.ucv.ro

Abstract - **In this paper, the authors address the current state of development of one of the conventional electric vehicles, already used and which will penetrate massively in the transport of persons. Are approached particularities that arise in the design permanent magnet synchronous motor, as an integral part of the kit fitted to an electric bicycle to transport a person.**

We started from the premise that for a type of motor there may be several design methods because the design data from which it starts can be different, imposed by the practical use of that motor, the differences consisting less in relations contained and more in their sequence. Have resulted such some specific aspects of the design calculation of this type of permanent magnet synchronous motors for driving a bicycle, which were highlighted in particular. One such important specific aspect that is highlighted in this paper is to identify by computation of the type of used permanent magnet. For this the authors used two complementary methods of calculation: the indirect method and the direct method. Also, the authors present the results of their own analysis and modelling "Matlab-Simulink" of the control system for the operation of a permanent magnet synchronous machine as a motor for driving an electric bicycle wheel.

Keywords - *modelling; permanent magnet synchronous motors; block function parameters; operating regimes; control system*

I. Introduction

The machine on which the experiments and numerical models were conducted is represented by a low power motor P2N=500 W, with 46 magnetic poles and 51 stator slots, which has a nominal voltage of line UN1=36 V, frequency of supply voltage at rated speed f1=50 Hz, number of phases m1=3 phases, insulation class F, degree of protection IP-44 (Fig. 1) [11], [12]. The ultimate aim is its optimization [7] so were studied two construction type variants for which other nominal data were indicated namely: rated speed nN [rpm] and nominal torque MN [Nm] (Fig. 1) [11], [12].

This permanent magnet synchronous motor made with technology ”brushless” comes mounted in wheel centre (tire) (20”, 26” or 28”) with rays (spokes) strengthened (reinforced) and is part of the kit Tucano (Spain) with which is equipped the electric bike on which all experiments were performed. The modular housing of the motor facilitates the rapid and relatively easy disassembly of it.

This motor operates with an efficiency of over 80%, to some size and a very low weight [7], [8]. It is designed to enter into a standard 100 mm fork and allows the installation of disc brake rotor. It is also provided with a new type of computer with power selector of the motor, with three levels: ECO – MED – MAX.

The 16 A computerized controller - the control centre of system - receives signals from the pedal sensor (PAS) or Accelerator and transmits the power required to motor. It incorporates the PPS system, the progressive power delivery that helps to control the bicycle and avoid slippages.

The kit contains a Lithium-Polymer Battery 36 V, 10 A (2.5 kg). It allows a duty travel between 30 and 60 km, are easily manageable and economical (there is the possibility of providing additional battery allowing the doubling autonomy of the new electric bicycle). The battery is protected by double safety circuit and has a charging indicator.

The smart charger loads the battery (1-5 hours depending on the download). Once charged the battery, it passes to indicate the variable loading, to maintain the same charge, thus avoiding overloading. The PAS pedal sensor is installed on the front axle of pedal. It detects user pedalling and sends a signal to controller in order to activate the motor. It may require for installation a connecting rod extractor tool and signal cutting brake levers. They are an essential safety element if the accelerator pedal is installed. Is cut the motor signal as soon as are pressed the brakes, preventing, for example, the user to further accelerate in emergency. The battery holder and controller (driver) installed in the rod seat is the ideal place where is placed the battery without changing the centre of gravity of the bicycle as well as the controller and key locking mechanism.

Fig.1. The studied synchronous motor with permanent magnets

II. Computation Of Required Permanent Magnet

From the construction type analysis of motor, have emerged several remarks as follow.

Because was wanted that the motor to provide a high moment of inertia, was preferred to be inner stator (stator winding automation is possible in this case) and outer rotor [2]. Some required characteristic geometric dimensions are shown in Fig. 2, obtained by the authors through numerical modelling aided by the dedicated software Flux 2D [11], [12].

The motor is designed to operate over a variable speed drive system by adjusting frequency, this system controlling and the internal angle, it is not necessary the presence of the starting cage. If these motors use permanent magnets with high value of coercive magnetic field strength, such as those based on samarium (this case study) or neodymium, the presence cage is no longer necessary for its shielding role and can give it up. In this constructive solution, the permanent magnets are placed on the outside of the rotor, directly to the air gap, achieving a major reduction in mass and an appropriate value of the moment of inertia. Also, the motor power being higher, because of the technological difficulties of achieving large magnets, the poles of the rotor are obtained by placing a number of adjacent smaller size magnets [3].

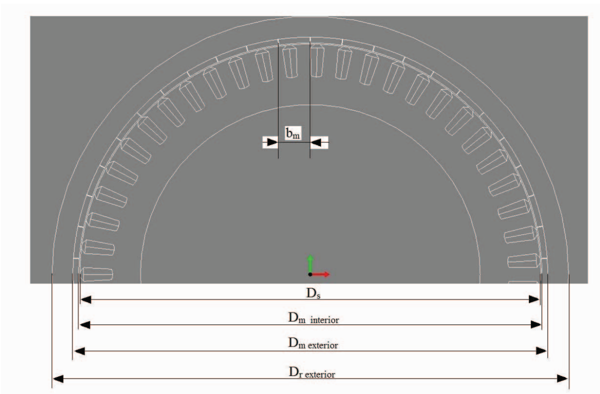

Fig 2. The half cross section of permanent magnet synchronous motor

In Fig. 2. the values of the diameters are:

D_s=199mm;
$D_{m\ interior}$=201mm;
$D_{m\ exterior}$=204.2mm;
$D_{r\ exterior}$=222.2mm.

It is known that the design algorithms, appropriate to different constructive solutions, differ mainly by the relations sizing, depending on the chosen construction type variant, by the placement of permanent magnets on the rotor. There is however a size namely the volume of the permanent magnet required, which does not depend on the geometry of the rotor.

Thus in the first instance the authors have proposed to identify by calculation the type of permanent magnet used. For this purpose were used two methods: indirect and direct method.

A. *Indirect method of computation the required permanent magnet*

Knowing that the sizing relations differ from a constructive solution to another, for motor sizing have adapted relations used for sizing motors with Alnico type magnets, although later it is found that this motor was made with rare earth magnets.

The required permanent magnet volume is determined by the relation (1):

$$V_m = \frac{2}{\pi^2} \frac{k_\Phi \cdot k_{ad}\left(1+\varepsilon\right)}{f_1 \cdot \xi \cdot k_{1m} \cdot B_r \cdot H_c \cdot \eta_m \cdot \cos\phi_m} k_m \cdot P_{2N} \quad (1)$$

where:

k_Φ - the shape coefficient of the permanent magnetic field is determined by the relation (2):

$$k_\Phi = \frac{\Phi_m}{\Phi_{m1}} = \frac{\pi^2}{8} \frac{\alpha_i}{\sin\frac{\pi}{2}\alpha_i} \quad (2)$$

α_i - polar coverage coefficient;

k_m - motor overloading factor is determined by the relation (3):

$$k_m = \frac{P_{2m}}{P_{2N}} \quad (3)$$

Shape coefficients of the magnetic field on the two axes are determined by the relation (4) and (5):

$$k_{ad} = \frac{\pi\alpha_i + \sin\left(\pi\alpha_i\right)}{4\sin\left(\frac{\pi}{2}\alpha_i\right)} \quad (4)$$

$$k_{aq} = \frac{\pi\alpha_i - \sin\left(\pi\alpha_i\right) + \frac{2}{3}\cos\left(\frac{\pi}{2}\alpha_i\right)}{4\sin\left(\frac{\pi}{2}\alpha_i\right)} \quad (5)$$

The coefficient k_{1m} is determined by the relation (6):

$$k_{\mathrm{Im}} = \sqrt{1 + \frac{1}{\varepsilon^2}\left(\frac{k_{ad}}{k_{aq}}\right)^2} \quad (6)$$

where:

ε - the degree of excitation of the motor is determined by the relation (7):

$$\varepsilon = \frac{E_0}{U} \quad (7)$$

ξ - the utilization coefficient of the permanent magnets is determined by the relation (8):

$$\xi = \frac{\Phi_0 \cdot F_{d\max}}{\Phi_r \cdot F_c} = \frac{E_0 \cdot I_{d\max}}{E_r \cdot I_c} \quad (8)$$

Permanent magnet volume resulted according to the measurements:

$$V_{m\,mas} = N_m\left(l_m \cdot b_m \cdot h_m\right) = N_m\left(S_m \cdot h_m\right) \tag{9}$$

where:

N_m = 46 - the number of permanent magnets and the dimensions of a single permanent magnet are:

- the permanent magnet length is:

$l_m = 24$ *mm*;

- the permanent magnet width (Fig. 2) is:

$b_m = 13$ *mm*;

- the permanent magnet height is:

$h_m = 2.5$ *mm.*

It is evaluated the report

$$\frac{V_{m\_\mathrm{Alnico}\_A4}}{V_{m\,mas}} = \frac{16.57\cdot 10^{-5}}{3.588\cdot 10^{-5}} = 4.617\ .$$

Then is determined the recalculated volume of required permanent magnet.

Thus for reducing the required permanent magnet volume in the report calculated previously, was chosen *by testing,* a magnet from *the fourth class* represented by *permanent magnets based on rare earths*, obtained by sintering and which having medium remanent inductions, coercive magnetic field strengths and maximum magnetic energies very high.

The type of permanent magnet used is *R1 sintered* [2], the composition of which includes expensive materials: Cobalt (Co) and Samarium (Sm), deficient materials worldwide. It has the characteristics: B_r=0.7 T – the remanent induction; H_c=480 kA/m – the coercive magnetic field strength; $(BH)_{\max}$=96 kJ•m^{-3} – the maximum magnetic energy per unit volume of magnetic material; μ_{rev} = 1.05- recovery coefficient.

It is evaluated the report and *it is calculated* $V_{m\,R1\,sintered}$:

$$\frac{\left(B_r\cdot H_c\right)_{R1\mathrm{sintered}}}{\left(B_r\cdot H_c\right)_{\mathrm{Alnico}A4}} = \frac{0.7\cdot 480\cdot 10^{3}}{1.3\cdot 52\cdot 10^{3}} = 4.97$$

The calculating error of the required permanent magnet volume is calculated from the relation (10):

$$\Delta V_m = \frac{V_{m\,mas} - V_{mR1\mathrm{sintered}}}{V_{m\,mas}}\cdot 100\left[\%\right] \tag{10}$$

B. Direct method of computation the required permanent magnet

This method is based on the premise that permanent magnets used are with rare earth, resulted in the indirect method presented in the previous paragraph and also adapts the existing sizing relations for another embodiment namely disc rotor motors with rare earth magnets.

Thus Table I compares the calculation coefficients values, the calculated required volume of permanent magnet by the two methods and that obtained from measurements (cf. Fig. 3), and calculation errors that resulted.

From the analysis presented in this paper, it is clear that the two methods of calculation of the required permanent magnet are complementary, especially if are not known the construction details for a particular motor of this type.

TABLE I.
VOLUME COMPARISON OF MEASURED AND CALCULATED REQUIRED PERMANENT MAGNET

Parameters	Indirect method	Direct method
k_Φ	1.198	1.0378
α_i	0.97	0.8
k_m	1.8	
k_{ad}	0.786	0.8152
k_{aq}	0.747	0.6726
k_{lm}	1.903	1.9003
ε	0.65	0.75
ξ	0.35	
$V_{mR1\mathrm{sintered}}$ x 10^{-5} [m^3]	3.333	3.356
$V_{m\,mas}$ x 10^{-5} [m^3]	3.588	
ΔV_m [%]	7.114	6.4663

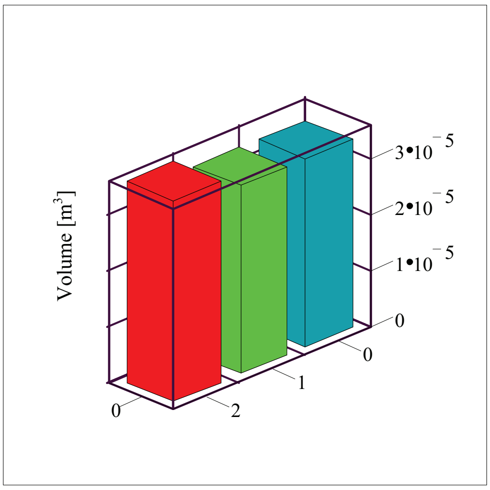

Fig. 3. The volume of required permanent magnet: 0 – indirect method; 1 – direct method; 2 – from measurements.

Also it was verified that for this type of sintered permanent magnet R1, low residual induction (B_r=0.7 T) and high coercive magnetic field strength (H_c=480 kA/m) to have the desired air gap induction is needed a constructive solution of the concentration of produced magnetic flux, and each of the N_m=46 magnets will be large cross-section S_m=312 mm^2 and of small height h_m=2.5 mm.

III. MATLAB-SIMULINK MODELLING OF VECTORIAL NUMERICAL CONTROL OF STUDIED MOTOR SUPPLIED BY A VOLTAGE INVERTER

The modern microprocessors, by their great working speed, can provide a real-time control much more complex of the motor "brushless", taking into account its dynamic electromagnetic regimes. Underlying this process is the vector control method, which takes into account the instantaneous position of flux vector from the machine [1], [2], [3], [4]. This is the case of studied synchronous motor with permanent magnet excitation by rare earth, without damping cage but with magnetic anisotropy, $L_d \neq L_q$ as can be seen from Table II, extracted from the calculation methodology for the design of permanent magnet synchronous motor for driving a bicycle, [11].

A. *Computation of block function parameters*

The model was realized with Matlab software, using the blocks from the library Simulink. In MS_MP block, the block for permanent magnet synchronous motor, were introduced the studied motor parameters for the two variants of computation (Variant I and Variant II) of Table II. The computation algorithm of block function parameters for the permanent magnet synchronous motor required a complex computation methodology whose main steps are outlined below.

Computation of total inertia moment to the motor shaft

For the computation of total inertia moment to the motor shaft, its rotor is assimilated to a cylindrical shell-type elementary corps, for which is used the equation (11):

$$J_r = \frac{1}{2} m_r \frac{\left(D^2_{r\,\mathrm{exterior}} + D^2_{r\,\mathrm{interior}}\right)}{4} \left[kg \cdot m^2\right] \quad (11)$$

where: $D_{r\ \mathrm{interior}}$ - the inner diameter of the rotor, viewed from the centre of symmetry of the motor; $D_{r\ exterior}$ - the outer diameter of the rotor, viewed from the centre of symmetry of the motor;

$$m_r = m_{jr} + m_m \left[kg\right] \quad (12)$$

m_{jr}- the rotor yoke mass calculated with (12):

$$m_{jr} = \gamma_{Fe} \cdot V_{jr} = \gamma_{Fe} \cdot \left(\frac{\pi D^2_{r\mathrm{exterior}}}{4} - \frac{\pi D^2_{m\mathrm{exterior}}}{4} \right) \cdot l_r \left[kg\right] \quad (13)$$

l_r- the rotor length;

$$m_m = \rho_{R1\mathrm{sinterized}} V_{m\ mas} \left[kg\right] \quad (14)$$

V_{mmas}- the permanent magnet volume according to measurements;

$\rho_{R1\mathrm{sinterized}} \approx 8200\left[kg/m^3\right]$ - value chosen from [2] for rare metal alloy R2, the same country of origin (England).

Computation of friction and own-ventilation mechanical torque

$$M_{mec+v} = \frac{P_{mec+v}}{\Omega_N} \left[N \cdot m\right] \quad (15)$$

where: Ω_N- the rated angular speed; P_{mec+v}- the friction and own-ventilation mechanical losses calculated with (16):

$$P_{mec+v} = 0{,}8 \cdot 2p \cdot \left(\frac{v}{40}\right)^3 \sqrt{\frac{l_r}{19}} \cdot 10^3 \left[W\right] \quad (16)$$

where:

$$v = \frac{\pi D_{r\mathrm{exterior}} \cdot n_N}{60} \left[m/s\right] \quad (17)$$

Computation of the constant of friction and own-ventilation mechanical torque

$$k_{M_{mec+v}} = \frac{M_{mec+v}}{\Omega_N} \left[(N \cdot m)/(rad/s)\right] \quad (18)$$

Computation of the electromagnetic torque

$$M_s = \frac{P_2}{\Omega_s} = \frac{p \cdot P_2}{2\pi \cdot f_1} \left[N \cdot m\right] \quad (19)$$

where: M_s- the torque developed by the motor, calculated with (20):

$$M_s = \frac{P_2}{\Omega_s} = \frac{p \cdot P_2}{2\pi \cdot f_1} \left[N \cdot m\right] \quad (20)$$

Computation of the constant of electromagnetic torque

$$k_M = \frac{M}{I_{fN}} \left[\frac{N \cdot m}{A}\right] \quad (21)$$

It should be noted that the authors have represented graphically by using their own programs developed in Mathcad 7.0 programming environment all motor operating characteristics. Thus these graphical representations showed that the rated power P_{2N} is obtained from a certain internal angle δ_N. Then directly, by customizing the adapted expressions [11] for the internal angle δ_N were obtained the nominal current and the nominal torque developed by the motor.

TABLE II.
BLOCK FUNCTION PARAMETERS - PERMANENT MAGNET SYNCHRONOUS MOTOR

Parameters	**Variant I** *ZsI*=48 **slots**	**Variant II** *ZsII*=54 **slots**
The equivalent resistance of the phase, R [Ω]	0.7182	0.7427
Direct-axis inductivity, L_d [H]	$2.7031 \cdot 10^{-3}$	$3.0656 \cdot 10^{-3}$
Quadrature-axis inductivity, L_q [H]	$7.7219 \cdot 10^{-3}$	$7.9327 \cdot 10^{-3}$
The number of pairs of poles, p	8	9
Total moment of inertia at the motor shaft, Jr [$kg \cdot m^2$]	0.0114	
The constant of the electromagnetic torque, k_M [Nm/A]	1.7985	2.0309
The constant of the friction torque, k_{Mmec+v} [$Nm/(rad \cdot s^{-1})$]	$3.8279 \cdot 10^{-4}$	$3.0245 \cdot 10^{-4}$

As an example, it is shown in Fig. 4 the MSMP block interface, Variant II.

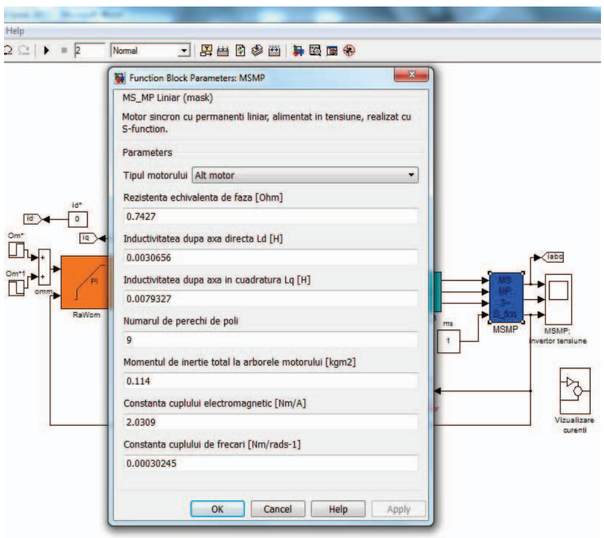

Fig.4. The MSMP block interface, *Variant II*.

In Fig. 5 is presented the complete model developed by the authors, in the programming environment Matlab [3], [5], [6], [10], for Variant II of studied permanent magnet synchronous motor.

Fig. 6 shows the running MSMP model, *Variant II* for no load regime.

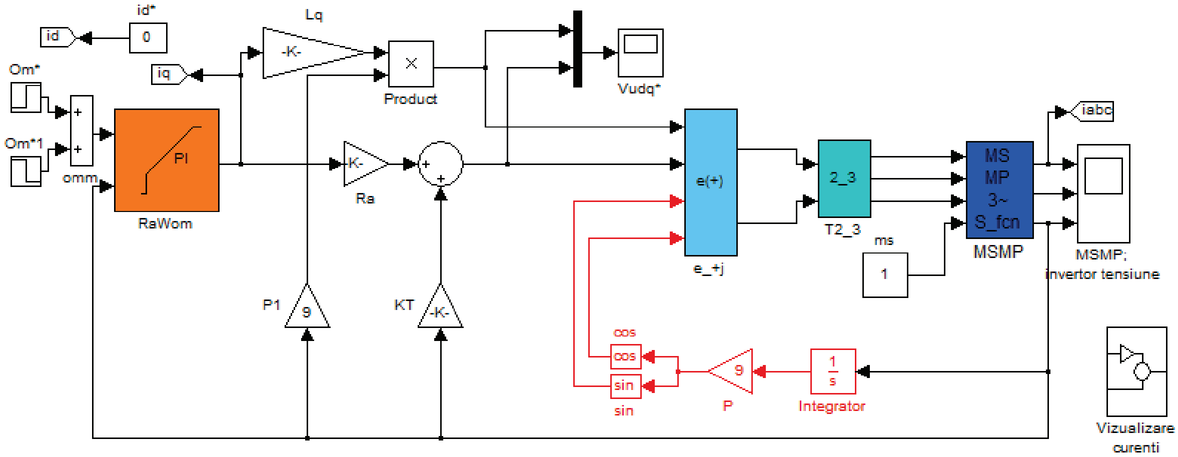

Fig. 5. The complete model accomplished with "Matlab" of PMSM, *Variant II*.

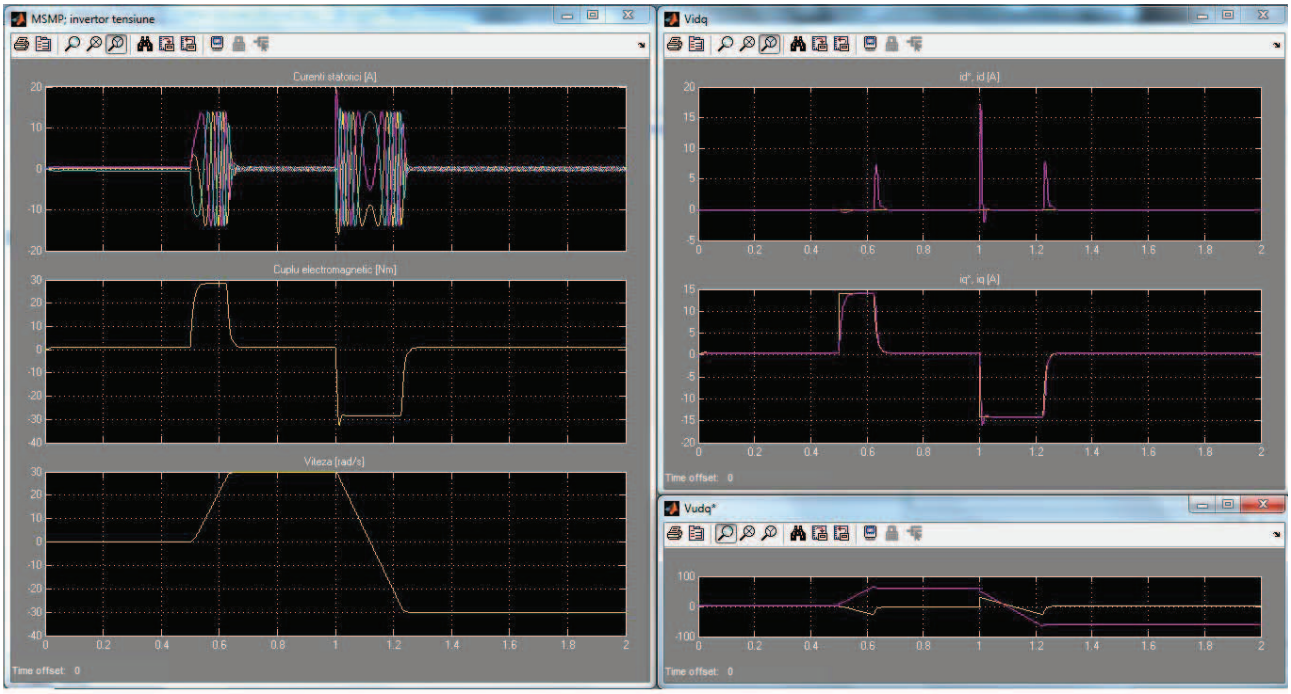

Fig. 6. Plots of stator currents, electromagnetic torque, speed, preset and real direct current, preset and real quadrature current, direct and quadrature voltages

IV. Conclusions

The complete methodology for computation conceived sequentially and the modelling of the control of this permanent magnet synchronous motor (special construction) were applied to the two mentioned constructive variants (Variant I and Variant II) chosen so because the existing constructive solution Z_{sreal}=51 slots, is characterized by the arithmetic mean of the number of stator slots of these.

We can estimate so that the characteristics of real constructive variant are between the corresponding two variants of comparative analysis.

By means of this modelling is checked that, these motors, used in high performance applications in terms of the speed adjustment range [3], [9] with the sine-division stator windings are energized at least until the nominal speed of the imprinted sinusoidal current and then to reaching the voltage, with rectangular voltage pulse. The generation of these currents and voltages can be done by numerically in this case, because are required very good final performances.

References

[1] A. Fransua and R. Măgureanu, Electrical Machines and Drive Systems, Ed. Th. & Technical Press, Oxford, 1984.

[2] R. Măgureanu and N. Vasile, Permanent magnet synchronous motors and variable reluctance, Technical Publishing House, Bucharest, 1982.

[3] R. Măgureanu and N. Vasile, Brushless synchronous servomotors, Technical Publishing House, Bucharest, 1990.

[4] R. Le Doeuff and J. Roberts, coord. Modelling and Control of Electrical Machines. New Trends, IMACS'91.

[5] Boldea and S. A. Nasar, Vector Control of AC Drives, CRC Press, 1992.

[6] D. O. Kisck, Vector control of AC machines, ICPE Publishing House, Bucharest, 1997.

[7] C. Ghiță, Electromechanical converters. Vol. II. Synchronous machine and machine dc. ICPE Publishing House, Bucharest, 1999.

[8] B. Moulton, Application des aimants aux machines électriques, Notes de cours 2005, ENS Cachan, Antenne de Bretagne.

[9] N. Papachatzis, Nouveaux produits software pour la commande des entraînements électriques (in Romanian), Thèse de docteur, Université Technique de Iassy, Iassy, 2005.

[10] V. Ivanov, Mathcad and MATLAB Applications, Craiova Universitaria Publishing House, Craiova, 2007.

[11] N. Digă, Design and 2D numerical modelling of permanent magnet synchronous motor, for driving a bicycle, Research Report no. 1, Electrical Engineering Doctoral School, Polytechnic University of Bucharest, June 20, 2013.

[12] S. M. Digă, C. Stoica, N. Digă and M. Brojboiu, "Considerations on 2D numerical modelling of permanent magnet synchronous motors for driving electric bicycles", The Fourth International Symposium on Electrical and Electronics Engineering - ISEEE 2013, 11-13 October 2013, Galați, Romania, IEEE Catalog Number CFP1393K-USB, ISBN 978-1-4799-2441-7.