Analysis of Reluctance Synchronous Motors Static Stability Using Routh-Hurwitz Criterion

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Abstract - **This paper analyzes the static stability of the reluctance synchronous motors, by means of the Routh-Hurwitz criterion. In this purpose, there is made a brief introduction which presents the types of disturbances occurring during the operation of these motors. Two types of disturbances are possible: low value disturbances, defining the so-called static stability and finite value disturbances, defining the dynamic stability. In the first case, analyzed in the paper, the disturbances are relatively low and that is why the changes around a stable operation point are small; this way a linearization of the machine differential equations is possible. As a consequence, the stability study is simplified because the linearized system can be integrated by analytical methods or can be analyzed by means of the stability criteria. Further on the mathematical model used i presented. The starting point in this phenomenon analysis is the mathematical model of the reluctance synchronous motor written in the two axes theory without considering the saturation. These equations are then processed, in certain conditions, in order to obtain the operational mathematical model. The Routh-Hurwitz criterion is applied to this model and this way the conclusions have resulted. The reluctance synchronous motors may have an unstable operation when they are supplied by frequencies lower than 50 Hz. This instability means rotor oscillations around the synchronism speed; these oscillations disturb the mechanism operation and they even could lead to the machine de-synchronization. This phenomenon, encountered especially at the machines having high rotor asymmetry, is not caused by resistant torque shocks, by imperfections of the system or by supply voltage variations, but it is a phenomenon specific to the reluctance synchronous motor; sometimes it can occur even at the rated frequency.**

Keywords - *reluctance synchronous motor, static stability, mathematical model, design*

I. INTRODUCTION

Reluctance synchronous motors (RSM) are permanently constrained by certain electrical and mechanical disturbances. These disturbances cause dynamic regimes which may end by a new steady state, by a permanent regime of oscillating around the synchronism speed or by an unstable regime when the synchronism is lost [7], [11], [12].

Two types of disturbances are possible:

- low value disturbances, defining the so-called static stability;
- finite value disturbances, defining the dynamic stability.

In the first case, the disturbances are relatively low and that is why the changes around a stable operation point are small; this way a linearization of the machine differential equations is possible. As a consequence, the stability study is simplified because the linearized system can be integrated by analytical methods or can be analyzed by means of the stability criteria (Routh-Hurwitz, Nyquist etc.) [6], [15], [16], [18].

The second case refers to the finite disturbances which influence the reluctance synchronous motors operation. These disturbances cause dynamic regimes which can finish by a new steady state, by a continuous regime of oscillations around the synchronism speed or by an unstable regime in which the synchronism is lost [2], [9], [14], [17].

It is necessary to make the numerical integration of these equations and to study the system answer to different disturbing quantities. The study depends not only on the initial conditions, but also on the disturbance magnitude and character.

In addition, in dynamic regime, according to [5]:

- the dynamic stability decreases at the same time with the supply voltage decrease (when the decrease is under a certain limit, the synchronism is lost);
- the reluctance synchronous motors has a different behavior over different values of the torque shock, at the same inertia moment;
- the synchronism loss is conditioned by the value of the applied torque shock;
- the previous conclusions emphasize the fact that the reluctance synchronous motors static stability is dependent both on the disturbance magnitude and character and on the initial conditions;
- with the help of some simulations like the ones presented before it is possible to establish the dynamic stability limit for each reluctance synchronous motors, which is an important work instrument for the designers of such type of motors.

This paper develops an analytical demonstration regarding the first case, the static stability.

II. MATHEMATICAL MODEL

The starting point in this phenomenon analysis is the mathematical model of the reluctance synchronous motor written in the two axes theory without considering the saturation [1], [8], [13], [19]:

$$u_d = R_s i_d + \frac{d\psi_d}{dt} - \psi_q \frac{d\beta_B}{dt}$$

$$u_q = R_s i_q + \frac{d\psi_q}{dt} + \psi_d \frac{d\beta_B}{dt}$$
$$0 = R_D i_D + \frac{d\psi_D}{dt} \qquad (1)$$
$$0 = R_Q i_Q + \frac{d\psi_Q}{dt}$$
$$m = \frac{3}{2} p(\psi_d i_q - \psi_q i_d)$$
$$\frac{J}{p}\frac{d^2\beta_B}{dt^2} = m - m_r$$
$$\psi_d = L_d i_d + L_{md} i_D$$
$$\psi_q = L_q i_q + L_{mq} i_Q$$
$$\psi_D = L_D i_D + L_{md} i_d$$
$$\psi_Q = L_Q i_Q + L_{mq} i_Q$$

Forwards the first two equations of the system (1) are processed considering that $u_d = -U_m \sin\theta$ and $u_q = U_m \cos\theta$:

$$-U_m \sin\theta = R_s i_d + \frac{d\psi_d}{dt} - \psi_q \frac{d\beta_B}{dt} \qquad (2)$$
$$U_m \cos\theta = R_s i_q + \frac{d\psi_q}{dt} + \psi_d \frac{d\beta_B}{dt}$$

The position angle β_B and the internal angle θ are linked by the relation:

$$\beta_B = \omega t - \frac{\pi}{2} - \theta . \qquad (3)$$

This relation, by differential, becomes:

$$\frac{d\beta_B}{dt} = \omega - \frac{d\theta}{dt} , \qquad (4)$$

or, by a new differential:

$$\frac{d^2\beta_B}{dt^2} = -\frac{d^2\theta}{dt^2} . \qquad (5)$$

In case of small oscillations, the differentials of these equations around a stable operation point ($\psi_{d0} = \psi_d$, $\psi_{q0} = \psi_q$, $i_{d0} = I_d$, $i_{q0} = I_q$ şi $\beta_{B0} = \beta_B$), also considering (5), have got the form:

$$-U_m \cos\theta_0 \Delta\theta = R_s \Delta i_d + \frac{d}{dt}(\Delta\psi_d) - \omega\Delta\psi_q + \psi_{q0}\frac{d}{dt}(\Delta\theta) \qquad (6)$$
$$-U_m \sin\theta_0 \Delta\theta = R_s \Delta i_q + \frac{d}{dt}(\Delta\psi_q) + \omega\Delta\psi_d - \psi_{d0}\frac{d}{dt}(\Delta\theta)$$

In a similar way, the equation 5 of the system (1) gets the form:

$$\Delta m = \frac{3p}{2}(\psi_{d0}\Delta i_q + i_{q0}\Delta\psi_d - \psi_{q0}\Delta i_d - i_{d0}\Delta\psi_q) \qquad (7)$$

The stability of a reluctance synchronous motor, according to [3], increases at the same time with load. As a consequence, the most difficult operation regime, from the stability point of view, is the no-load operation regime, which will be analyzed forwards.

In this case it can be written:

$$i_{q0} = I_{q0} = 0$$
$$\psi_{q0} = \Psi_{q0} = 0$$
$$i_{d0} = I_{d0} = \frac{U\sqrt{2}}{\sqrt{R_s^2 + X_d^2}} = \frac{U_m}{Z_d}$$
$$\psi_{d0} = \Psi_{d0} = \frac{I_d U_m}{Z_d} \qquad (8)$$
$$\theta = \theta_0 = -arctg\frac{R_s}{X_d}$$
$$\cos\theta_0 = \frac{X_d}{Z_d}$$
$$\sin\theta_0 = -\frac{R_s}{Z_d} .$$

In the conditions (8), the relations (6) and (7) become:

$$-\frac{U_m X_d}{Z_d} = R_s \Delta i_d + \frac{d}{dt}(\Delta\psi_d) - \omega\Delta\psi_q$$
$$\frac{U_m R_s}{Z_d} = R_s \Delta i_q + \frac{d}{dt}(\Delta\psi_q) + \omega\Delta\psi_d - \frac{U_m L_d}{Z_d}\frac{d}{dt}(\Delta\theta) \qquad (9)$$
$$\Delta m = \frac{3p}{2}\frac{U_m}{Z_d}(L_d \Delta i_q - \Delta\psi_q) .$$

III. Operational Equations

By applying Laplace transform, the above equations become [13]:

$$-\frac{U_m X_d}{Z_d}\Delta\bar{\theta} = R_s \Delta\bar{i}_d + s\Delta\bar{\psi}_d - \omega\Delta\bar{\psi}_q$$
$$\frac{U_m}{Z_d}(R_s + sL_s)\Delta\bar{\theta} = R_s \Delta\bar{i}_q + s\Delta\bar{\psi}_q + \omega\Delta\bar{\psi}_d \qquad (10)$$
$$\Delta\bar{m} = \frac{3p}{2}\frac{U_m}{Z_d}(L_d \Delta\bar{i}_q - \Delta\bar{\psi}_q) .$$

The Laplace transform is now applied to the equations 3 (with ψ_D given by the equation 9) and 7 of the system (1), in the hypothesis that the machine was in steady state in the first moment ($\left.\frac{di_D}{dt}\right|_0 = 0$, $i_D = i_Q = 0$); it is obtained:

$$0 = (R_D + sL_D)\bar{i}_D + sL_{md}\bar{i}_d , \qquad (11)$$

respectively:

$$\overline{\psi}_d = L_d \bar{i}_d + L_{md} \bar{i}_D . \quad (12)$$

When replacing $\bar{i}_D$, obtained with (11), in (12), it is obtained:

$$\overline{\psi}_d = \left(L_d - \frac{sL_{md}^2}{R_D + sL_D} \right) \bar{i}_d = L_d(s) \bar{i}_d , \quad (13)$$

where there has been used the notation:

$$L_d(s) = L_d - \frac{sL_{md}^2}{R_D + sL_D} = L_d \frac{1 + sT_d}{1 + sT_{do}} , \quad (14)$$

where T_d is the short circuit time constant:

$$T_d = \frac{L_d L_D - L_{md}^2}{R_D L_D} \quad (15)$$

and T_{d0} is the no-load time constant:

$$T_{d0} = \frac{L_D}{R_D} . \quad (16)$$

Processing the equations 4, 8 and 10 of the system (1), in the same way, it is obtained:

$$\overline{\psi}_q = L_q(s) \bar{i}_q , \quad (17)$$

where:

$$L_q(s) = L_q - \frac{sL_{mq}^2}{R_Q + sL_Q} = L_q \frac{1 + sT_q}{1 + sT_{qo}} \quad (18)$$

with

$$T_q = \frac{L_q L_Q - L_{mq}^2}{R_Q L_Q} , \quad (19)$$

respectively:

$$T_{q0} = \frac{L_Q}{R_Q} . \quad (20)$$

For the case of small oscillations (13) and (17) become:

$$\begin{aligned} \Delta\overline{\psi}_d &= L_d(s) \Delta\bar{i}_d \\ \Delta\overline{\psi}_q &= L_q(s) \Delta\bar{i}_q \end{aligned} \quad (21)$$

Now it is possible to centralize the operation equations of the reluctance synchronous motor, written in operational, (10) and (21), for the case we mentioned:

$$\begin{aligned} -\frac{U_m X_d}{Z_d} \Delta\overline{\theta} &= R_s \Delta\bar{i}_d + s\Delta\overline{\psi}_d - \omega\Delta\overline{\psi}_q \\ \frac{U_m}{Z_d}(R_s + sL_s)\Delta\overline{\theta} &= R_s \Delta\bar{i}_q + s\Delta\overline{\psi}_q + \omega\Delta\overline{\psi}_d \\ \Delta\overline{\psi}_d &= L_d(s)\Delta\bar{i}_d \\ \Delta\overline{\psi}_q &= L_q(s)\Delta\bar{i}_q \\ \Delta\overline{m} &= \frac{3p}{2}\frac{U_m}{Z_d}(L_d \Delta\bar{i}_q - \Delta\overline{\psi}_q) \end{aligned} \quad (22)$$

IV. ROUTH-HURWITZ CRITERION

In order to analyze the stability of the reluctance synchronous motors, the Routh-Hurtwitz criterion has been applied .

This is also named the criterion of the coefficients, being an algebraic criterion for evaluating the stability of an automatic system, without solving the characteristic of the differential equation that expresses its dynamics.

There is considered a method for detecting the positive real roots or the complex-conjugated roots with positive real part of a polynomial which, in the last analysis, is the characteristic equation of the equivalent transfer function.

The two authors published their papers in a period when the problem of the automatic systems stability did not exist (Routh - 1877, Hurwitz - 1895) but only a pure mathematical interpretation of the condition in which a n-order polynomial has only negative roots or complex-conjugated roots with negative real part.

Since this condition is just the stability condition, if the polynomial is even the characteristic equation of the system, the Routh-Hurwitz method may be taken as a stability criterion.

Thus, according to [10] a system of which characteristic equation is:

$$a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0 = 0 \quad (23)$$

is stable if the following conditions are fulfiled:

- there are all the powers of s and their coefficients have the same sign;
- determinants:

$$D_0 = |a_1|;$$

$$D_1 = \begin{vmatrix} a_1 & a_0 \\ a_3 & a_2 \end{vmatrix}; \quad (24)$$

$$D_2 = \begin{vmatrix} a_1 & a_0 & 0 \\ a_3 & a_2 & a_1 \\ 0 & a_4 & a_3 \end{vmatrix},$$

have the same sign as a_0.

The first condition, for the real values of the reluctance synchronous motor parameters, is fulfilled, the coefficients a_4, a_3, a_2, a_1 and a_0 always being positive [4].

The second condition involves that the three determinants are also positive, which means to consider the the most restrictive condition is $D_2 > 0$ or, equivalently:

$$a_3(a_1 a_2 - a_0 a_3) - a_1^2 a_4 > 0 . \tag{25}$$

In order to apply this criterion, the equations (22) have been written again, in specified conditions, as the equation (23) [4].

Thus, (22) can be written successively:

$$(a_6 s^6 + a_5 s^5 + a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0)\Delta\bar{\theta} = 0 \tag{26}$$

where

$$\begin{aligned} a_6 &= Kc_4 \\ a_5 &= Kc_3 \\ a_4 &= Kc_2 + b_4 \\ a_3 &= Kc_1 + b_3 \\ a_2 &= Kc_0 + b_2 \\ a_1 &= b_1 \\ a_0 &= b_0 \end{aligned} \tag{27}$$

where

$$K = \frac{2JL_q}{3p^2\left(\dfrac{U_m}{\omega}\right)^2}\left(\frac{Z_d}{X_d}\right)^2 \cong \frac{2JL_q}{3p^2\left(\dfrac{U_m}{\omega}\right)^2} \tag{28}$$

and b_0, …, c_4 are the coefficients of the transfer function:

$$G(s) = \frac{b_4 s^4 + b_3 s^3 + b_2 s^2 + b_1 s + b_0}{c_4 s^4 + c_3 s^3 + c_2 s^2 + c_1 s + c_0} . \tag{29}$$

The solution of the equation (26), in time domain, may be written as:

$$\Delta\theta = g_1 e^{s_1 t} + g_2 e^{s_2 t} + g_3 e^{s_3 t} + g_4 e^{s_4 t} + g_5 e^{s_5 t} + g_6 e^{s_6 t} \tag{30}$$

s_1, ... , s_6 being the roots of the characteristic equation:

$$a_6 s^6 + a_5 s^5 + a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0 = 0 . \tag{31}$$

For current values of the reluctance synchronous motors parameters, according to [13], a_5 and a_6 are negligible over the other parameters.

As a consequence, the characteristic equation becomes:

$$a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0 = 0 ,$$

therefore exactly the equation (23).

With its help the major critical frequency has been then computed.

This has the form:

$$f_c = \frac{k_c}{2\pi L_{mq}}\sqrt{R_s R_Q\left(1 - \frac{L_q}{L_d}\right)} , \tag{32}$$

where:

$$k_c = \sqrt{mn} , \tag{33}$$

$$m = 1 + \frac{L_q}{L_d} + \frac{T_{d0} + T_{q0}}{T_{da}} \tag{34}$$

$$n = 1 - \frac{\left(1 + \dfrac{L_q}{L_d}\right)^2 T_{q0} + T_d + \dfrac{L_q}{L_d} T_{d0} + 2T_{qa}}{T_{d0} + T_{q0} + T_{da} + T_{qa}} \tag{35}$$

and

$$T_{da} = \frac{L_d}{R_s} ; \tag{36}$$

respectively:

$$T_{q_a} = \frac{L_q}{R_s} . \tag{37}$$

V. Conclusions

By applying the Routh-Hurwitz criterion there have been obtained the conclusions detailed forwards.

The reluctance synchronous motors may have an unstable operation when they are supplied by frequencies lower than 50 Hz. This instability means rotor oscillations around the synchronism speed; these oscillations disturb the mechanism operation and they even could lead to the machine de-synchronization.

This phenomenon, encountered especially at the machines having high rotor asymmetry, is not caused by resistant torque shocks, by imperfections of the system or by supply voltage variations, but it is a phenomenon specific to the reluctance synchronous motor; sometimes it can occur even at the rated frequency.

Starting from the observation that the RSM have a stable operation in a range which is larger when the major critical frequency is close to zero, the following conclusions emerge from the analysis of the above relations:

- in order to obtain a large stability range the R_s value must be as low as possible;
- from the same reason, the value of L_{mq} must be as high as possible;
- the stability is higher when the ratio $\frac{L_q}{L_d}$ is closer to 1 (inconvenient situation from the synchronizing torque point of view).

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