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Computing of Phase Shift for Coupled Oscillators through a Two-Port Passive Network

Mihai Iordache*, Lucia Dumitriu*, Dragos Niculae*, Lucian Mandache**, Jean-Marie Paillot***, David Cordeau***

* University "Politehnica" of Bucharest, Romania, mihai.iordache@upb.ro, lucia.dumitriu@upb.ro, dragos.niculae@upb.ro

** University of Craiova, Romania, lmandache@elth.ucv.ro

*** University of Poitiers, Angoulême, France, jean.marie.paillot@univ-poitiers.fr, david.cordeau@univ-poitiers.fr

***Abstract*— Voltage controlled microwave oscillators are used in digital communication systems. Pairs of coupled and synchronized voltage controlled oscillators are used to control the phase in microwave antenna arrays as an alternative to other methods. A particular phase shift can be obtained by choosing the free-running frequencies of the oscillators in the array. The aim of this paper is to analyze such a pair of coupled oscillators by targeting the phase shift between their output voltages, the coupling circuit being a passive two-port passive network. For this purpose, we use our computer-aided analysis tool ANCSYAP (ANalog Circuit Symbolic Analysis Program) capable to build the symbolic expressions of desired network functions or global parameters (as input/output/transfer impedances and admittances, fundamental or hybrid parameters) and to find the resonant frequencies of any two-port network as function of global parameters. The analysis is performed both in time domain and in frequency domain and in this way we can compute the shift phase analytic expression. To prove the procedure, we include two relevant examples.**

I. Introduction

A very good directivity of microwave antennas for digital communication systems is obtained with antenna arrays with phase control. An alternative method to accomplish the phase control is based on the microwave voltage controlled oscillators (VCO) [1-8, 11-13, 17-21]. For such an oscillator, the level of an input voltage controls the frequency of the output.

The radiation pattern of the antenna array is steered in a particular direction by controlling the phase gradient between the signals applied to adjacent elements of the array. The required phase shift between two adjacent elements can be obtained by detuning the free-running frequencies of the outermost oscillators in the array [2, 3, 7, 11-13]. In [4-8, 18-20] it is shown that the resulting interstage phase shift does not dependent on the number of oscillators in the array.

We consider in this paper a pair of Van der Pol oscillators coupled through a two-port passive linear network, used in synchronized antenna array applications (fig. 1). We performed a detailed computer-aided analysis both in time domain and in frequency domain to find the phase shift between the output voltages of the oscillators, as main characteristic imposed by the application requirements. For this purpose, we use our computer-aided analysis tool capable to build the symbolic expressions of desired network functions or global parameters (as input/output/transfer impedances and admittances, fundamental or hybrid parameters) and to find the resonant frequencies of any two-port network as function of global parameters [10-12, 17-20].

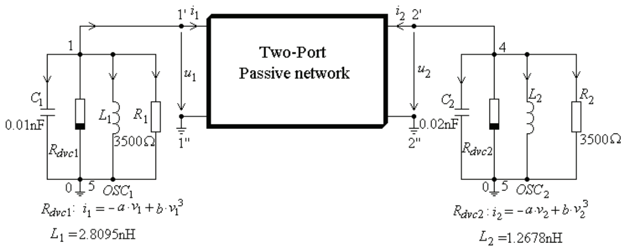

Fig.. 1. A pair of Van der Pol oscillators coupled through a two-port passive linear network.

The active part of each Van der Pol oscillator can be modeled as a voltage controlled resistor (fig. 1) [1, 3, 6-9]. This latter can be modeled through two linear resistors with negative slopes. In this manner, the model circuit becomes linear and it can be analyzed using the complex representation [3, 6-12].

The nonlinear characteristics of the two voltage-controlled nonlinear resistors are expressed as:

$$\begin{aligned} i_1 &= -av_1 + bv_1^3 \\ i_2 &= -av_2 + bv_2^3 \end{aligned} \tag{1}$$

where a is the negative conductance necessary to start the oscillation and b a parameter used to model the saturation phenomenon.

Assuming sinusoidal voltages of the form $v(t) = A \cdot \sin(\omega_0 t)$, the currents of the nonlinear resistors become:

$$i(t) = \left(-a + \frac{3}{4}bA^2\right)A\sin(\omega_0 t) - b\frac{A^3}{4}\sin(3\omega_0 t) \tag{2}$$

Therefore, by neglecting the third harmonic, the nonlinear resistors can be modeled by linear resistors with the conductances:

$$G_{01} = -a + \frac{3b}{4}A_1^2; \quad G_{02} = -a + \frac{3b}{4}A_2^2 \tag{3}$$

where A_1 and A_2 are magnitudes of the voltages v_1 and v_2.

The complex admittances, in the sinusoidal behavior, corresponding to the two oscillators have the following expression:

$$\underline{Y}_1 = j\omega C_1 - a + \frac{3b}{4}A_1^2 + \frac{1}{j\omega L_1} + \frac{1}{R_1};$$
$$\underline{Y}_2 = j\omega C_2 - a + \frac{3b}{4}A_2^2 + \frac{1}{j\omega L_2} + \frac{1}{R_2}. \quad (4)$$

The equations in the admittances, in the sinusoidal behavior, of the coupling two-port passive linear network have the expression:

$$\underline{I}_1 = \underline{Y}_{11}\underline{U}_1 + \underline{Y}_{12}\underline{U}_2$$
$$\underline{I}_2 = \underline{Y}_{21}\underline{U}_1 + \underline{Y}_{22}\underline{U}_2 \quad (5)$$

Some examples of the pair of oscillators coupled through a two-port passive linear circuit are presented in section II more detail. In this way we prove how the coupling circuit parameters have a deeply influence on the phase shift between the output voltages of the two Van der Pol oscillators. We extend our study on the case where the coupling circuit is replaced by two-port passive complex networks.

II. Two Coupled Van Der Pol Oscillators Through a Two-Port Passive Nonlinear Network

The base of the oscillator analysis is represented by VCO's that have different free-running frequencies and are able to lock at a common frequency to coupling circuits, [1, 2, 4, 8, 17 - 20]. Two oscillators coupled by a resonant network can be synchronized at the same frequency [7, 19, 20]. But the synchronization is deeply dependent on the coupling network, [1-8, 11-13]. Coupled microwave oscillators have been modeled as simple Van der Pol oscillators [1, 3, 6, 7, 15-21]. This model provides satisfactory results for many applications. Also the simplicity of the equations is very helpful.

We consider, as the first example, two oscillators coupled through a *RLCM* network, shown in fig. 2.

The topologies of both oscillators are identical, but the parameters of the circuit elements are chosen to obtain different free-running frequencies. We intend to find the synchronization frequency which depends on the resonant frequencies of oscillators and of the coupling circuit.

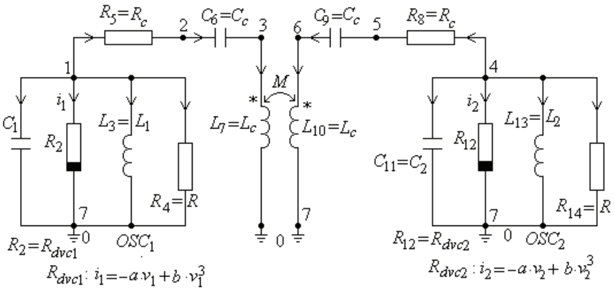

Fig. 2. Two parallel resonant circuits coupled through a *RLCM* network.

We consider the following numeric values of the circuit parameters in Fig. 1: a = 0.008 S, and b = 0.00071 S/V^2 , C_1 = 0.01 nF, C_2 = 0.02 nF, C_c = 2.53 pF, L_1 = 2.8095 nH, L_2 = 1.2678 nH, L_c = 20.0786 nH, M = 15.05895 nH, R = 3500 Ω and R_c = 113.5 Ω. The resonant frequencies for the two oscillators are:

$$f_{01} = \frac{1}{2\pi\sqrt{C_1 L_1}} = 950\,\text{MHz},$$
$$f_{02} = \frac{1}{2\pi\sqrt{C_2 L_2}} = 1000\,\text{MHz} \quad (6)$$

The complex admittances, in the sinusoidal behavior, corresponding to the two oscillators and to the coupling *RLCM* circuit have the following expression:

$$\underline{Y}_1 = j\omega C_1 - a + \frac{3b}{4}A_1^2 + \frac{1}{j\omega L_1} + \frac{1}{R_1} = \frac{0.2\cdot 10^{-10}\left(\omega^2 j - 0.56\cdot 10^9\omega - 0.36\cdot 10^{20} j\right)}{\omega};$$

$$\underline{Y}_2 = j\omega C_2 - a + \frac{3b}{4}A_2^2 + \frac{1}{j\omega L_2} + \frac{1}{R_2} = \frac{0.2\cdot 10^{-10}\left(\omega^2 j - 0.14\cdot 10^9\omega - 0.38\cdot 10^{20} j\right)}{\omega};$$

$$Y11\_c := \frac{-0.3\ 10^{-11}\ I\,(-0.3\ 10^{-9}\ I\,\omega + 0.6\ 10^{-19}\ \omega^2 - 1.)\ \omega}{1. - 0.4\ 10^{-28}\ I\,\omega^3 - 0.2\ 10^{-18}\ \omega^2 + 0.6\ 10^{-9}\ I\,\omega}$$

$$Y12\_c := \frac{0.2\ 10^{-30}\ I\,\omega^3}{1. - 0.4\ 10^{-28}\ I\,\omega^3 - 0.2\ 10^{-18}\ \omega^2 + 0.6\ 10^{-9}\ I\,\omega}$$

$$Y21\_c := \frac{0.2\ 10^{-30}\ I\,\omega^3}{1. - 0.4\ 10^{-28}\ I\,\omega^3 - 0.2\ 10^{-18}\ \omega^2 + 0.6\ 10^{-9}\ I\,\omega}$$

$$Y22\_c := \frac{0.253\ 10^{-11}\ I\,\omega\,(1. - 0.6\ 10^{-19}\ \omega^2 + 0.3\ 10^{-9}\ I\,\omega)}{1. - 0.4\ 10^{-28}\ I\,\omega^3 - 0.2\ 10^{-18}\ \omega^2 + 0.6\ 10^{-9}\ I\,\omega} \quad (7)$$

The resonant frequency of the *RLCM* coupling circuit is:

$$f\_Y11 := [0., 0.23259082\,10^8, 0.30737334\,10^8, 0.61411659\,10^8]$$

$$f\_Y12 := [0., 0., 0., 0.61537578\ \ 10^8, 0.23211489\ \ 10^8]$$

$$f\_Y21 := [0., 0., 0., 0.61537578\ 10^8, 0.23211489\ 10^8]$$

$$f\_Y22 := [0., 0.23259082\ 10^8, 0.30737334\ 10^8, 0.61411659\ 10^8] \quad (8)$$

We can denote that the values of the frequencies corresponding to the complex transfer admittances $\underline{Y}_{11}$ and $\underline{Y}_{22}$ ($\underline{Y}_{12}$ and $\underline{Y}_{21}$) coincide, because the two-port passive linear network is symmetric one.

In fig. 3 the waveforms for the output voltages corresponding to the two oscillators are shown, and in fig. 4 the Fourier characteristic of the two waveforms is presented.

Performing a Spice simulation (or Matlab simulation based on the state equations, [10-18]) we get the magnitudes values A1 = 2.2393 V, A2 = 3.201 V (Fig. 2), and the synchronization frequency fs = 1.0056 GHz for the initial conditions vC1(0) = 2.0 V and vC2(0) = 2.0 V (Fig. 3). According to the assumption presented in Section I, in sinusoidal behavior, the two voltage-controlled nonlinear resistors were substituted by two linear resistors with the conductances given by the expressions (3). Therefore, the circuit in fig. 2 can be analyzed by the complex representation method [10, 11, 16-21].

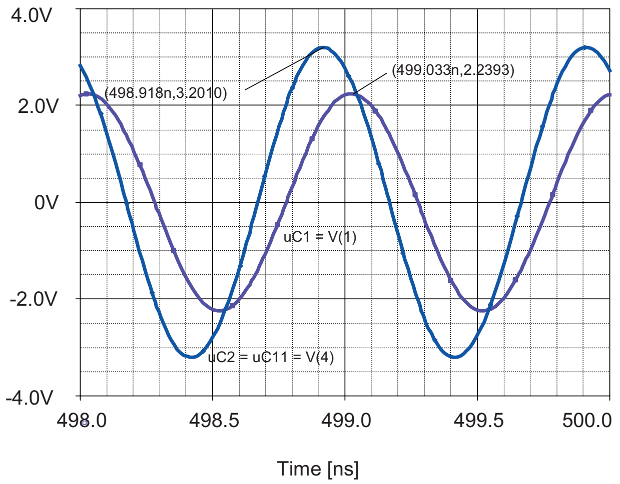

Fig. 3. Waveforms of the oscillator voltages: $v_{C1} = V(1)$ and $v_{C2} = V(4)$.

The nodal equations in complex form for the considered circuit are:

$$\begin{aligned} \underline{Y}_1\underline{V}_1 &= -\underline{Y}_{11}\underline{V}_1 - \underline{Y}_{12}\underline{V}_2 \\ \underline{Y}_2\underline{V}_2 &= -\underline{Y}_{21}\underline{V}_1 - \underline{Y}_{22}\underline{V}_2 \end{aligned} \tag{9}$$

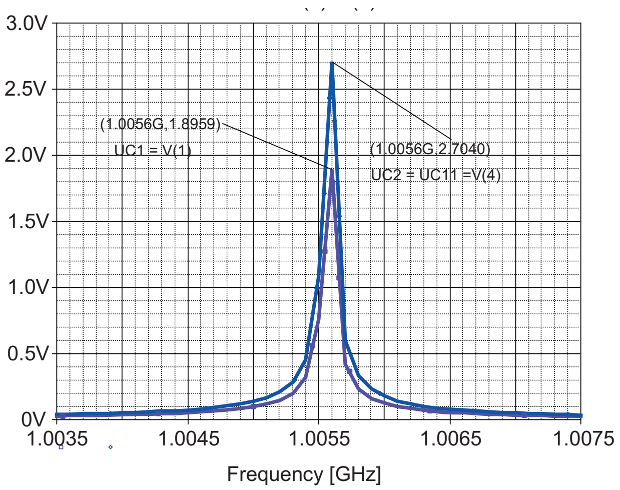

Fig. 4. Determining of the synchronization frequency.

If we denote $\underline{X} = \dfrac{\underline{V}_1}{\underline{V}_2} = \dfrac{A_1}{A_2} e^{j\varphi_{21}}$ and take the numeric values of the circuit parameters specified above, then the equation (9) becomes a system of two nonlinear algebraic equations with the unknowns $\underline{X}$ and ω. Solving these algebraic equations, we obtained the following results:

$$\underline{V}_{os1}/\underline{V}_{os2} = \underline{X}_{c\_analytic} = 0.54536265 - 0.44048094j \Rightarrow$$
$$\begin{cases} \varphi_{21\_analytic} = \alpha_{v_{os1}} - \alpha_{v_{os2}} = \arg\left(\underline{V}_{os1}/\underline{V}_{os2}\right) = -38.947° \\ X_{analytic} = V_{os1}/V_{os2} = 0.70103 \end{cases}$$
$$\omega_{s\_analytic} = 0.631898 \cdot 10^{10} \text{ rad/s}; \; f_{s\_analytic} = 1.0056 \text{ GHz}. \tag{10}$$

In order to compute the phase shift, using a Spice or Matlab simulation, we can use the following relation:

$$\varphi_{21\_Spice} = f_s \cdot \left(t_{os1\_m} - t_{os2\_m}\right) \cdot 360 \tag{11}$$

where f_s - is synchronization frequency (see fig. 4) and t_{os1_m} (t_{os2_m}) represents the time moment when the first (second) oscillator voltage is maximum (see fig. 3). Replacing the numeric values, the result is:

$$\varphi_{21\_Spice} = -39.2787° \tag{12}$$

Taking into account the magnitudes of the output voltages, the ratio of these magnitudes has the value:

$$X_{Spice} = \frac{V_{0s1m}}{V_{os2m}} = \frac{2.2393}{3.201} = 0.70093 \tag{13}$$

We can denote the good agreement between the results obtained by Spice simulation and by our procedure, the error being very small:

$$\begin{aligned} \varepsilon_X &= \frac{\left(X_{analytic} - X_{Spice}\right) \cdot 100}{X_{Spice}} = 0.2098\%; \\ \varepsilon_{f_s} &= \frac{\left(f_{s\_analytic} - f_{s\_Spice}\right) \cdot 100}{f_{s\_Spice}} = 0.06042\%; \\ \varepsilon_{\varphi_{21}} &= \frac{\left(\varphi_{21\_analytic} - \varphi_{21\_Spice}\right) \cdot 100}{\varphi_{21\_Spice}} = -0.8445\%. \end{aligned} \tag{14}$$

We consider, as second example, the circuit represented in fig. 1, when the coupling passive linear circuit has the structure in fig. 5.

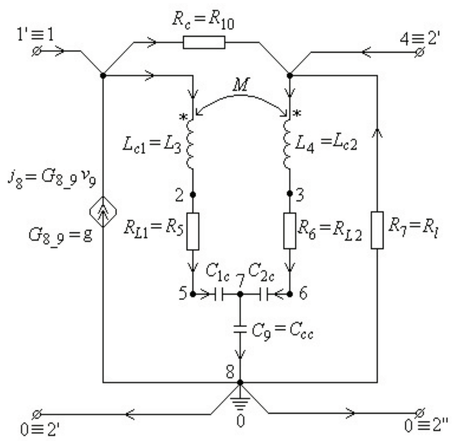

Fig. 5. Two-port passive network structure.

Performing a Spice simulation (or Matlab simulation), for the numeric value parameters: R_c = 200 Ω, $C_{c1} = C_{c2} = C_c$ = 5.0 pF, C_{cc} = 6.5 pF, $L_{c1} = L_{c2} = L_c$ = 10.6 nH, M = 3.71 nH, $R_l = R_{load}$ = 250 Ω, $R_{L1} = R_{L2} = R_L$ = 1.352 Ω, g = 0.0002 S, a = 0.008 S, and b = 0.00071 S/V^2 , we get the magnitudes values A_1 = 1.605 V, A_2 = 2.4449 V (fig. 6), and the synchronization frequency f_s = 1.1522 GHz (fig. 7), with the initial conditions $v_1(0)$ = 2.0 V and $v_2(0)$ = 1.0V.

Because the circuit in fig. 5 has a high complexity, the expressions of the transfer admittances ($\underline{Y}_{11}$, $\underline{Y}_{12}$, $\underline{Y}_{21}$ and $\underline{Y}_{22}$) in full-symbolic form are too complicated ones. The transfer admittances in function of the angular frequency ω have the expressions:

$$Y11\_c := -\frac{0.005(-0.8\,10^{-50}\,\omega^4 + 0.2\,10^{39}\,I\,\omega^3 + 0.8\,10^{30}\,\omega^2 - 0.1\,10^{19}\,I\,\omega - 0.2\,10^{10})}{(-0.4\,10^{-30}\,\omega^2 + 0.3\,10^{-22}\,I\,\omega + 0.2\,10^{10})\,(1. - 0.3\,10^{19}\,\omega^2 + 0.5\,10^{11}\,I\,\omega)}$$

$$Y12\_c := -\frac{0.005\,(0.8\,10^{-50}\,\omega^4 - 0.8\,10^{-40}\,I\,\omega^3 - 0.8\,10^{-30}\,\omega^2 + 0.4\,10^{-20}\,I\,\omega + 0.2\,10^{-10})}{(-0.4\,10^{-30}\,\omega^2 + 0.3\,10^{-22}\,I\,\omega + 0.2\,10^{-10})\,(1. - 0.3\,10^{-19}\,\omega^2 + 0.5\,10^{-11}\,I\,\omega)}$$

$$Y21\_c := \frac{0.00002(-0.2\,10^{-47}\,\omega^4 + 0.2\,10^{-37}\,I\,\omega^3 + 0.1\,10^{-27}\,\omega^2 - 0.8\,10^{-18}\,I\,\omega - 0.3\,10^{-8})}{(-0.4\,10^{-30}\,\omega^2 + 0.3\,10^{-22}\,I\,\omega + 0.2\,10^{-10})\,(1. - 0.3\,10^{-19}\,\omega^2 + 0.5\,10^{-11}\,I\,\omega)}$$

$$Y22\_c := \frac{0.00002(0.4\,10^{-47}\,\omega^4 - 0.5\,10^{-37}\,I\,\omega^3 - 0.3\,10^{-27}\,\omega^2 + 0.2\,10^{-17}\,I\,\omega + 0.6\,10^{-8})}{(-0.4\,10^{-30}\,\omega^2 + 0.3\,10^{-22}\,I\,\omega + 0.2\,10^{-10})\,(1. - 0.3\,10^{-19}\,\omega^2 + 0.5\,10^{-11}\,I\,\omega)} \quad . (15)$$

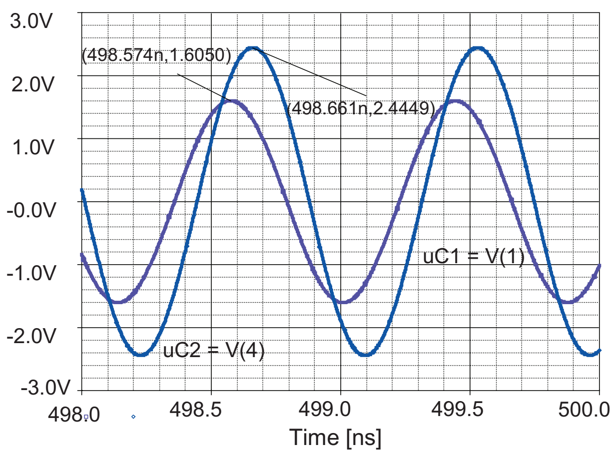

Fig. 6. Out voltage waveforms of each oscillator.

Due the circuit in fig. 7 is a symmetric one, from the relations (15) it comes out that the transfer admittance $\underline{Y}_{12}$ is equal to $\underline{Y}_{21}$.

The current controlled nonlinear resistor currents vs time is shown in fig. 8.

By solving the equations (9) and by Spice or Matlab simulation, see relation (11), the results have the following structure:

$$\begin{aligned} &f_{s\_Spice} = 1.1522\ \text{GHz};\\ &\underline{X}_{analytic} = 0.5312 - 0.38546j;\\ &X_{analytic} = 0.65635;\ X_{Spice} = 0.65647;\\ &\omega_{analytic} = 72.3445\cdot 10^8\ rad/s;\\ &\omega_{Spice} = 72.358\cdot 10^8\ \text{rad/s};\\ &\varphi_{21\_analytic} = -35.9832°;\ \varphi_{21\_Spice} = -36.0869°. \end{aligned} \qquad (16)$$

From the relations (16) we can denote that the results obtained by Spice simulation are very close to the ones obtained by our procedure.

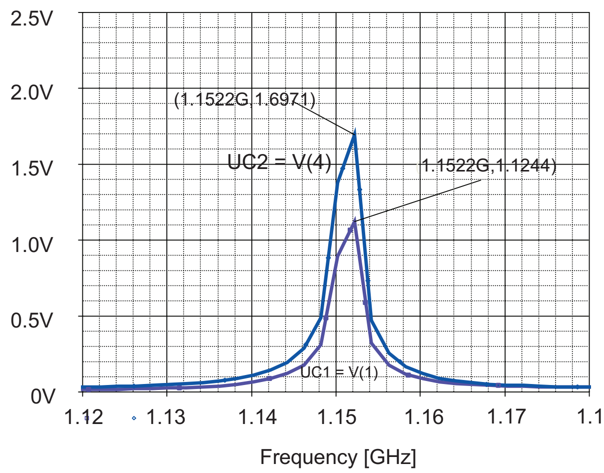

Fig. 7. Synchronization frequency obtained by a Fourier analysis.

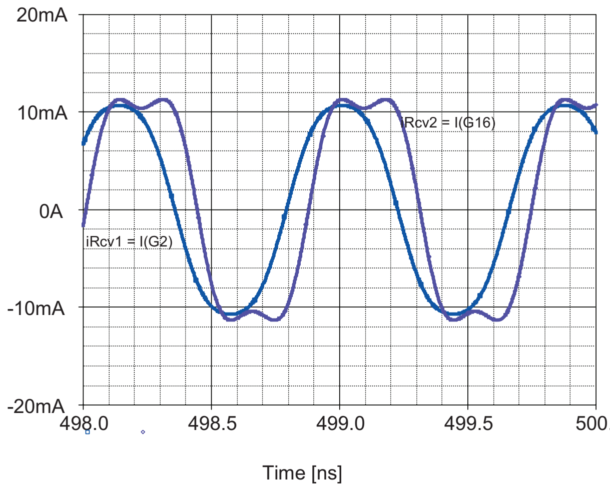

Fig. 8. Nonlinear current controlled resistor currents iRcv1 and iRcv2 vs time.

III. Conclusion

The phase shift between output voltages of each oscillator and the way how oscillators work are very important in orienting the radiation pattern, in a phased antenna array, in a certain direction. Researches are performed so that a particular phase shift can be obtained by choosing the free-running frequencies of the oscillators in the array. In this paper different working behaviors of analysis were applied in order to compare the results obtained and also, for a better understanding of the influence the parameters have in the oscillators' synchronization. The main limit we encounter is that the analysis is made around the synchronization frequency. Thus, there is a risk of malfunction outside this region.

Using our software tool ANCSYAP, we performed the symbolic analysis of an antenna array in order to compute in symbolic, numeric-symbolic or numeric form the coupling circuit parameters. This software has been enhanced with dedicated routines for generating the admittances, impedances, *H* hybrid and fundamental parameters of the coupling networks, which are modeled by passive linear two-port circuits.

The symbolic or numeric-symbolic (when it is considered as symbols only a part of the parameters associated to the passive couple circuits) expressions of the coupling circuit parameters are useful in writing the dynamic equations of an array of coupled oscillators for the automatic design of these devices.

The phase shift between the output voltages of the coupled oscillators can also be computed analytically, see relation (13), but, in this case, we must do very many simplifications because the equations describing the oscillators are nonlinear.

REFERENCES

[1] Mihaela Ionita, M. Iordache**,** Lucia Dumitriu*,* D. Cordeau, J. M., Paillot, "Generation of the Coupling Circuit Parameters for the Coupled Oscillators Used in Antenna Arrays", Proceedings of XIIth International Conference on Synthesis, Modeling and Simulation Mehods and Applications to Circuit Design (SMACD 2012), Seville, Spain, pp. 237-240, September 2012.

[2] J. Lynch and R. York, "Synchronization of oscillators coupled through narrowband networks", IEEE Transactions on Microwave Theory and Techniques, vol. 49, No.2, pp. 237-249, February 2001.

[3] K. Kurokawa, "Injection locking of microwave solid-state oscillators", Proceedings of the IEEE, vol. 61, No. 10, pp. 1386-1410, October 1973.

[4] N. Tohme, J.M. Paillot, D. Cordeau and P. Coirault, "Analysis of the frequency locking region of coupled oscillators applied to 1-D antenna arrays", 38th European European Microwave Conference, Amsterdam, Oct. 2008, pp. 353-361.

[5] P. Liao and R. York, "Beam scanning with coupled vcos," IEEE Proc.Antennas Propagation Soc. Int. Symp., vol. 2, pp. 836–839, June 1994.

[6] R. York and T. Itoh, "Injection and phase-locking techniques for beam control," IEEE Trans. Microwave Theory Tech., vol. 46, pp. 1920–1929,Nov.1998.

[7] F. Hutu, D. Cordeau and J.M. Paillot, "A 2.4 GHz active phase shifter architecture for smart antennas control", IEEE MTT Microwave Symposium, Boston, USA, pp. 821-24, June 2009.

[8] Nidaa Y. Tohme, *Etudes comportementales et réalisation d'un réseau d'oscillateurs couplés et de déphaseurs actifs appliqués à la command d'un réseau d'antennes linéaire*, Thèse de Doctorat, Poitiers, 2006.

[9] R. York, "Nonlinear analysis of phase relationships in quasi-optical oscillator arrays," IEEE Trans. on Microwave Theory and Techniques, vol. 41, Oct. 1993, pp. 1799–1809.

[10] Lucia Dumitriu, M. Iordache*,* I. Matei, "Symbolic tool for analysis and design of analog linear circuits", Revue Roum. Sci. Techn.- Électrotechn. et Énerg., Tome 45, no. 2, Bucarest, 2000, pp. 253-265.

[11] M. Iordache, Lucia Dumitriu, I. Matei, *ANCSYAP - ANalog Circuit SYmbolic Analysis Program*, User Guide, Library of Electrical Department, Politehnica University of Bucharest, Bucharest, 2010.

[12] Mihaela Ionita, Contribution of the study of synchronized differential oscillators used control antenna arrays, PhD dissertation, Dept. Elec. Eng. Politehnica Univ. of Bucharest, March, 2012.

[13] T. Health, "Beam Steering of Nonlinear Oscillator Arrays through Manipulation of Coupling Phases", IEEE Trans. Antennas and Propag., N0 52 (7), 2004, pp. 1833-1842.

[14] J. Lynch, Analysis and design of systems of coupled microwave oscillators, PhD desertation, Dept. Elect. Comput. Eng. Univ. California, Santa Barbara, CA, May 1995.

[15] Oana Sîrbu, M. Iordache, L. Mandache, Lucia Dumitriu, „Novel Approach for Electromagnetic Actuators Analysis in Transient Behavior", Journal: Advances in Electrical and Computer Engineering, Vol: 12, Issue: 1, Year: 2012, ISSN: 1582-7445, e-ISSN: 1844-7600, pp. 103-108.

[16] L. L. Sharfe, T. R. Behreus,"A First Course in Electrical and Computer Engineering with MATLAB – Programs and Experiments", Addison-Wesley Publishing Company Reading, Massachusetts, 1990.

[17] Iulia Dumitrescu, Contributions to the autonomous analog circuit analysis, PhD dissertation, Dept. Elec. Eng. Politehnica Univ. of Bucharest, March, 2012.

[18] Mihaela Ionita, D. Cordeau, J. M, Paillot, S. Bachir, M. Iordache, "A CAD Tool for an Array of Differential Oscillators Coupled Through a Broadband Network", *International Journal of RF and Microwave Computer-Aided Engineering,* Vol. 23, Issue 2, pp. 178-187, ISSM: 1096-1099-047X, March 2013

[19] D. Cordeau, Mihaela Izabela Ionita, J. M. Paillot, M. Iordache, "New Formulation of the Equations Describing the Locked States of Two Van der Pol Oscillators Coupled Through a Broadband Network – Application to the Design of Two Differential Coupled VCOS", Journal of Circuits, Systems, and Computers (JCSC), ISSN; 0218 – 1266, Volume 30, Issue: 8 (2012), pp. 1587 – 1605
W. Stutzman and G. Thiele, *Antenna Theory and Design*, New York: John Wiley and Sons, 1981.

[20] Iulia Dumitrescu, S. Bachir, D. Cordeau, J. M. Paillot, M, Iordache, "Modeling an Characterization of Oscillator Circuits by Van der Pol Model Using Parameter Estimation", Journal of Circuits, Systems, and Computers (JCSC), ISSN; 0218 – 1266, Volume 21, No. 5 (August 2012), pp. 1587 – 1605.

[21] M. M. Hassoun and K. S. McCarville, "Symbolic analysis of large-scale networks using a hierarhical signal flowgraph approach", Analog Integrated Circuits and Signal Processing, Vol. 3, pp. 31-42, 1993.